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ESSAYS ON THE HICKS-SRAFFA SUPERMULTIPLIER CONSIDERING AUTONOMOUS EXPORT AND THE LABOR MARKET

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In *memorian* of my
Grandfather, Carlito Araujo

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DE ARAUJO OLIVEIRA, J. G. **Ensaios do Supermultiplicador Hicks-Sraffa Considerando Exportações Autônomas e o Mercado de Trabalho.** 2023.Tese de Doutorado (Doutor), Universidade de Brasília, Brasília, 2023.

RESUMO

A presente tese está dividida em quatro capítulos, sendo cada um deles independentes, mas que possuem objetivos em comum; são eles: analisar como foi formado o Supermultiplicador Sraffiano (SSM) e como este se comporta considerando as exportações como uma proposta alternativa ao consumo autônomo e incluindo o mercado de trabalho. O primeiro capítulo, busca apresentar o contexto histórico da formação do SSM e como têm sido tratado o debate nacional e internacional com respeito ao tema. Para isso, expôs-se que a teoria original foi baseada nos trabalhos de Sraffa e Garegnani; no entanto, só foi formalizada em meados dos anos de 1990. O modelo publicado, na época, propõe que o crescimento econômico deve, necessariamente, considerar que o consumo autônomo dos capitalistas é responsável pela determinação do avanço das economias. Contudo, essa proposta não satisfez a maioria os pensadores críticos do pós-Keynesianismo que, a partir do trabalho de Lavoie em 2015, têm gerado um extenso debate. As argumentações e réplicas visam verificar se, de fato, a teoria é eficiênciente ao considerar este componente autônomo como um garantidor do crescimento sustentável e da convergência da capacidade de utilização para seu nível normal. O segundo capítulo, propõe uma nova abordagem considerando dois diferentes componentes autônomos, as exportações e o consumo, gerando três diferentes casos: (I) quando o crescimento das exportações (g_X) é igual ao crescimento do consumo autônomo (g_Z); (II) $g_X > g_Z$; e (III) $g_X < g_Z$. Através desse novo modelo, analisou-se as condições de estabilidade e a existência de uma bifurcação de Hopf para os três casos; contudo, apenas nos dois últimos é possível garantir a estabilidade e como consequência apresentar a bifurcação. Além disso, é provado através das simulações numéricas a robustez do modelo proposto no referido capítulo e, com o uso da abordagem computacional, pode-se garantir os resultados do ciclo endógeno. O terceiro, busca contribuir com a literatura propondo que a oferta de trabalho, no longo prazo, não necessariamente é perfeitamente elástica, de forma que, em um caso especial, ela pode ser perfeitamente inelástica. Essa hipótese gera uma significante diferença com abordagem tradicional pós-Keynesiana que, primordialmente, não considera que a oferta de trabalho pode ser limitada a valores menores que a plena capacidade de utilização dos fatores no longo prazo. Com isso, pode-se verificar como se dá o comportamento em ambos os lados, da produtividade e da demanda, sob a hipótese de trabalho restrito. Como resultado, essa modificação mostrou que, na forma original, o SSM

não consegue sustentar crescimento sem gerar excesso de demanda e aumentar a escacez da mão-de-obra. O quarto capítulo vêm flexibilizar a hipótese do mercado de trabalho na versão original do modelo, mostrando como a elasticidade da oferta de mão de obra (e assim, não considerando nenhum dos casos extremos acima), afeta a produtividade e a demanda agregada, impondo que a dinâmica da oferta da mão de obra é restritiva. Sendo assim, como pode-se verificar, apesar de independentes, todos os quatro capítulos versam sobre o mesmo tema e possuem o objetivo em comum.
Palavras-chaves: Pós-Keynesiano; Estabilidade; Bifurcação de Hopf; Supermultiplicador Sraffiano.
JEL: E12; C62; B24

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ABSTRACT

The present Thesis is divided into four chapters, each of which are independent, but have common objectives; they are: to analyze how the Sraffian Supermultiplier (SSM) was formed and how it behaves considering exports as an alternative proposal to autonomous consumption and including the labor market. The first chapter seeks to present the historical context of the formation of the SSM and how the national and international debate on the subject has been treated. For this, it was exposed that the original theory was based on the work of Sraffa and Garegnani; however, it was only formalized in the mid-1990s. The model published at the time proposes that economic growth must necessarily consider that the autonomous consumption of rentiers is responsible for determining the progress of economies. However, this proposal did not satisfy all critical thinkers of post-Keynesianism who, from Lavoie's work in 2015, have generated an extensive debate. The arguments and replies aim to verify if, in fact, the theory is efficient in considering this autonomous component as a guarantor of sustainable growth and the convergence of the utilization capacity to its normal level. The second chapter proposes a new approach considering two different autonomous components, exports and consumption, generating three different cases: (I) when the growth of exports (g_X) is equal to the growth of autonomous consumption (g_X); (II) $g_X > g_Z$; and (III) $g_X < g_Z$. Through this new model, the stability conditions and the existence of a Hopf bifurcation were analyzed for the three cases; however, only in the last two is it possible to guarantee stability and demonstrate the bifurcation. Furthermore, the robustness of the model proposed in this chapter is proved through numerical simulations and, with the use of the computational approach, the results of the endogenous cycle can be guaranteed. The third chapter seeks to contribute to the literature by proposing that labor supply, in the long-run, is not necessarily perfectly elastic, so that, in a special case, it may be perfectly inelastic. This hypothesis generates a significant difference with the traditional post-Keynesian approach that, primarily, does not consider that the labor supply can be limited to values lower than the full capacity of using the factors in the long term. With this, it is possible to verify how the behavior occurs on both sides, productivity and demand, under the hypothesis of restricted work. As a result, this modification showed that, in its original form, SSM cannot sustain growth without generating excess demand and increasing labor shortages. The fourth chapter makes the labor market hypothesis in the original version

of the model more flexible, showing how the elasticity of labor supply (and thus, not considering any of the extreme cases above), affects productivity and aggregate demand, imposing whatever dynamics of labor supply is restrictive to the model. Thus, as can be seen, despite being independent, all four chapters deal with the same theme and have a common objective.
Keywords: Post-Keynesian; Stability; Hopf-Bifurcation; Sraffian Supermultiplier.

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Notations

L^* Feasible quantity of labor

Y^D Effective demand

Y^S Productivity of the supply side

Y_p Income/productivity in full employment

e_L Wage elasticity of labor demand

g_{Y^D} Growth rate of the aggregate demand

g_{Y_p} Growth rate of the potential productivity

g_A Growth rate of the total autonomous component

g_K Growth rate of the capital

g_L Labor growth rate

g_W Growth rate of wages

g_X Growth rate of exports

g_Y Growth rate of the income

g_Z Growth rate of autonomous consumption

h Investment accelerator

A Total autonomous component

C Total consumption

I Total Investment

K Capital stock

L Labor

M Total imports

W Gross wages

X Total exports

Y Total income

Z Autonomous consumption

c Marginal propensity to consume

e Labor-capital ratio

g Growth rate of the economy

l labor-output ratio

m Marginal propensity to import

n Population growth rate

q Tobin's q

u Capacity utilization

v capital-output ratio

w per capita wage

α Relevance of Z and X to the model

γ Exogenous investment variable

μ Normal capacity utilization

ω Wage-share

GENERAL INTRODUCTION

This PhD Thesis is divided into four distinct but related chapters. Each one has individual objectives intending to contribute to the Growth Theory based on the Hicks-Sraffa Supermultiplier (hereafter SSM). In this vein, here, we cote the purpose of each chapter, to design the evolution of the model and the relation between them.

The first presents a historical overview and the technical definition of the autonomous component, arguing if the original definition given by Serrano (1995a) is the correct one. His definition, in p. 1, says that: the component came from the rentiers, does not generate capacity utilization and is income free. Authors like Skott, Oreiro, Punzo and others did not agree with this and argued that the model may deepen the Harrodian instability[1]. However, members some Departments of Economics in Brazil[2], confront their arguments and expanded the model including monetary policies, international trade, government activities and so on.

Based on this debate, we explore how the SSM was constructed and how the international and national dispute has been taken. For such analysis, we delivered a historical panorama of the framework and its critiques. Considering the results of our investigation, we verified the evolution of the discussion, especially focusing on the theoretical and empirical shreds of evidence presented by each research group. At the end of the chapter, we discuss the efficiency and the fragility of the SSM. The main contribution of this revision is to ensure a solid theoretical basis for the development of our new approach including international trade and labor market.

The second seeks to introduce the autonomous export to the model and analyses its behavioral dynamics. Thus, we drawled a panorama showing the importance of the exports and why this is a possible autonomous alternative component. After that, it developed an extension of the SSM considering a simple Open Economy without government activities and proved the stability condition of our new framework using the Routh-Hurwitz criteria. Our effort guarantees the relevance and efficiency of the automaticity of exports instead of autonomous consumption (hereafter AC). Using the same criteria, we proved the existence of a bifurcation to each designed case. Thus, we show a numerical simulation of the model using the R Software and applying the deSolve technique.

[1] See Skott (2017); Oreiro, Silva and Santos (2019); Skott, Oreiro and Santos (2021); and others.
[2] See Lavoie (2015); Lavoie (2019); Serrano and Freiras (2017); Serrano, Freitas and Bhering (2019); and others.

Recently, Dvoskin and Landau (2022) and Oreiro and Santos (2021) indicated that the SSM could have a limited cycle. These authors proved the existence of the bifurcation, which is a necessary and sufficient condition to demonstrate the cycle. This chapter differs from their papers. We proved mathematically the Hopf[3] criteria and we expand the work by including the computational analysis. Besides, our framework did not disprove or criticize Serrano's theory but reinforced the view that the existence of an autonomous component is essential to the model and does converge capacity utilization to its normal level. The technique used here advances the literature by proving, for the first time, the essential nature of the model[4].

The third chapter discusses the potential inconsistency in the treatment of the supply side under the Supermultiplier approach. More specifically, it shows that, in the case where labor is the restricting factor of production, the accelerator mechanism[5] does not provide a satisfactory framework with which to analyze investment behavior, as it drives the economy towards over-accumulation of capital – a form of dynamic inefficiency. Although countervailing effects are considered, including exogenous labor-augmenting technological progress, they are not sufficient to fully offset tendencies towards excessive capital accumulation. The chapter also stresses that long-period output will not be purely demand-driven in a Supermultiplier-type model when labor is the binding constraint under a Leontief technology. Instead, it will be determined through the interplay of aggregate supply and demand. The above findings imply that the usual results of Supermultiplier-type models are highly dependent on the conventional assumption that capital is the restricting factor.

Then, we demonstrated how both demand and productivity behave when we consider the labor supply inelastic, which is a special case, but the post-Keynesian researchers seem to avoid the theme. The fact is, if we consider the labor supply restricted, the framework tends to increase labor shortage and create excess demand. This issue became more relevant during the COVID-19 pandemic[6], which expand the labor shortage around the world. In this kind of model, it is interesting that did not matter if the labor is restricted or not, the analysis

[3] The Hopf bifurcation demonstrates that we have a singular point that create an unstable environment into the model. For more information, we recommend Gandolfo (1996) Part III, CH 3.

[4] Most of the critiques around the SSM is that the model guarantee the Harodian Instability. This chapter reforces the stability condition of the model, but also ensure a possibility to converge to an unstable path, which differs from all the previous literature.

[5] This principle was also proposed by Aftalion and John Clark in 1913, but their principle reinforces the marginalist theory and is related to the Growth theory proposed by Samuelson. For more information, we recommend Simonsen (1988).

[6] For more information, please, read the following International Monetary Fund Discussion paper: Tudela, Clymo and Munro (2022) and Duval, Oikonomou and Tavares (2022).

seems to continue to work without being affected by the work-force. This chapter opens the possibility to continue the research by developing a model which includes the labor activity in the assumptions. In central, we contributed here by expressing a new paradigm and opening a new question: Is the investment function presented by Serrano correct?

In the fourth, we demonstrate, graphically, how the non- and perfect elasticity is inefficient even in a long-run perspective, like the Hicks-Sraffa Supermultiplier. In this vein, we develop a model using this approach and considering a Leontieff Productivity type of function, but considering the influence of the value of the elasticity between 0 and infinity, we did not assume any of these extreme points. The observed results guarantee the importance of the elasticity to the model, and also the average wage growth rate. This result confronts to all the post-Keynesian argument about the accommodation of the labor supply because of they assume that the framework is perfectly elastic. Therefore, we also proves that the model cannot be perfectly inelastic like in the neo-Classical vision. Our model stands in a more generalize, and we believe realistic, assumption that we have to assume some level of the elasticity and also look to all other exogenous variables of the model that affect the results.

Our contributions express new insights (but important ones) into the SSM and we presented in this Thesis the limitations of the approach. However, these limits, especially the ones which include the labor-market analysis, seem to involve all the post-Keynesian and neo-classical perspectives, since both theories assume extreme points in their long-period models. In the modern world, considering the facilities of immigration and home office jobs, the possibility to consider a rigid limitation is unreasonable and generates a huge fragility to the frameworks. After this Thesis, we intend to continue to explore this issue, developing a more generalized model expressing the concerns about this proposal from all post-Keynesian perspectives.

CHAPTER 1 - RECENT CONTROVERSIES OF THE SRAFFIAN SUPERMULTIPLIER: THE INTERNATIONAL AND BRAZILIAN DEBATE[7]

The present chapter provides a detailed investigation of how the Sraffian Supermultiplier (hereafter SSM) was developed, and the critiques around the theme. The theory initially presented by Serrano in 1995 and linked to the neo-Kaleckian approach by Lavoie (2015), was essentially based on the works of Sraffa and Garegnani. According to the last mentioned paper, the model is supported by autonomous consumption delivered from the rentiers side, which drives capacity utilization to its normal level. Therefore, authors such as Skott, Oreiro, and others, pointed out that this assumption does not fit the literature. In this vein, our investigation works on the historical development of the theory showing the evolution of the Supermultiplier in an international and Brazilian debate. This chapter is central to this entire Thesis, since, here, we delimitate the contributions of this new approach. However, we also pointed out the fragilities of the model, raising the main questions which will be answered in this Thesis.

1. Introduction

Here we dealt with two objectives, the first one is to present the development of the SSM, which started with Serrano's (1995a and 1995b). The author was inspired by the works of Sraffa (1960) and Garegnani (1962), their approaches look at both the demand and supply sides of the economy. The second one is to present the international and Brazilian debate around the theme. It is important to reach out to the importance and presence of Latin American researchers in the debate. On one side, we have a Brazilian working group formed by Serrano, Summa, Bhering, and others which are responsible to disseminate the idea of this framework. On the other side, Oreiro, Dávila-Fernandez, Santos and other Brazilian researchers who disagree with the long-term solution presented by the model, especially the ones which defend the New Developmentalism ideas.

[7] I would like to thank to all the participants of the "UnB Webnar" and the "*IX Encontro Científico do PPGE da UNESP*" for their helpful coments and suggestions. I am also in debt with Professor Mauro Boianovsky for his orientations. Obviously, they bear no responsibility for points presented here.

Serrano (1995a, 1995b) [8] described the contribution as an alternative result to the growth theory by considering both the demand and supply sides. According to him, the Supermultiplier results are:

> (i) long-period effective demand determines normal productive capacity, and (ii) autonomous component of final demand (those expenditures that are neither financed by contractual wage income nor can create capacity) generate induced consumption via the multiplier and induced (capacity creating) investment throught the accelerator. (SERRANO, 1995a, p. 1)

The last definition shows how the approach behaves and, in part, is closer to the definition presented by Hicks (1950) (see Wood, 2018)[9]. Nonetheless, according to McCombie (1985), the Supermultipliers type of model only works from an Open Economy perspective. This issue was not approached in Serrano's 1995 original article but it is central to the international debate as we can see in Skott (2017).

However, this formulation has some important contributions to the neo-Ricardian[10] theory offering contributions in the long run perspective and capital accumulation. These accomplishments can be linked to the neo-Kaleckians[11] and post-Keynesian line of thinking, as presented by Lavoie (2015). According to the last author, one of the central views tapped by these schools it is that Autonomous Consumption (hereafter AC) guarantees the convergence of the capacity utilization to its normal level, but, it is important to emphasis that, for the neo-Kaleckians, this is not the central concern.

This argument was sustained by De-Juan's (2005) article, which tested the robustness of the model using computational simulation to prove that, in a long-run perspective, the AC does converge the capacity utilization to its normal level, agreeing with the theoretical results presented by Serrano, and reinforcing the Garegnanis' ideas. Notice that De-Juan's

[8] The first appearance of the Sraffian Supermultiplier is in Serrano's Thesis, so called "The Sraffian Multiplier", defended in 1995.

[9] Harrod (1951, p. 267) says: "16. Mr. Hicks's system total activity is related to autonomous plus induced investment by the ordinary multiplier principle in the short run; but in the long run it is related directly to autonomous investment by a 'Supermultiplier'", which agrees with Serrano's designed investment function: $I = hY$.

[10] We are not differentiating the neo-Ricardians and Sraffians, since, in our view, they have important theoretical differences, but in this context, we are only looking to the critique of neoclassical economics, which are quite identical.

[11] The neo-Kaleckian approach started with the works by Rowthorn (1981) and Dutt (1984), by considering that the capacity utilization does not necessarily is in its normal level in the short and medium period. In this vein, they claim that the investment function is defined by the capacity utilization and profit-rate. The post-Kaleckian approach started by the work of Bhaduri and Marglin (1990), defining that the neo-Kaleckian model only could define their investment function when the capacity utilization is in its normal level, and this function is defined by the capacity utilization and the profit-share. A synthesis of both theories can be found in Hein (2014), Chapter 6.

simulation was defended in 1989, according to his Thesis, but it confirms the SSM mathematical formalization published in 1995 only. The first author mentioned here did not publish the paper until 2005 and updated the work quoting Serrano (1995a) as is presented in his conclusions.

The SSM became interesting to support the neo-Kaleckian model when Lavoie (2015) considered the autonomous demand in his article. This argument does contribute to both Kaleckian and Keynesian perspectives, since consumption is an important component of the demand-driven approach, especially when we consider the rentiers consumption part. For him, “there is no doubt that most post-Keynesians have not paid enough attention to the autonomous or semi-autonomous components of household consumption or household investment” (Lavoie, p. 197, 2015). Based on all these arguments, our intention here is to explain how the autonomous part can or cannot drive the economy, which is an important issue in this chapter.

The AC is defined by Serrano (1995a, p. 1) as “lumping together both rentiers' consumption and the part of 'investment' that does not have any capacity generating effects”, and since it does not create capacity, this component has to be independent of the income. This definition confronts the current one specified by the literature. According to Rose (2018), the definition should be a consumption related to the planned expenses of unemployed workers. In this case, the income automaticity is explained by considering that workers will not earn their income, but consume the one saved to survive. However, the SSM does not consider the workers' savings [see Serrano (1995a), Serrano and Freitas (2017), Lavoie (2015), and others] and assume that the AC could be a non-modelled financial market, government expenditures or exports (in the case of an Open Economy). Rose’s definition is plausible but is by far to be the only one considered, Skott (2019) presents some possible alternatives, including foreign income is defended by some Structuralism researchers. Nikiforos (2018) pointed out that such components hardly can be considered autonomous in a long-run perspective which creates serious problems in an empirical valuation.

In this vein, authors who defend Structuralism and/or the Harrodian Instability, have argued that the consideration of this autonomous component harms the theory and also guarantees the instability of the model. Skott (2019, p. 4) says: “In fact, it may be difficult to think of any consumption component of the rich that is truly autonomous”; for him, all consumption depends on income. Following his point of view, these components, except for the government activities and foreign income, are extremely volatile and hardly could stabilize the economy. Each one of these mechanisms will guarantee the Harrodian Instability [see Skott, Santos and Oreiro (2022)]. The international and Brazilian debate around the SSM is the core of this chapter.

This chapter is divided into five sections: the first one is this introduction, which presents the objectives and justification of the research, by briefly presenting a historical overview which is expanded below. The second one shows the historical evolution of the SSM and the link with the neo-Kaleckian approach. The third part presents the critiques around the AC component and the debate started by Skott (2017). In the fourth part, we present Brazilian theoretical and empirical economic discussion. The final section is to discuss grounded on the historical perspective.

2. The Sraffian Supermultiplier from Garegnani to the neo-Kaleckians

Garegnani (1962) explored in his theoretical approach, the determination of the aggregate product over time and connected it with the theory of accumulation developed by Sraffa (1960). The latter, re-started to look at to surplus concept[12], which re-constructs the theory of distribution and the relative prices based on the classical synthesis. On one hand, he looks to the productivity side, on the other, Garegnani shows that both (aggregate production and accumulation processes) are driven by demand, showing a long-run perspective of the effective demand delivered by Keynes and Kalecki. His central work is to understand the development problem of Italy and can be found in the SVIMEZ 1962 report. He was concerned on how the effective demand can explain the limited growth of the country, proposing that the Sraffian Surplus theory is an alternative to overcome such a problem and these concepts was central to Serrano's Thesis.

It is obvious that the contact with Professor Garegnani was extremely important to develop Serrano's framework, as he presents in his Thesis Acknowledgement section:

> During the long gestation period of this work (its main ideas began to take shape in late 1989) I have been fortunate to have the help and support of several colleagues and friends. I would like to start by thanking Dr. G. C. Harcourt for his friendly and liberal supervision (and his detailed comments on a first draft of this work) and Mr. K. Coutts who was my Faculty advisor at Cambridge. I have benefitted from many stimulating discussions with Prof. P. Garegnani first in Cambridge and then later in Rome, where I also had very useful discussions with Prof. R. Ciccone. (SERRANO, 1995b, Acknowledgement section)

[12] Sraffa shows that the assumption of perfect competition must be disregarded and looked at to the monopolistic digression which produces a surplus in the economy. For this theme, we do recommend Obrinsky (1983) book CH 8 named "Sraffa and the Surplus Revival".

As we can see, the author "drank from the fountain" to obtain his results. The approaches mentioned before the quotation analyze the effective demand by considering the surplus in the long run and criticizing the marginalism theory. Thus, to extend those contributions, Serrano (1995a, 1995b) presented the SSM, and his framework stands from two principals' assumptions. The first one is that the model works in a long-run perspective, where the effective demand determines the standard productivity capacity. The second considers part of the rentiers consumption as autonomous, which does not create capacity, and works with an investment accelerator. Those assumptions were delivered by the author, to solve the instability of the warranted growth rate presented by Harrod (1939)[13], and Domar (1947). Studying both results, Cesararro (2015) indicates that Harrod's problem was solved by the new result, but it was only possible in the case of considering the autonomous component of the theory. For him, p. 169; "Serrano approaches this question (Harrodian problem) by noting the surprising neglect of the autonomous/non-capacity-creating components of aggregate demand in the Post-Keynesian literature".

Nonetheless, it is only possible to understand the development of the Supermultiplier if the definition of Harrod's (1965, p.70) multiplier is clear, thus: "the ratio of increment income (= increment of output) required to make people save an amount equal to the increment of investment is called the Multiplier"[14]. Such a definition agrees with the Keynesian multiplier, where the determination of the income is faced with the savings rate, increasing economic activity. In fact, according to Harrod's book, "Mr Keynes" got it right about this definition. Kaldor (1940, p. 78), says: "that economic activities always tend towards a level where savings and investments are equal", which proves the point presented by Harrod.

This concept leads to the formal concept presented by Hicks (1950) about the Supermultiplier, which, for him, is a multiplier with accelerator characteristics. Such a view was used in the original formulation considering that expected autonomous demand is driven by the marginal propensity to consume and the accelerator came from the investment share. Thereby, the Hicks Supermultiplier, according to McCombie (1985)[15], works better when

[13] Skott (2010) presents the resurgence of the Harodian instability problem into the neo-Kaleckian debate. In his point of view, the model designed by Dutt (1984) does not guarantee a sustainable long-term growth since it is hard to prove that in this kind of model the capacity utilization will converge to its normal level, even considering steady-state values, producing an instability.

[14] From this point of view, what we present is that the model consists of the Kaynesian multiplier, see Keynes (1936, p. 166).

[15] "The most satisfactory basis of the export-led growth theory is the operation of the Hicks' Supermultiplier", McCombie (1985).

an Open Economy is considered. Determining the basis of the export-led growth theory. However, the SSM model only considers a closed economy, ignoring this fact.

The approach designed by Serrano assumes a long-run perspective and avoids the Harrodian Instability. By solving the knife-edge problem, he proves the stability of the model, even considering excess capacity. His argument was structured in the following formulas:

$$Y = \min\left\{\frac{K}{v}; \frac{L}{l}\right\} \tag{1}$$

$$DA = Y = C + I \tag{2}$$

$$C = Z + cY \tag{3}$$

$$I = hY \tag{4}$$

Where (1), (2), (3), and (4) are a Lontieff productivity function, Effective (aggregate) Demand, Aggregate Consumption, and Aggregate Investment, respectively. Z, c, h, K, v, L, l are the Autonomous Component, Marginal Propensity to Consume, Investment Accelerator Mechanism, Capital Stock, Capital-Output ratio, Labor, and Labor-Output ratio, respectively. After some mathematical manipulations, the result is:

$$Y = \frac{Z}{1-c-h} \tag{5}$$

Where $\frac{1}{1-c-h}$ is the Sraffian Supermultiplier[16] and, in this case, the growth of the economy is provided by the autonomous component and the behavior of the accelerator mechanism, determining both product and aggregate demand level.

His result opened doors to extensions and empirical analysis. One important result was presented De-Juan (2005), which tested the model computationally, and proved that AC does converge the capacity utilization to its normal level. All the articles presented above are responsible for the beginning of the SSM approach, therefore, the theme was forgotten in the first 2000' decade. However, it returned as an alternative to the neo-Kaleckian framework by Lavoie (2015). He updated Dutt's model by introducing the autonomous component and also showing the convergence of capacity utilization to its normal level. This contribution was inspired by Serrano and Freitas's 2015 paper.

[16] The HSSM was presented, at first, by Serrano (1995a). However, Bortis (1997) also developed, independently, an identical result. Serrano and Freitas (2017) claimed his pioneering spirit.

The model was initially developed by Rowthorn (1981) and Dutt (1984) to analyze market imperfections (like Monopoly), unemployment of factors, and others. This approach explores such assumptions in the short, medium, and, in some cases, the long run, always considering the level of capacity utilization of less than one [see Amadeo (1986)], which differs from the Kaldorian perspective. The last standpoint assumes full capacity utilization and a perfectly competitive market. Many economists have defended the neo-Kaleckian theory as the most reliable alternative to post-Keynesianism, such as: Dutt, Lavoie, Hein, and Cesaratto, among others. However, the theory has been rigorously criticized, especially, by the Harrodians, Structuralisms, and New Developmentalism, especially by Skott, Oreiro, Punzo, and others.

In defense of the perspective, Lavoie (2015) analyzed the stability of the adjustment of capacity presented by Serrano and Freitas (2015, 2017). The first author developed an alternative response to the criticism[17], by considering the non-capacity creating autonomous expenditures. In such a paper, he also proved that the level of capacity utilization leads to its normal level and the Keynesian stability stands in the short-run. These results invalidate Skott's (2012) initial proposal, guaranteeing that the Harrodian instability does not present to be overly strong. According to Lavoie:

> Some Sraffian economists have long been arguing that the presence of non-capacity-creating autonomous expenditures provides a mechanism that brings back the model to normal rates of capacity utilization, while safeguarding the main Keynesian message and without going back to classical conclusions. (LAVOIE, 2015, Abstract)

His approach expands the canonical investment function delivered by Dutt (1984). Furthermore, Allain (2014, 2019) reinforced the proposal by developing a framework, where the Harrodian thought is stabilized by considering the autonomous expenditure in the Kaleckian model and concluding that his approach is the right candidate as an autonomous aggregate demand component. Improving these results, Serrano, Freitas, and Bhering (2019) show that Instability does not occur in the SSM, since autonomous non-capacity creation impacts the investment in an initial situation of imbalance between the capacity and the demand, affecting both growth rates. All articles since Skott (2012, 2017), and Lavoie (2015) presented here, generated a systematic debate around the model. The discussion has questioned if the new framework can or cannot guarantee the equalization between the

[17] At this time, the post-Kaleckians researchers, like Bhaduri, Skott and others were the major neo-Kaleckian critics.

normal and the actual capacity utilization, and also if the model controls the Harrod Instability [Skott (2019) argued these issues which are replied by Lavoie (2017)].

Until here, the original approach and extensions were only concerned with a closed economy and without government activity; however, the model was also expanded to an Open Economy. Summa (2016) presents a heterodox macroeconomics "New Consensus" intending to respond to mainstream monetary economics. His article introduced inflation to the model and analyzed its monetary policy implications, in a structure considering international trade. He proved that the monetary system affects productivity capacity and functional income distribution.

Nah and Lavoie (2017) extended the framework by introducing the real exchange rate and dividing the export into two parts. The first one is an initial autonomous export, which states stationary, and the second one is an export function concerning the real exchange rate. By these introduced concepts, on page 3 of their paper, they confirm that wage-led growth "can be limited by the sensitivity of the real exchange rate to changes in income distribution". Although, all these models still consider AC as an essential concept to ensure that capacity utilization goes or goes not to its normal level. The central problem, as was presented by Skott (2019), is that the rentiers do not, or hardly will have such a component. The next section is destined to show how the recent controversy has been driven.

3. The Recent Controversies around the Theory

The previous section shows the development of the SSM and its interaction with the neo-Kaleckian framework. As we saw, the necessity to consider the autonomous component of the demand is crucial and determines economic growth in the original approach. However, to the second line of thinking, the AC led the capacity utilization to its normal level, and guaranteed stability conditions in the long run, reinforcing the robustness of the theory and as a response to the post-Kaleckian criticism. However, the new formulation started to be criticized by some heterodox economists, such as Skott, Oreiro, Punzo, Santos, Dávila-Fernàndez, and others. They agree that the autonomous demand in the proposed condition does not sustain the stability of the neo-Kaleckian model, and should not be considered part of rentiers consumption.

Thus, to continue our exposition, we follow the definition of the AC presented by Rose (2018). According to her, this component should only be delivered from workers' consumption when unemployment is considered and not from rentiers. In this vein, unemployed people should consume their planned savings until they find jobs. Serrano's original approach characterized the AC as part of the consumption of the rentiers, as we presented in the section above, which is unusual in the literature. This issue became the central point of the discussion started by Lavoie (2015; 2017) and Skott (2019)[18]. On one side, Lavoie used the SSM as an alternative in defense of the neo-Kaleckian model, on the other, Skott shows that, if the autonomous part is considered, the model tends to be unstable and agrees with the Harrodian Instability defended by the author. Here, we present the criticism around the Supermultiplier and why the debate is historically relevant to construct this new framework on Heterodox Economic Thinking from a post-Keynesian perspective.

The first analysis made by Skott (2019) about the AC, is:

> Capitalists—or more generally, the rich—can draw on their wealth and need not be constrained by current income. Indeed, it may seem reasonable to assume that the rich leave some components of their consumption untouched in bad times. But that is not sufficient to make these components autonomous in the sense of the literature. [...] It may be difficult to think of any consumption component of the rich that is truly autonomous. (SKOTT, 2019, p. 4)

In the sentence above, he seems to agree, in part, with the definition presented by Rose, which says that AC can only be a part of the workers' expenses. Therefore, Serrano's assumption not only indicates that rentiers consumption is in part autonomous, but also does not create capacity (which means that is income free), and this is another issue to be pointed out here. Lavoie seems to agree with this definition since he does not argue with which the concept in his 2016 reply.

Skott (2019) concludes that both theoretical and empirical evidence of the Kaleckian model is weak, even considering the autonomous component. For him, the framework did not correctly specify the investment function and in the short run, did not satisfy the stability condition, resulting in the so-called "knife-edge problem"[19]. However, his arguments were disproved by Lavoie's (2016) reply.

Following the argument presented by Lavoie (2016) and Serrano and Freitas (2017) the justification for the use of the autonomous component in the theory, is that the AC brings

[18] The original work was published as an early draft in 2016, Metroeconomica only organized the issue in 2019.
[19] The "knife-edge problem" sustains that, for the model to be stable, the warranted and natural growth rates must be equal. This is a challenging task, since those variables are determined only by exogenous variables (savings, capital-output and the population growth rates). Such "problem" was solved by Solow (1956) and Kaldor (1956), composing the first two solutions to the Growth Theory. From one side, the neo-classical proposal and the other determine the income distribution theory.

the capacity utilization to its normal level, and guarantees the stability of the model. Serrano and Freitas (2017) linked both theories (SSM and neo-Kaleckian) to provide a more satisfactory closure to the heterodox framework. Thus, to defend the theory, it was shown that Skott tried to present three central points which supposedly invalidate the argumentation of Lavoie's (2015) article, but he did not succeed.

> Summarizing, Skott (2016) makes three claims. First, when calibrating with plausible parameter values, the stabilizing mechanism is unlikely to operate. Second, in the analysis of the simple model, I have omitted the second stationary solution. Third, and this is presented as his most damning condemnation, he seems to imply that the constraints that need to be imposed on the simple and the complex models are in contradiction with each other. (LAVOIE, 2017, p.195)

In his 2017 paper he also did augmented by saying that "Skott takes overly seriously the worth of the little models that we build for exposition and heuristics purposes", justifying each critique made by the second author. For the first claim, Lavoie, in his own words, indicates that the model is a prototype. Second, the author agrees with Skott, that he omitted the second stationary solution, but it is justified since it seems that he wrongly assumed a positive solution. The correct result is null and because of this was omitted. Finally, after all the exposition, the mentioned paper proved, from a graphical solution, the complexity of the model.

However, those initial aspects pointed out by Skott, seem to elucidate some other critiques around the AC, especially considering that such a component will lead the capacity utilization to its normal level. According to Dávila-Fernandez, Oreiro and Punzo (2019)[20]:

> If it is not true that autonomous consumption brings capacity utilization to its normal level (as in fact it is the case), [...] The simple introduction of non-capacity generating autonomous demand is no sufficient condition to solve the inconsistency problem nor to bring capacity utilization to its normal level. (DÁVILA-FERNANDEZ, OREIRO AND PUNZO, 2019, 316)

These arguments reflected the fragility of the model. The authors do agree with the points made by Skott, and they choose to reinforce the debate calling inconsistencies the possible mistakes presented by Lavoie. In response, the last author named his article "Inconsistencies in the note of Dávila-Fernández, Oreiro and Punzo" in 2019 to reply to their critiques. He decomposed their note, showing, in his own words, "All three of their claims are wrong" (Lavoie, 2019, p. 320). These debates became the central discussion for the post-

[20] The article was posted in a volume in 2019, the early draft was published online in October 2017.

Keynesian framework, being the theme of a few volumes of international prestigious journals, such as Metroeconomica volume 70, issue 2, 2019 and Review of Keynesian Economics volume 8, issue 3, 2020.

However, even considering this as an initial debate, both sides have shown that it is hard to defend an AC being part of rentiers consumption; autonomous to the income; and that it does not generate capacity. Besides, the capacity is generated by the necessity to employ labor in the economy, reducing the sub-utilization of the capital, as was presented by Amadeo (1986) when he shows the behavior of capacity utilization in the income distribution and capital accumulation process. Based on the presented above, considering the AC as a private part of the rentiers, which does not generate capacity, is nonsense to the theory of growth for some authors, but it makes sense for others. This debate still occurs and a winner has not been defined yet. In the next section, we will present the Brazilian participation in the debate.

4. The Brazilian Debate

As was presented above, the SSM is the PhD Thesis Serrano's contribution, in 1995. Such principle, as was pointed out by Lavoie, was unfairly neglected for 20 years, but revived by Lavoie's (2015) work. The first critique around the theme in response to Lavoie (2015) was the Dávila-Fernández, Oreiro and Punzo (2019), where the two first authors are Brazilian economists. The central argument presented by the authors is the impossibility of the model to converge capacity utilization to its normal level by using the AC growth rate as support. Therefore, Lavoie responded to the authors by concluding that they did not understand his approach. In this episode, he replied directly to the authors, but the authors did not respond to his arguments.

Furthermore, Serrano, Freitas and Bhering (2019)[21] indirectly replied to the instability concerns presented by Skott (2017, 2019) and Dávila-Fernández, Oreiro and Punzo (2019) papers. The first authors published a paper which concludes that the model cannot be unstable in the case of considering the autonomous component, such as in the SSM. Their conclusion says:

> [...] if there is an autonomous demand component that does not create capacity in the model, as shown by the Sraffian Supermultiplier, demand-led growth at the rate at which this component grows is fundamentally (or statically) stable. (SERRANO, FREITAS AND BHERING, 2019, 280).

[21] The first draft published online was in 2018.

The authors reinforce the arguments presented by Serrano and Freitas (2015, 2017), which inspired Lavoie to link the SSM and the neo-Kaleckian framework. They believe that their approach can be an alternative Closure to the Heterodox Growth Theory.

This discussion opened a debate between the researchers who follow the Sraffian/neo-Ricardian line of thinking and the ones who follow the new Developmentalists school and/or Harrodianism in Brazil. On one side, we have a stronger defence by the academic staff concentrated in some Departments of Economics in Brazil and, on the other, the intellectual attack by isolated economists around Brazil. The debate extended the framework considering an Open Economy, and monetary and fiscal policies. Summa (2016) developed an extension of the SSM to the case of an Open Economy and considered monetary policies, which raised the hypothesis of a "New Consensus"[22]. This approach was published only a few months after Lavoie's 2015 article. His analysis shows the effects of inflation on the growth rate of the economy, considering the SSM, and in his conclusion, he shows that the limits of the demand-led growth path are the choices about their external economic policies made by the countries, where the international inflation has an important effect on the national productivity growth.

His article raised an important question of how inflationary targeting affects the growth rate, allowing the author to develop another work, which was published in the special edition of the Review of Keynesian Economics dedicated to the SSM. This paper named "Stagnation and Unnaturally Low-Interest Rate: a Simple Critique of the Amended New Consensus and the Sraffian Supermultiplier Alternative" written by Serrano, Summa and Moreira (2020) reinforces the arguments presented by Summa (2016) and shows that the inflation and the real exchange rate does affect the growth, which is determined by the AC growth rate.

Such monetary principles are defended by the new Developmentalist's school of thinking. However, they do not agree with Summa and the group about the mechanism which imposes an autonomous component to define growth. In this vein, Oreiro, Silva and Santos (2020), replied to their argument of a "New Consensus" indicating, besides the monetary side, that all the growth theory developed by the SSM has weakness and, instead the theory

[22] "The New Consensus model with inflation targeting is based on the following theoretical structure: (i) the effective output depends on the real interest rate (stimulating investment spending), (ii) the existence of an accelerationist Phillips curve and (iii) a Taylor rule, relating the Monetary Authority response via nominal interest rate to deviations of inflation from its target and output from its potential" (Summa, 2016, p. 310).

defended by the Sraffians, the Developmentalist Macroeconomic Theory is a more plausible alternative to determine the road for the Heterodox approach. In their own words:

> Finally, we argue that the Kaldorian models of growth, the basis of the so-called developmentalist macroeconomics; and stock-flow consistent (SFC) models appear to be much more promising alternatives for the development of heterodox theories of growth and income distribution than the Sraffian Supermultiplier approach. (OREIRO, SILVA AND SANTOS, 2020, p. 529).

Those arguments and critiques were reinforced and increased by the recent paper of Skott, Oreiro and Santos (2021). Their argument is that; the autonomous component is possible in the short-run but definitely is not in the long period [like was shown by Skott (2019)]. Thus, they show mathematical arguments and computational simulation of the model to reinforce that in a long-run perspective, the SSM is unstable, and agrees with the Harrodian Instability. They also introduced government expenditures, one of the possible justifications for the AC, but even for this case, they obtained the same result.

As we can see, the debate is not finished yet. Therefore, all these authors (in favor or not) agree that one possible autonomous component of the model (maybe the most plausible), is the exports. The next section presents the central discussions.

5. Final Remarks

The first section of this chapter, after the introduction, shows the development of the SSM and its interaction with the neo-Kaleckian approach, especially presenting the use of the Hicks (1950) definition of the Supermultiplier. The SSM theory claims that the AC component is indispensable to delivering his solution for the “knife-edge problem”. For him, the autonomous component explains how the product/income behaves and gives the theory a stable result in a long-run period avoiding and correcting the Harrod-Domar problem.

Thus, when Lavoie (2015) introduced the concept to the neo-Kaleckian model, the autonomous component passes to justify how the capacity utilization tends to its normal level and reveals the SSM as a solid alternative to the Heterodox Theory of Growth. Therefore, all the authors who defended this approach did not present an unquestionable definition of the AC and, according to one presented by Rose (2018), the AC would not be derived from rentier consumption, but from the necessity of the worker’s class to survive the unemployment, or as was interpreted here, in the case of retirement, when they would consume their planned savings. In this vein, the AC is a questionable variable by the definition given by Serrano (1995a, 1995b), which supported the controversy attacked by Skott and other authors, when the model is integrated into the neo-Kaleckian framework by Lavoie (2015).

Based on these arguments, in section three, we presented the initial debate around AC, which did not cheer up part of the post-Keynesians. Skott (2016) started the debate around the theme, showing that the investment function presented by Lavoie led the model to the Harrodian Instability, and the autonomous component does not agree with the one defended by the current literature. Besides, to Dávila-Fernández, Oreiro and Punzo (2019), AC also cannot guarantee that capacity utilization will tend to its normal level. The authors also defended that the autonomous component hardly came from the rentier side.

The international debate around this theme raises some important questions, two of them are: How do well define the AC to the theory? Is it possible to consider the AC as was supported by Rose (2018)? However, in this case, the model would consider the differences between class savings and a mechanism of retirement. Another solution is government intervention to guarantee income to the workforce in the case of unemployment. This concept has been the centre of the discussion and divides among the heterodox authors, even the one who composes the same post-Keynesian School.

Thus, section four presents the Brazilian debate about the SSM. On one side, the neo-Ricardian group and defends the SSM as an alternative to the heterodox approach. For them, the SSM answer all the problems involving the growth theory, especially correcting the knife-edge problem. On the other, some important economists, especially those related to the new developmentalism group, constantly attack the theory, intending to prove the model's weakness, by showing that does not correct the knife-edge problem and results in the Harrodian Instability. For them, it is unlikely that the AC came from the rentier side or that this assumption even exists. Besides, these authors confront the idea that the AC would converge the capacity utilization to its normal level. The methodology divergence between the authors presented in this chapter expresses some fragilities of the model. Our intention in this Thesis is to generate some alternative solutions to these issues, especially the ones dealing with international trade and the labor market.

CHAPTER 2 - ENDOGENOUS CYCLES IN AN EXTENDED MODEL WITH AUTONOMOUS CONSUMPTION AND EXPORTS[23]

The central objective of this chapter is to present an autonomous component alternative to the model of endogenous cycles. For us, instead of the rentier consumption, the most plausible AC is the exports. According to Skott (2019) and Nah and Lavoie (2017) the growth can be led by the international trade which reinforces both Thirlwall's Law and Prebisch-Singer export-led results. Here, we demonstrate mathematically and graphically the long-run stabilization and the convergence of the capacity utilization to its normal level, if the new component is considered. After an exhaustively research, we concluded most of other authors has not proved the endogenous cycle to this kind of approach, although they inform in the title that it exist. This is the mains contribution here. Our effort here demonstrate that existence which affect the dynamic of the model and their theoretical interpretations of the convergence of the capacity utilization to its normal level.

1. Introduction

In the previous chapter, the autonomous consumption is an impasse to the theory, however, any authors presented above invalidate the existence of an autonomous demand component. The effective demand were originally developed by Keynes (1936) and Kalecki (1937) in two distinguished works, and they found the same result. According to then, their models only works in a short-run perspective; coting Keynes (1923, p. 65): "in the long-run we are all dead". Therefore, a few years later, economists such as Joan Robinson, Nicholas Kaldor, Garegnani and others tried to improve the effective demand theory by designing a long-period alternative.

This new ideas opened a huge debate between the two Cambridges, England and US. The most important result, named first generation of income distribution theories, is the Cambridge Equation (also known as Kaldor-Robinson Theorem). This theory explains the dynamics of the economic growth and how to sustain the capitalism without oppressing the workers class. The second and third generation observed the field in all three cases, short-, medium- and long-run perspectives, but presenting an investment function led by the capacity utilization [see Rowthorn (1981), Dutt (1984) and Bhaduri and Marglin (1990)]

[23] This Chapter was co-authored by Professors Ricardo Araujo and Helmar Nunes.

The capacity utilization explains the market imperfections, like monopoly or oligopoly, since we have to consider idle capital. The first generation is an especial case when their new approach (neo- or post-Kaleckian Theory[24]) considers perfect competitive market, but their intention is to show a more generalized Theorem which approximate to the real world. One of the most important theorist of the second generation is Marc Lavoie. He realizes the strong criticism of the third generation. Their arguments are related to the weak explanation about how the capacity utilization can converge to its normal level in the long-run presented by the neo-Kaleckians.

Thus, Lavoie (2015) introduced the autonomous consumption to the model and proved the necessary and sufficient conditions to the convergence. His model has been target by the Harrod followers, especially Peter Skott, but the debate is not over. Their works oppened the space for a new framework, known as Sraffian Supermultiplier Model (previously defined as SSM), which is the essence of this Thesis.

Our focus here is on the examination of the SSM by considering the exports as an autonomous component, instead of the Autonomous Consumption (previous defined as AC). In this case, two different propositions are considered: the first one - income growth is equal to the exports growth indicating that the AC tend to disappear in the long-run; the second is an inverse situation and the exports tend to be eliminated in a long-run perspective. One of the main concerns of this kind of model is the stability, to avoid this problem, we proved it by using the Routh-Hurwitz criteria.

Usually the authors only proves the Stability to sustain their argument, and some mathematical formalizations does not include the analysis of extreme points. In some cases, if we force the result of a singular exogenous variable, the model can obliterate the stable condition, such a result is named Hopf-Bifurcation[25]. The Routh-Hurwitz criteria are able to obtain the bifurcation, which indicates a dot where the model starts to be unstable. This point is proved here for each case presented in the previous paragraph. Finally, we demonstrate the robustness of the theory by approaching a numerical simulation for each case using the deSolve path in software R.

This chapter is divided into four parts. The first one is this brief introduction. The second presents the development of the model and the mathematical proof of the stability

[24] The neo-Kaleckians economists are known as the second generation, and the post-Kaleckians are the third generation. We invite the reader to read Hein (2014) Chapter 6 where he defines each Economic School.
[25] To deepen in the mathematical approach we recommend the reading of Gandolfo (1996).

and bifurcation. After this part, we approached the numerical simulation. The last section is the concluding remarks.

2. The Dynamic System of Our Extended Model Considering Autonomous Export

The following model considers international trade, but we are not concerned with government activities, which differs from Oreiro and Santos (2021) analysis. Our intention here is to demonstrate an alternative autonomous component to the model. We agree and cote Thirlwall (2002, p. 83) sentence:

> Exports are the only true component of demand in an economic system, in the sense of demand emanating from outside the system. This is very important to bear in mind. The major part of consumption and investment demand is dependent on the growth of income itself". (THIRLWALL, 2002, p. 83)

Delimitated the limits of the effective demand and showed what we want to analyze here, our approach create a slight change on the initial settings presented by Serrano (1995a), as we can see below:

$$Y = C + I + X - M \tag{1}$$

$$C = cY + Z \tag{2}$$

$$I = hY \tag{3}$$

$$M = mY \tag{4}$$

Equation (1) is the GDP. Equation (2) presents the consumption function which is formed by a marginal propensity to consume (c) times the GDP, and summed to the exogenous AC (Z), this expression is the same one of the original formulation of the SSM. Equation (3) is the total investment; this formula is determined by an investment accelerator (h) multiplied by the total income. Equation (4) is the import and is equal to a monetary marginal propensity multiplied by the GDP. Substituting (2), (3) and (4) in (1), we determine the SSM considering an Open Economy. Hence:

$$Y = \frac{Z+X}{1-c-h+m} \tag{5}$$

The presence of autonomous export in the model raises the GDP, as well as the presence of the exports. These results are obtained when we apply the partial derivative of

GPD with respect to X and m: $\frac{\partial Y}{\partial X} = \frac{1}{1-c-h+m} > 0$ if $1 > c + h + m$; and $\frac{\partial Y}{\partial m} = \frac{-(Z+X)}{(1-c-h+m)^2} < 0$. For simplification, let us take $A = Z + X$, thus:

$$Y = \frac{A}{1-c-h+m} \tag{5.1}$$

From (5.1) we demonstrate the growth rate of the economy. Deriving Y with respect to the time and, then, dividing both sides by the GDP, we have:

$$\hat{Y} = \hat{A} + \frac{\dot{h}}{1-c-h+m} \tag{6}$$

The accent represents the growth rate of each variable and Equation (6) is the growth rate of the income. The economic activities increases when the autonomous component or the investment accelerator is raising, we expect that all other variables are constants. However, in our model, it is possible to determinate the growth rate of A. For simplification, from now on we will assume the growth rate by the letter g subscripted by the variable in question. First, we have to apply the Napierian logarithm in both sides, and derivate with respect to time.

$$g_A = \frac{\dot{Z}}{Z+X} + \frac{\dot{X}}{Z+X} \tag{7}$$

Multiplying the right side by one, we can re-write the equation (7) as:

$$g_A = \frac{\dot{Z}}{Z+X}\left(\frac{Z}{Z}\right) + \frac{\dot{X}}{Z+X}\left(\frac{X}{X}\right) \tag{7.1}$$

From (7.1) we may consider $\alpha = \frac{Z}{Z+X}$, so:

$$g_A = \alpha g_Z + (1-\alpha) g_X \tag{8}$$

Equation (8) is the growth rate of the autonomous component. Where α shows the relevance of Z and X to the model: if α approaches to one, the model tend to exclude the exports; and if it is closer to zero, the autonomous consumption tends disappear.

Furthermore, α is dynamic and for our system we have to determine its equation. Deriving with respect to time and rearranging the formula, we have:

$$\dot{\alpha} = \alpha(1-\alpha)(g_Z - g_X) \tag{9}$$

Equation (9) is the autonomous component dynamic formula and will be central to obtain the stable condition. Substituting (8) in (6), thereby:

$$g_Y = g_A + \frac{\dot{h}}{1-c-h+m} \tag{6.1}$$

For Serrano and Freitas (2017) the accelerator of investment is an endogenous function limited by the difference between the actual capacity utilization (u) and the natural capacity utilization (μ), as we can see in (10):

$$\dot{h} = \gamma(u-\mu)h \tag{10}$$

Substituting (10) in (6.1), we have:

$$g_Y = g_A + \frac{\gamma(u-\mu)\,h}{1-c-h+m} \tag{6.2}$$

The level of the capacity utilization is determined by $u = \frac{Y}{Y_p}$, where Y_p is the potential productivity. Originally, Kaldor (1956) already showed that $g_{Y_p} = g_K$. In this vein, the differential equation of u can be obtained by taking the logarithm in both sides and then applying the time derivative, resulting in: $\hat{u} = g_Y - g_K$. Assuming $g_K = \frac{hu}{v}$ and g_Y from Equation (6.2), we obtain:

$$\dot{u} = \left[g_A + \frac{h\gamma(u-\mu)}{1-c-h+m} - \frac{hu}{v}\right]u \tag{11}$$

Equation (11) shows the rate of change of capacity utilization. From Equations (9), (10), and (11) we determine the steady-state value of u, h, and α. Considering $\dot{h} = 0$ in (10), thus:

$$u^* = \mu \tag{12}$$

The star in each variable represents the steady-state equilibrium. Once we have (12) and considering $\dot{u} = 0$, we obtain:

$$h^* = \frac{g_A v}{\mu} \tag{14}$$

Nonetheless, the steady-state equilibrium value of α, depends if: $g_Z = g_X$; $\alpha^* = 0$; or $\alpha^* = 1$. For each result we will define if the model deals with export- or consumption-led mechanism. From (9), (10) and (11), we have the following system:

$$\begin{cases} \dot{h} = h\gamma(u-\mu) \\ \dot{u} = \left[g_A + \frac{h\gamma(u-\mu)}{1-c-h+m} - \frac{hu}{v}\right]u \\ \dot{\alpha} = \alpha(1-\alpha)(g_Z - g_X) \end{cases} \tag{15}$$

The system (15) determines our Jacobian Matrix properly:

$$\begin{bmatrix} \dot{h} \\ \dot{u} \\ \dot{\alpha} \end{bmatrix} = \begin{bmatrix} \gamma(u-\mu) & h\gamma & 0 \\ \frac{\gamma(u-\mu)(1-c+m)u}{(1-c-h+m)^2} - \frac{u^2}{v} & g_A + 2h\gamma u\sigma - \frac{2hu}{v} & 0 \\ 0 & 0 & (1-2\alpha)(g_Z - g_X) \end{bmatrix} \tag{16}$$

Using the Matrix (16) and considering the steady-state values, the model has three cases, each one considers a different optimal value of α. Thus, it is suggested as possible three propositions with which we will demonstrate the stable condition and bifurcation.

PROPOSITION 1: Considering that our model presets an equalization between aggregate autonomous component, exports and income growth rates, in this case: $g_Z = g_X = g_A$. If that matter, the only possible value for α is 0.5. In this case, considering the Matrix (16); h^*; and u^* we cannot prove the stability or bifurcation conditions of the model.

Proof: Appendix A – a: shows the mathematical manipulation, but in short, we have a line and a column composed only by zeros. Thus the determinant cannot be found which means that the S_3 of the characteristic polynomial is equal to zero.

PROPOSITION 2: Considering the economy impulsed by the autonomous export; i.e. $\alpha^* = 0$. In this case, the economy in a long-run will exclude autonomous consumption and the only truly exogenous component will be the exports like in Thirlwall (2002). Thus, considering the steady-state values of h^* and u^*, our model is stable and the bifurcation exists.

Proof: Appendix A – b: presents the mathematical manipulation which proves the stable condition and Hopf-bifurcation. In this case, we can guarantee the stability if $g_X > g_Z$, and $\gamma > 0$. The bifurcation does exist only and only if:
$\gamma \geq \frac{(g_Z - g_X)(g_X^2 - g_Z + g_X)}{[g_X 2v\sigma(g_Z - g_X) - g_X^2 \mu]}$.

The central issue here is that our framework guarantees an alternative autonomous component that agrees to the current literature. Thirlwall (2002), and also Skott (2019) express their positive opinion about the exogeneity of the exports. Oreiro and Santos (2021), and Dvonski and Landau (2022) had the same conclusion, but they did not presented proof of the bifurcation. For the last two works, AC does not exist, they considered the government expenditures and exports as autonomous. However, it seems that mathematically speaking, does not matter if we consider government expenditures or AC, if we only have 2 similar components in the model we will always have the same result. Here, we proved the existence of an endogenous cycle which will be graphically demonstrated in the numerical simulation.

PROPOSITION 3: Considering the economy impulse by the autonomous consumption; ie. $\alpha^* = 1$. In this case, the economy in a long-run perspective will exclude the exports. This result is much closer to the ones found by Serrano, Freitas, Summa and others. Considering the steady-state values of h^*, and u^* our model is stable and the bifurcation exist.

Proof: Appendix A – c: presents the mathematical manipulation which proves the stable condition and Hopf-bifurcation. In this case, we can guarantee the stability if $g_Z > g_X$, and $\gamma > 0$. The bifurcation does exist only, and only if:

$\gamma \geq \frac{(g_X - g_Z)(g_Z^2 - g_X + g_Z)}{[g_Z 2 v\sigma(g_X - g_Z) - g_Z^2 \mu]}$.

The third case express the inverse of the one presented in Proposition 2. Here the exports will disappear in the long-run perspective and the growth will be driven by the AC. Therefore, the cycle was proved in both propositions and we now have the necessary conditions to approach the Computational Numerical Simulation.

3. Numerical Results, Discussion and Graphical Interpretation

Looking to understand in a detailed form how the model behaves usually the literature uses graphical interpretation. In this section we demonstrate the results graphically. Thus, this part is divided in 2 sub-sections. The first one presents the simulated graphics of 2 and 3 cases of our 3D dynamic system. The second sub-section shows the graphics of each case which proves the Hopf-Bifurcation, by considering the steady-state of each variable and the gamma as the bifurcation variable.

3.1 Simulating the 3D Dynamic System

The present numerical simulation of the 3D Ordinary Differential Equation system was approached using the Software R and the following paths: deSolve; phaseR; scatterplot3d; and latex2exp. The values to our case are hypothetical, therefore, all of them are based in close values for emerging economies, such as the Latin America countries. The parameters used are specified in Table 1:

Table 1 - Parameters

Second Case		**Third Case**	
Parameters	Value	Parameters	Value
g_X	0.06	g_X	0.05
g_Z	0.05	g_Z	0.06
v[26]	2	v	2
μ	0.8	μ	0.8
γ	0.167	γ	0.167

[26] This value is equal to the one presented by Feu (2001) for Brazil and OECD countries. The value of 0.6 in u is based on Oreiro and Santos (2021).

c	0.7	c	0.7
m	0.4	m	0.4

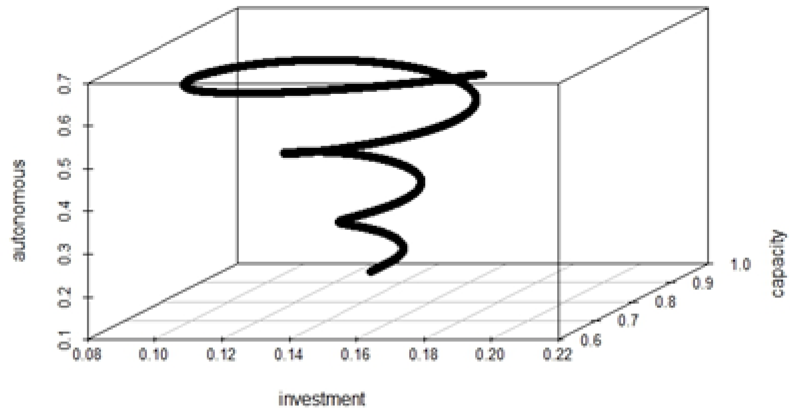
To solve the problem it is necessary to define the steady-state values of each dynamic variable, which is presented in Table 2.

Table 2 - State Values

Variable	Value
h	0.192
u	0.6
α	0.7

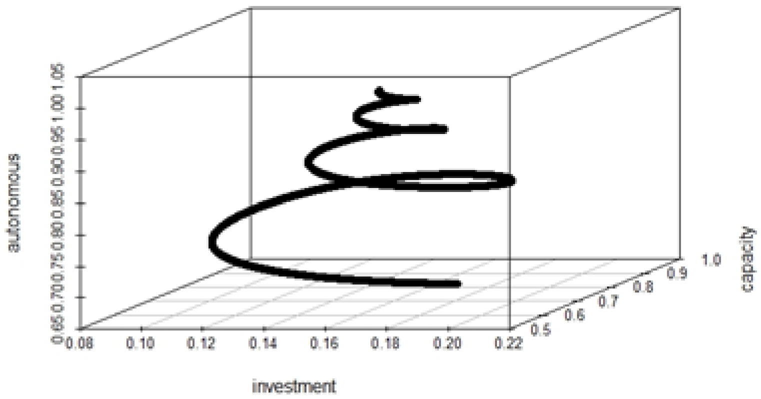
Since we intend to observe the long-run dynamic effects, we choose that the time is 240 units, each unit can be equal to a month resulting in almost 20 years. The first graphic shows the behavior of the investment accelerator, capacity utilization, and autonomous proportion to the case of $g_Z < g_X$, thus:

Figure 1 - Endogenous Cycle: gz<gx



As we can see, as the growth rate of the exports are bigger than the growth rate of the autonomous component, the AC tend to disappear in the long-run. Figure 2 shows the graphic when $g_X < g_z$:

Figure 2 - Endogenous Cycle: gx<gz

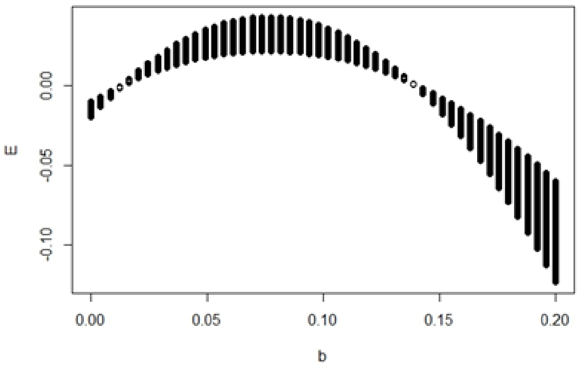


In this case, the AC will disappear with the exports. Therefore, it seems that the economy is converging to be Closed and eliminates the international market in our model. Both results shows that our system is a cycle and it is possible to prove a bifurcation. The next sub-section demonstrate such a result.

3.2 The Hopf-Bifurcation

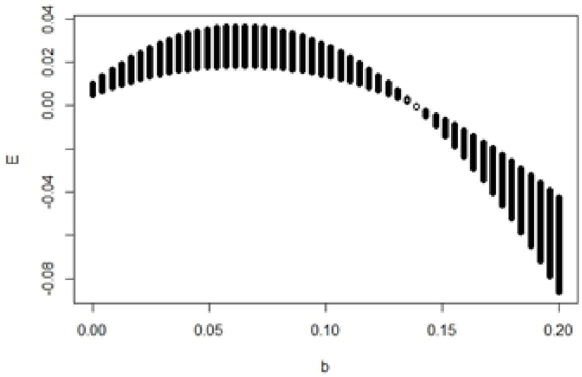
To show the Hopf-Bifurcation, first, we have to assume the steady-state values to each dynamic variable, thus, according to the parameters used above, we have: $E = 0.0000593315702479339$; $h = 0.15$; $u = \mu = 0.8$; and $\alpha = 0.7$. The variable chosen to be the bifurcation one is γ. Figure 3 shows the interval when γ guarantees E positive or negative, thus:

Figure 3 - Limits of the Bifurcation: gz<gx



Here $b = \gamma$ only for simplification on the Software basis. As we can see, between 0.13298 and 0.14020 approximately $E > 0$. Figure 4 shows the bifurcation to the case of $g_A = g_Z$. In this case, the following values considered are:
$E = 0.0000657520661157025$; $h = 0.15$; $u = \mu = 0.8$; and $\alpha = 0.7$.

Figure 4 - Limits of the Bifurcation: gx<gz



In this case, we don't have the minimum value, but if $\gamma \cong 0.1308$ them $E < 0$. So, it is proved that exist a bifurcation to each case. Next section presents the Concluding Remarks.

Concluding Remarks

This chapter approximate the SSM with the mathematical formalization. Here, we have developed the model considering both consumption and exports been autonomous and design the dynamics for three different aspects. The case where $g_A = g_X = g_Z$ is not relevant for our analysis, since the stability condition cannot be guaranteed and probably our approach will obtain the Harrodian Instability result. Therefore, we have two other possibilities: the first one is when the economy is driven by the exports and the second one when the economy is driven by the AC (original Serrano's formulation).

The first case agrees to both Serrano's and Skott's formulation, since both researchers did not avoided the idea that the exports can be an autonomous component. The mathematical proof guarantee the stability. In addition, we also proved the Hopf-Bifurcation to this case and this is the main accomplishment of this chapter. Notice that, after a huge investigation, we did not found any other research proving it. Besides, we also demonstrated the robustness of the model and where the bifurcation exist by using a numerical simulation and graphical interpretation.

The second is also stable and also have a bifurcation, however, the definition of the AC seems to be an impasse between the neo-Ricardians and the Harrodians. Besides the effort made by the authors, it is unreasonable to believe that rentiers can have an AC. In a post-Keynesian perspective, Thirlwall (2002) demonstrate an interesting point when he considers the exports (or the international income) autonomous. Besides, even in a mainstream perspective, Rose (2018) indicates that the only unemployment insurance is truly autonomous. In this vein, it is a hard task to defend the original Serrano's (1995a and

1995b) idea, since our model proves that international trade can be considered an alternative to the autonomous component and will also converge the capacity utilization to its normal level, in part agreeing with Skott's works.

CHAPTER 3 - A THEORETICAL NOTE ON THE SUPERMULTIPLIER APPROACH IN A LABOR-CONSTRAINED ECONOMY[27]

This chapter discusses a potential inconsistency in the treatment of the supply side under the Supermultiplier approach. More specifically, it shows that, in the case where labor is the restricting factor of production, the accelerator mechanism does not provide a satisfactory framework with which to analyze investment behavior, as it drives the economy towards over-accumulation of capital – a form of dynamic inefficiency. Although countervailing effects are considered, including exogenous labor-augmenting technological progress, they are not sufficient to fully offset tendencies towards excessive capital accumulation. The chapter also stresses that long-period output will not be purely demand-driven in a Supermultiplier-type model when labor is the binding constraint under a Leontief technology. Instead, it will be determined through the interplay of aggregate supply and demand. The above findings imply that the usual results of Supermultiplier-type models are highly dependent on the conventional assumption that capital our restricting factor.

1. Introduction

The canonical Hicksian-Sraffian Supermultiplier models (e.g., Serrano [1995a] and and Blecker and Setterfield [2019]) usually rely on Leontief-type technologies where the limiting factor of production is capital. That is, these approaches (often implicitly) assume a dual economy formulation in which labor supply is unconstrained: The "modern" sector (the one that is normally explicitly considered) can always rely on labor surplus supplied by a "peripheral", backward sector a la Lewis (1954).[28]

Using a basic Supermultiplier model, we show that, in the presence of a labor supply constraint, the investment accelerator mechanism embedded in this class of apparatus does not provide a fully satisfactory framework with which to analyze investment behavior. In particular, the accelerator implies that investment activity would continue as long as aggregate demand is rising. However, if a labor supply constraint prevents aggregate (goods) supply from accommodating to rising demand, over-investment (and over-accumulation of capital) would ensure, suggesting a myopic behavior on the part of firms. That is, once a

[27] Professor Jorge Thompson Araujo is the co-author of this paper.
[28] Blecker and Setterfield (2019), p. 35 *et seqs*.

labor-constrained economy is considered, a different approach to investment decisions is needed[29].

More broadly, once the capital-constrained economy assumption is jettisoned, and a physical limit on the size of the labor force is postulated, there is no guarantee that aggregate supply will accommodate itself to the long-term level of aggregate demand dictated by the Supermultiplier. Long-period output is no longer purely demand-driven, but rather determined through the interplay of aggregate supply and demand.[30]

Having labor as the constraining factor as opposed to capital is by no means a far-fetched assumption, theoretically and in the real world. In the Keynesian growth literature, this is reminiscent of Joan Robinson's (1962) "inflation barrier" and "restrained golden age" cases. In real-world circumstances, labor shortages can occur in the presence of major shocks such as a war or a pandemic such as COVID-19, and currently reflected in labor market tightness in advanced economies.[31] Other plausible forms of labor scarcity can occur at the sectoral or firm level, for example, particularly in developing countries, where the issue is not so much lack of labor but lack of skilled labor[32]. That said, the focus of this chapter is purely on the theoretical properties of Supermultiplier models.

Our framework is organized as follows. Section 2 introduces a basic Supermultiplier model, with Leontief technology and a labor supply constraint. It shows that under these assumptions aggregate supply does not play a merely accommodative role to demand and that in the long-run a situation analogous to the Robinsonian "restrained golden age" materializes. Section 3 further investigates the relationship between aggregate supply and demand, showing how the accelerator mechanism may lead to a situation of over-investment that does not self-correct; and how labor-augmenting technical progress can mitigate the excess demand brought about by the labor supply constraint. Section 4 concludes with some additional reflections on the adequacy of the accelerator mechanism as a theory of investment under the assumption of a constrained labor supply.

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[29] Lavoie (2015)'s formulation of the Supermultiplier model includes a neo-Kaleckian investment function. In this chapter, we are concerned about the canonical version, in which investments decisions follow an accelerator mechanism.

[30] The importance of supply-side constraints has been acknowledged by a few authors in the non-mainstream Keynesian tradition. For example, when discussing upward Harrodian divergence, Skott (2017) notes: "This process is subject to obvious supply side limits: there is an upper limit on the utilization rate, and prolonged periods of high growth will run into labor constraints" (p. 10). Marglin (2021) also discusses the role of supply constraints on long-run growth.

[31] Duval *et al*. (2022).

[32] Here, we abstract from heterogeneous skills and treat the labor force as homogeneous.

2. The Basic Model

The basic one-good economy setup discussed in this section includes a Leontief production function – with labor as the restricting factor – on the supply side, as per equations (1) and (2) below, where L and K are, respectively, quantities of labor and capital, l is the labor-output ratio and v is the capital-output ratio. The maximum feasible quantity of labor is given by L^*, and equation (3) should be interpreted as the set of feasible outcomes regarding labor employment. It also means that a dual-economy mechanism is not operating.

The demand side broadly follows the open-economy version of the Supermultiplier presented in, for example, Blecker and Setterfield (2019). The economy in question is assumed to be small, open and confined to a country or region[33]. This version bypasses the usual concern with conventional Supermultiplier models regarding the role of autonomous domestic demand in the determination of long-period output. Here, this role is played by the demand for exports (X^*) from the Rest of the World. In addition, C, I and M are, respectively, aggregate consumption, investment and imports, with c, h and m representing the marginal propensities to consume, invest and import. The investment function follows a basic accelerator mechanism. For simplicity, government activity is left out, without any loss of generality. The model is fully described by equations (1) – (9) below:

$$Y = Y^S = \min[\frac{K}{v}, \frac{L}{l}] \tag{1}$$

$$Y^S = \frac{L}{l} \tag{2}$$

$$L \leq L^* \tag{3}$$

$$Y = Y^D = C + I + X - M \tag{4}$$

$$C = cY \tag{5}$$

$$I = hY \tag{6}$$

$$M = mY \tag{7}$$

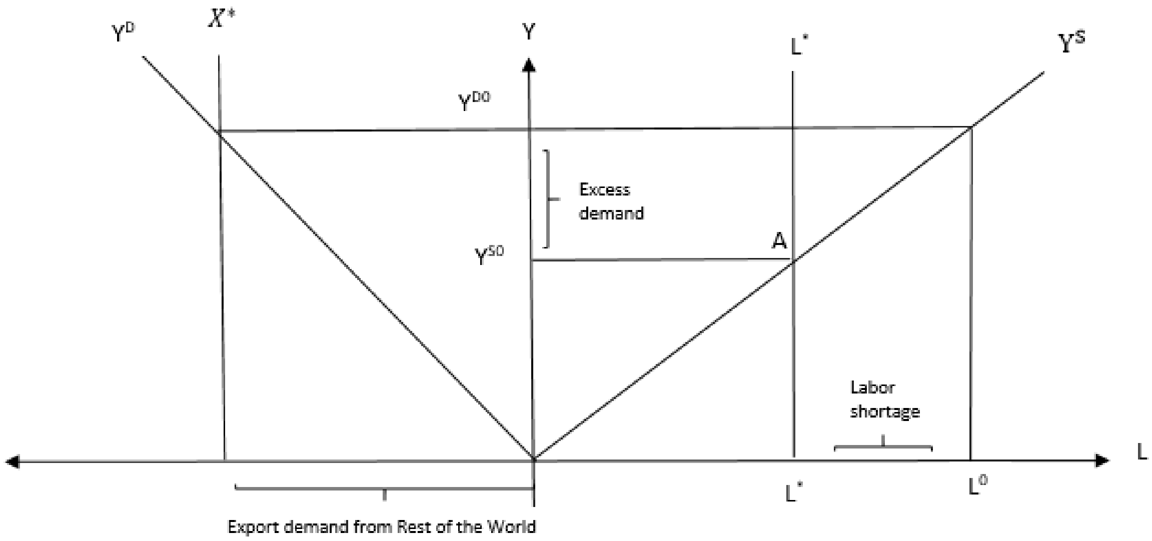
$$X = X^* \tag{8}$$

$$Y^D = \left(\frac{1}{1-(c+h)+m}\right)(X)^* \tag{9}$$

[33] This formulation works as long as this economy interacts with the Rest of the World through international trade. Exports are taken as exogenous by our model economy, but they are endogenous to the aggregate income level of the Rest of the World.

On the demand side, long-period output is determined by equation (9). However, there is no guarantee that the quantity of labor required to satisfy long-period aggregate demand Y^D would also satisfy equation (3). Full employment of the labor force would require $L=L^*$, but consistently with a Keynesian-type framework, there is no reason to expect that this should necessarily be the case. If $L<L^*$, involuntary unemployment would ensue. But the case we are more interested here is when the level of Y^D dictated by equation (9) implies $L>L^*$: In this case, equation (3) would not be satisfied, leading to a situation of excess demand and labor shortage, as illustrated in Figure 5 below.

Figure 5 - Long-Period Excess Demand with Labor Shortage



Elaborated by the authors

This presentation is analogous to (albeit not equivalent with) Joan Robinson's (1962) "restrained golden age" (see also Marglin [1984]). Robinson's "mythical" (her own words)[34] golden age concept was characterized by long-run full employment, in which the natural growth rate n (or labor force growth rate) would coincide with the warranted growth rate g. However, as she pointed out, there was no guarantee that this "felicitous" equality would materialize. She discussed several instances in which such equality would not hold. The case of interest here – the restrained golden age – is associated with $g > n$, leading to an excess demand for labor. The case discussed in this section is similar, although the model is not formulated in terms of growth rates, in contrast to her results. In addition, she proposes a

[34] See Robinson (1962, p. 52): "I used the phrase 'a golden age' to describe smooth, steady growth with full employment (intending thereby to indicate its mythical nature). Corresponding nicknames may be given to other possible phases of growth."

mechanism whereby rentiers restrain investment demand which eventually moves equilibrium to the point at which $g = n$.

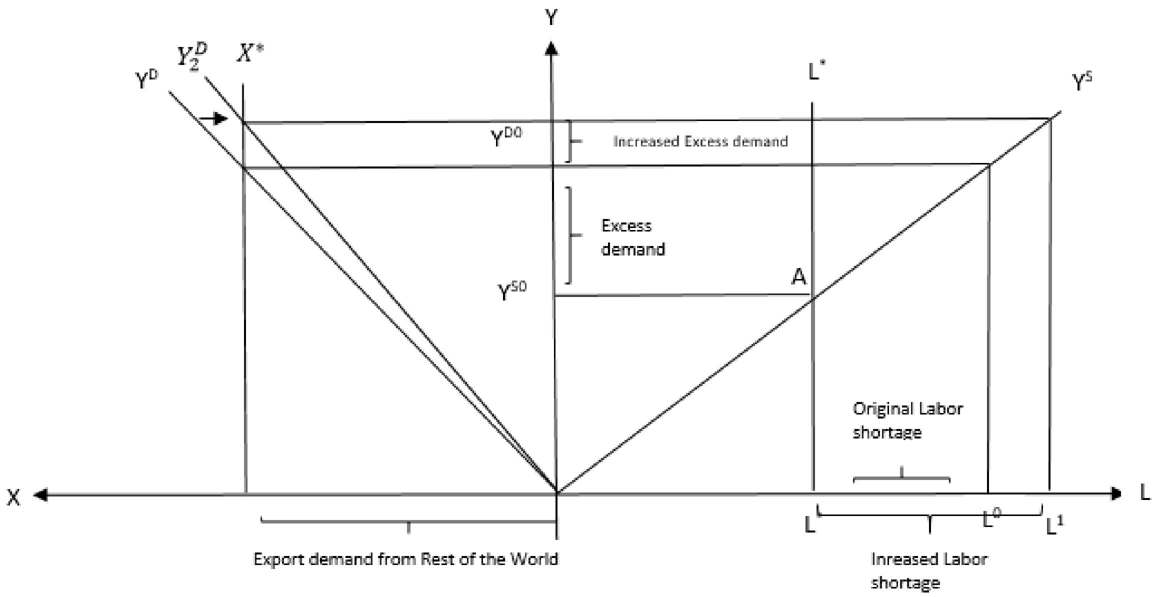
3. Re-Defining the Investment and Consumption Functions

This section focus into interactions between aggregate demand and supply, first observing into how the investment accelerator mechanism tends to lead to excessive capital accumulation in light of the physical constraint placed on labor supply, which could be interpreted as a form of dynamic inefficiency (Cf. Diamond [1965]); and then discussing possible countervailing factors, on both the supply and demand sides. It also briefly looks into an extension of the investment accelerator proposed by Serrano and Freitas (2017).

3.1 The Accelerator Mechanism and Over Accumulation of Capital

As it stands, the accelerator mechanism described in equation (6) above tends to generate – in and of itself – more capital than can be productively absorbed by the existing labor force. More precisely, the economy would tend to over-accumulate capital. With labor as the binding constraint, a growing excess of demand will worsen the shortage of labor, as in Robinson's restrained golden age case (see Figure 6).

Figure 6 - Rising Aggregate Demand with Worsening Labor Shortage

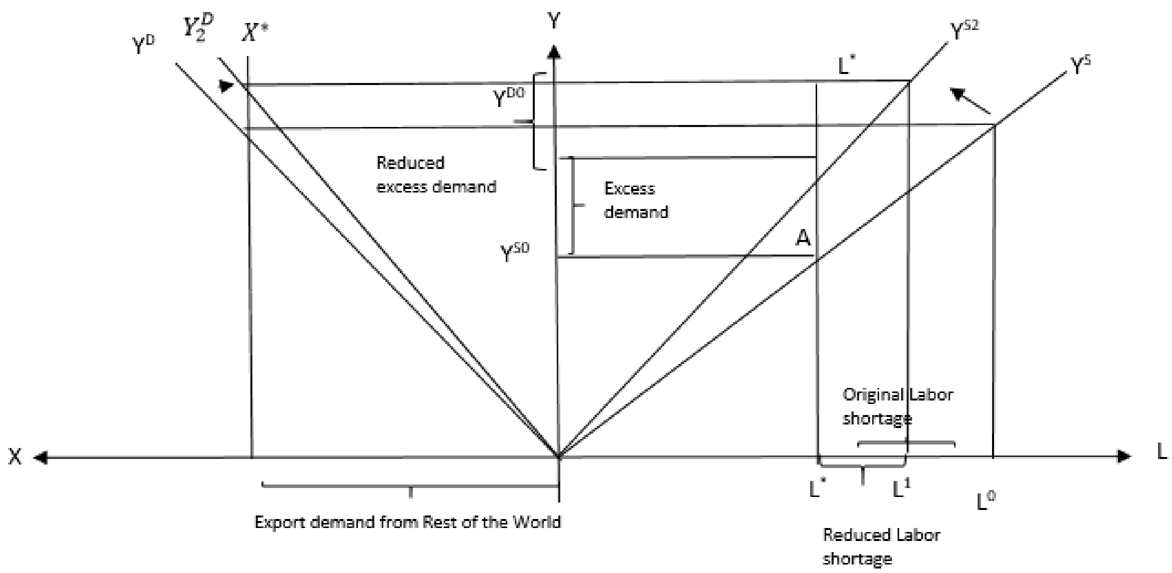


Elaborated by the authors

3.2 Countervailing Mechanisms: Labor-Augmenting Technical Progress and Weaker "Animal Spirits"

A first plausible mechanism is labor-augmenting technical progress, which operates by reducing the labor coefficient l. This is depicted as an increase in the slope of the aggregate supply curve Y^S (since $\frac{\partial Y^S}{\partial L} = \frac{1}{l}$), reducing the excess demand and the shortage of labor. Figure 7 shows this case of exogenous technical progress, which is reminiscent of Solow (1956).[35] However, there is no reason to expect that such exogenous technical progress would exactly suffice to eliminate the labor shortage. In fact, if investment accelerates faster than technical progress occurs, labor shortage will continue to manifest in the long-run.[36]

Figure 7 - Exogenous Technical Progress as a Partial Countervailing Factor

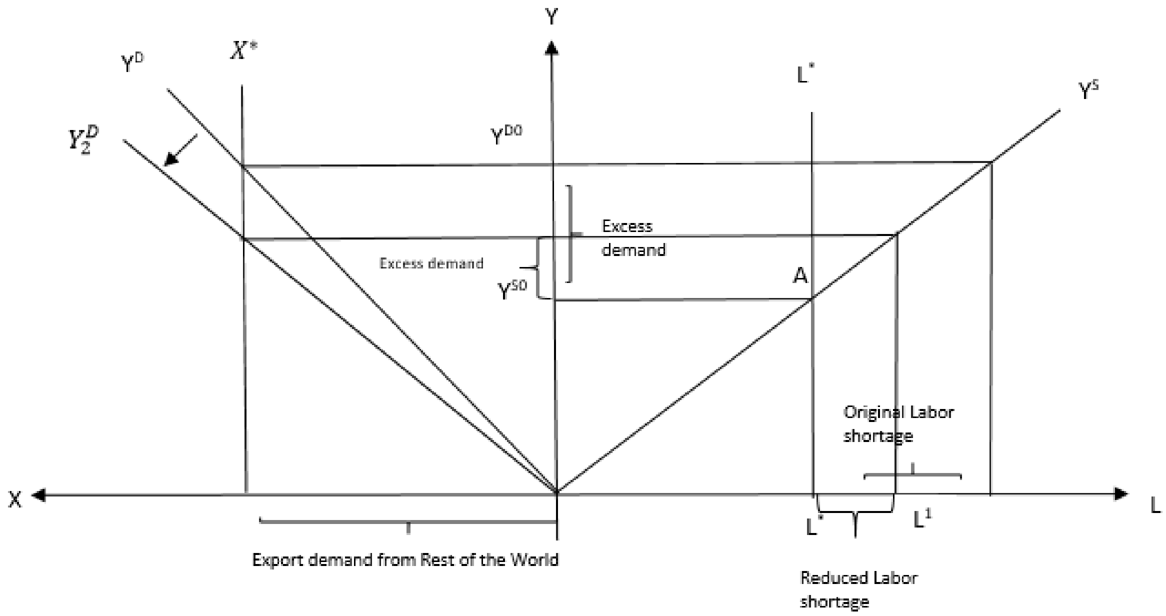


Elaborated by the authors

A second mechanism, now on the demand side, involves an exogenous reduction in the marginal propensity to invest – perhaps reflecting depressed "animal spirits" – leading to an investment slowdown and a deceleration in the demand for goods and labor. Thus, the slope of the aggregate demand curve would be less steep, causing a reduction in both excess demand and labor shortage, as shown in Figure 8.

[35] A plausible mechanism for endogenous technical progress can be introduced, but it is beyond the scope of this chapter.
[36] Another potential countervailing mechanism would be the endogenization of L as a function of human capital, knowledge or skills.

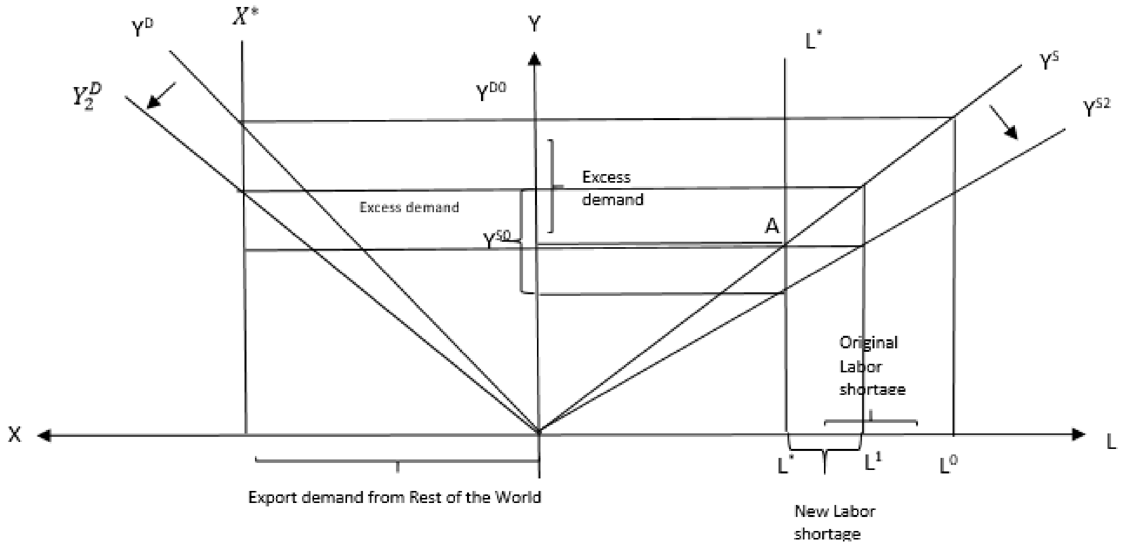
Figure 8 - A Lower Marginal Propensity to Invest as a Countervailing Factor



Elaborated by the authors

A third mechanism combines the previous two. Under the plausible assumption that higher labor productivity materializes through investment activity[37], the investment slowdown could ultimately be associated with a lower efficiency in production. That is, a higher *l*, implying a less steep slope for the aggregate supply curve Y^S, as shown in Figure 9.

Figure 9 - Investment Slowdown with Loss in Efficiency in Production



Elaborated by the authors

[37] As in models of embedded technical progress, such as Arrow (1962).

3.3 Revisiting the Investment Accelerator Function

In what follows, an equation of motion for the accelerator mechanism commonly used in the Supermultiplier literature (e.g., Serrano and Freitas [2017]; Blecker and Setterfield [2019]) is briefly examined. To see how this equation of motion operates, first take time derivatives of both sides of equation (9), as is presented below:

$$\dot{Y}^D = \frac{\dot{h}X^*}{(1-c-h+m)^2} \rightarrow \frac{\dot{Y}^D}{Y^D} = \frac{\dot{h}}{1-c-h+m}$$

The equation of motion for the accelerator *h* follows their model, namely:

$$\dot{h} = \gamma(u-\mu)h \tag{10}$$

Note that *u* stands for actual capacity utilization and μ for the "normal" level of capacity utilization, both expressed in terms of the available stock of capital. Denoting the growth rate of the aggregate demand, $g_{Y^D}\ as \left(\frac{\dot{Y}^D}{Y^D} = g_{Y^D}\right)$, it follows that

$$g_{Y^D} = \frac{\gamma(u-\mu)h}{1-c-h+m} \tag{11}$$

As a result, aggregate demand growth can be explicitly expressed as a function of the investment accelerator. Whenever the marginal propensity to invest is increasing, that is $\dot{h} > 0$, the economy will have ever-rising excess capacity. In this case, $g_{Y^D} > 0$, which would speed up the labor shortage described above.

Note that the Supermultiplier model in Serrano and Freitas (2017) also has capital as the binding factor. Their model implicitly assumes with perfectly elastic labor supply, which does not influence the equilibrium between the aggregate supply and demand in the product market. However, this formulation does not seem adequate for an economy that is labor-constrained.[38] In particular, it is clear what normal capacity utilization μ would mean in this context. It would make more sense to reformulate the investment function to take into account the evolution of labor unit costs, thus reflecting changes in real wages as well as in labor productivity. One possibility is to endogenize μ as a function of the employment ratio

[38] As in models of embedded technical progress, such as Arrow (1962). similar models, but stops short of proposing a formal mechanism to address them.

L/L^*, in such a way that firms would take into account labor scarcity when making their investment decisions.[39]

4. Concluding Remarks: Theoretical Implications and Areas for Further Research

The model studied in this chapter contains two potentially problematic implications for Hicksian-Sraffian Supermultiplier models. By displaying these issues very explicitly, the present analysis sheds further light on the limitations of the approach as a theory of the determination of output in the long period.

The first one is the investment accelerator function as traditionally formulated becomes inadequate for a labor-constrained economy. This is due to the fact that this investment function does not include a mechanism for making firms stop accumulating capital before the supply of labor turns binding. This clearly implies a myopic behavior on the part of the owners of capital. Furthermore, as discussed in the previous section, overaccumulation of capital then ensues, as investment activity keeps going despite the labor supply limits, which is analogous to the dynamic inefficiency case in neoclassical growth models. As noted above, a more adequate formulation would need to explicitly consider how firms react to changing unit labor costs.

The second one is when we consider labor as the binding constraint, the Supermultiplier model no longer provides a purely (or even largely) demand-driven framework for the determination of long-period output. In fact, aggregate supply plays a major – if not *the* main – role in determining output in the long period. A typical Supermultiplier model that combines an investment accelerator with a capital-constrained Leontief production function has an embedded mechanism for adjusting aggregate supply to aggregate demand. This instrument works as follows: Any increase in aggregate demand will lead to an increase in investment for a given marginal propensity to invest v. In turn, higher investment will increase the stock of capital – and thus the supply of goods produced in this economy – thereby accommodating the original rise in aggregate demand. Thus, in a very concrete sense, aggregate supply adjusts to whatever level is dictated by aggregate demand. This mechanism breaks down in a labor-constrained economy, simply because investment in physical capital does nothing to increase the supply of labor, which provides

[39] We are grateful to Gilberto Tadeu de Lima for making this point.

an upper limit to the ability of increases in aggregate demand to generate a corresponding increase in output.[40] [41]

Two areas for further research stand out. First, in line with the above discussion, more refined investment functions could be introduced, with the aim of avoiding the implied myopic behavior brought about by the accelerator in the context of labor supply constraints. For example, endogenizing μ with respect to the employment ratio, perhaps taking into account Goodwin-type labor market dynamics, would be a possibility, as noted above. Alternatively, a version of the Tobin's q theory of investment could be a candidate, since it contains a mechanism to prevent investments from continuing indefinitely (namely, investment activity will cease for $q<1$, in its most common formulation). Second, a less rudimentary presentation of the labor market would be in order. In particular, a labor supply shortage would lead to rising real wages, which would have cost, demand and distributional implications, potentially acting as an additional adjustment mechanism between aggregate demand and supply (Marglin [2021]).

[40] Of course, a labor-constrained Leontief technology does not preclude the case in which the Supermultiplier is sufficiently low to absorb only a fraction of the available labor force, leading to a situation of long-period unemployment.

[41] An open economy formulation would also allow for another possible adjustment mechanism through the labor market. Suppose that an explicit export function is introduced – so that, for example, X becomes a function of the real exchange rate. If labor shortage leads to rising unit labor costs, then the real exchange rate would tend to appreciate, making exports less competitive and inducing a demand adjustment through the external sector. Of course, the drawback of this reformulation would be to lose X as an autonomous component of aggregate demand.

CHAPTER 4 – SHIFFITING PERSPECTIVES: LOOKING AT THE LABOR SIDE IN A HICKS-SRAFFA SUPERMULTIPLIER[42]

The central objective of this chapter is to introduce the wage elasticity of labor demand into the Hicks-Sraffa Supermultiplier (HSSM). It is a well-known fact that some technological advancement excludes part of the workforce of the market, especially the less qualified ones, restricting the demand for workers, which makes it impossible for all economies to ensure full employment (sometimes even in the long-run). However, it is not restricted to less skilled. The development of Artificial Intelligence created the possibility of a computer performing small surgeries without any assistance from a human doctor. Unfortunately, the post-Keynesian tradition usually does not give proper attention to this market, only assuming that, in a long-run perspective, employment automatically adjusts to the capital. Thus, here, we expand the HSSM incorporating the wage elasticity of labor demand and the average salary growth rate to determine how such a mechanism influences the dynamics of this type of model. Then, we attest their stable conditions and estimate a numerical simulation to measure the computational support of our theoretical results.

1. Introduction

> […] what is the new theory about? It is presented to us, primarily, as a theory of employment; but before the book is ended, both author and reader are convinced that it is not only a theory of employment.
>
> John Hicks (1936, p. 238), Keynes' Theory of Employment

We are not trying to compare our work to the canonical book written by Keynes, but, like him, we have the intention to explain some concerns about the Labor Market[43]. His original framework was developed in a short-run perspective, and according to him "in the long-run we are all dead" (KEYNES, 1923, p. 65). However, the most prominent Macroeconomic Schools, delivered from his ideas, works over a long period, like Robinson (1953), Kaldor (1956), and Solow (1956). These authors had the objectives of understanding how the economy grows; explaining the product behavior and predicting future comportments to avoid a new crisis.

[42] This chapter is co-authored by Professor Joanilio Rodolpho Teixeira.

[43] His foundations are not identical to ours since our concern here is related to the labor power loss and capitalist empowerment, like in Marglin (1974).

Over the past decades, the above scholars have inspired other economists to pursue and expand their theories, such as Pasinetti, Lucas, Mankiw, Nelson, Dutt, Rowthorn, Lavoie, and others. Indeed, these prominent authors, among other issues, diverge on how to deal with Labor Market. On one side, the post-Keynesians believe that the labor supply is perfectly elastic inspired by Lewis's (1954) work, where the quantity will adjust to the subsistence wage or the immigration will interfere and correct the market; besides, the wages exceed the demand and opening opportunity to new industries. On the other side, the supply of employment is perfectly inelastic (neo-Classical view) where, in a long-run perspective, the quantity of work will be interfered with only by the population growth rate and the wages will be determined by their level of productivity. In this vein, for them, the demand for work is a function of real wages and cut in money wages will always drive the economy to full employment [see Vercherand (2014)].

Our intention here is to look at the demand for labor side. Since the 1980s, advancements in technology have substituted employment activities, especially the ones less qualified. Recently, even delicate works have been made by robots, like minor surgeries[44]. This new reality implies an increase in unemployment since the reallocation of these persons is not simple and hardly will happen in the short- or medium-run. Besides, if technological improvement continues, more and more humans will be excluded from the market[45]. Admittedly, the wage elasticity of labor demand will tend to a perfectly inelastic situation, only when wages are equal to zero; in this case, the capitalist will have control of the employees since the workers' power is declining, thus, the empowered class will have to choose between raising investments to sustain the advancements in technology or increase wages to support workers. Such a perspective creates a restriction from the demand side and reflects our days.

In our view, if the workers lose their force to bargain and they continue to be substituted by robots and AI, it is not a utopian situation to believe that this class will promote a revolution like the one proposed by Engels (2020). By the way, a laureates Nobel in Economics, Professor Daniel Kahneman, already warned society about the human incapacity

[44] If the reader wants to improve his/her knowledge about the theme, we recommend the works of Dyer-Witheford et al. (2019), Acemoglu and Restrepo (2019), Schwabe and Castellacci (2020), and Graham (2022). All of them have different perspectives but equal concerns about the same problem.

[45] Thus, even knowing that in the US the advancements in technology approximate the economy to full employment in the past, today, we are dealing with a different situation, where the machines and AI perfectly substitute the workforce in all areas. So, the effects on wages will be stronger and the labor power will be harmed. Some of these effects are explored by Webb (2019) and Acemoglu and Restrepo (2019), but they are not so radically.

to compete with AI [see Campos (2023)]. In this vein, the effects of such a mechanism will provide the capitalists with a powerful tool to eliminate any possible chance of worker's unions guaranteeing a minimum of subsistence to the labor force, increasing the informal sector, and making precarious the worker environment. Thus, to understand the implications of this elasticity and the effects on capital accumulation and investment we present a new approach including both the wage elasticity of labor demand and the growth rate of average wages in the capacity utilization dynamic function.

Thus, this chapter is divided into four parts after this introduction. Section two presents the justification of the present research and its importance to our days. Section three introduces into the theoretical framework the concept of wage elasticity of labor demand and the implications of a more flexible or rigid relation between firms and workers. In the fourth section it is presented the formal development of our new approach, proofing stability condition and limit cycle. Such section also indicates its limitations, defining the maximum and minimum value of the wage elasticity of labor demand which depends on the growth of both autonomous components and average wages. The fifth section, reinforces the results by approaching a numerical simulation of the dynamic system, ensuring the convergence of the capacity utilization to its normal level and comparing different situations. Then, the Concluding Remarks are available and the rigorous mathematical proves of two theoretical propositions are included in the appendix.

2. Wage Elasticity of Labor Demand, Concept, and Possible Implications

The wage elasticity of labor demand measures the sensitivity of employment to a change in wage. This concept is central to the allocation of the workforce on the productivity side. Recent technological advances have replaced employed people (at least in its formal sense); especially those less qualified. Today, we have some machines that not only answer phones and deliver simple human interactions, like sales but also perform surgery without human help [see Grahan (2022) and Dyer-Witheford, Kjøsen and Steinhoff (2019)]. The percentage of highly qualified workers is small and the most sensitive part is the lower skilled, thus affecting the labor demand and wages, as pointed out by Schwabe and Castellacci (2020).

These concerns come from the firm side and it is a fact that the decision to hire or fire a employee is linked to the firm's plans and varies over time since entrepreneurs are looking to increase their profits and none of them are truly altruistic [see Huergas and Arias (2019)]. On many levels, labor affects the effective demand, especially from the consumption perspective once the workers consume most of (or in some cases all) their income (wages). In this vein, considering a multiplier mechanism such as the Keynesian one, a strong reduction in consumption will affect growth and the determination of profits [see Pasinetti (1962)]. This is an important issue to be discussed when the relevant theoretical models are elaborated on.

Usually, post-Keynesian models assume the automatic adjustment of labor to capital[46]. However, in real economies, this will not always be the case. One example, recently, is the world experienced with the Coronavirus pandemic; and, nowadays, we are dealing with the consequences of COVID-19, especially concerning the reconstruction of economies worldwide. All countries were affected by the “stay at home” policies that were essential to stopping the advance of the pandemic. Even though most countries provided policies to reduce the effects on their hosted firms, some of them were not prepared to deal with the situation and had to choose to fire their staffs.

Such an event accelerated technological advancement and the macroeconomic effects have now been observed, especially dealing with the substitution between labor and robots. Prettner and Bloom (2020), Semuels (2020), and Acemoglu (2021) have shown, and predicted, the dynamic effects of the pandemic in relation to this particular behavior. What is especially alarming is the huge amount of unemployed workers and the inequality of income, since the power of labor is declining in comparison to capital. The advances in different types of technology, such as machine learning and AI[47], have been a concern of some economists, like Acemoglu and Restrepo (2019), even before COVID-19. Consequently, addressing the efforts to understand such effects from a long-run perspective is valid, and starting to give more attention to such a problem will avoid hard consequences for the workers affected now and in the near future, especially to design new policies.

Here, we expand the HSSM idea by considering labor as a biffing restriction. We believe that the limit came from the demand for it when advancements in technology substitute the workforce, excluding part of the labor. So, we made it flexible the assumption of wage elasticity of labor demand and included the growth of average wage in such a case. This expansion of the original framework will provide a modern structure, conducting it as

[46] Let us take this opportunity to explain that we are not generalizing such a thought. Thus, if the reader wants to improve the knowledge in the field, we recommend the reading of Palley (2012), Palley (2019), and Dutt (2020).

[47] In our view, for this and future works, the AI perfectly substitutes both less and highly skilled workforce. In this vein, this first model intends to incorporate the labor demand to the HSSM but in the future, we will expand the work by including government and the effects of technological change.

a new interpretation of reality in the 21st century, like the implications of AI. The next section presents the theoretical development of our new approach, including its stability and bifurcation proofs.

3. Approaching the Dynamic System

Here, we presents the mathematical formulation and proofs of our new dynamic approach. Let us assume a simple Leontieff productivity function:

$$Y = min\left\{\frac{uK}{v}; \frac{L}{l}\right\} \tag{1}$$

Equation (1) only is plausible if: $\frac{uK}{v} = \frac{L}{l}$. Thus, it implies that labor is an essential restriction to productivity growth. However, this is not exhaustively explored by the post-Keynesian literature, once they assume a perfect supply elasticity that will automatically adjust the employment level. Though Keynes (1936, CH. 21) assumes that workers' elasticity affects Macro behavior in a short-run perspective, he did not demonstrated it in the long-period. Our intention, is to show it for such case improving the analysis. Like him, for simplification, we will assume rigid prices and only salaries will vary. At the end of the present mathematical formulation, we demonstrate an appropriate approach by introducing the wage elasticity of labor demand to the capacity utilization dynamic formula. Thus, from (1), we have:

$$Y = \frac{uK}{v} \rightarrow Y = uK\left(\frac{Y}{K}\right)\left(\frac{L}{L}\right) \rightarrow Y = \frac{ueK}{l} \tag{2}$$

Where: $e = \frac{L}{K}$ and is constant according to Hicks Neutrality. Rearranging the equation, we obtain:

$$u = \frac{Yl}{Ke} \tag{2.1}$$

Applying the Napierian logarithm and time derivation in 2.1, we have:

$$\frac{\dot{u}}{u} = \frac{\dot{Y}}{Y} - \frac{\dot{K}}{K} - \frac{\dot{l}}{l} \tag{3}$$

Equation (3) expresses a new formulation of the capacity utilization, including the labor-productivity growth rate $\left(\frac{\dot{l}}{l}\right)$, so:

$$\frac{\dot{l}}{l} = g_L - g_Y \tag{4}$$

Equation (4) shows that l is impacted by both growth rates of labor $\left(g_L = \frac{\dot{L}}{L}\right)$ and productivity $\left(g_Y = \frac{\dot{Y}}{Y}\right)$. Substituting (4) in (3), and assuming $g_K = \frac{\dot{K}}{K}$, we have:

$$\dot{u} = (2g_Y - g_L - g_K)u \tag{3.1}$$

3.1 express a new formulation of the capacity utilization variation in time by including the workforce in its behavioral determination. The model designed above referrers the productivity side. Thereby, our demand is expressed by the HSSM in its simplest way, but considering an Open Economy. In this vein, we assume the following set of equations:

$$Y = C + I + X - M \tag{5}$$
$$C = Z + cY \tag{6}$$
$$I = hY \tag{7}$$
$$M = mY \tag{8}$$

Equation (5) is the effective demand which is equal to the income (Y), which is composed of the sum of total consumption (C), total private investment (I), exports (X), and imports (M). Equation (6), (7), and (8) was properly named, where c, h, and m are respectively their marginal propensities, and Z is the autonomous consumption. Substituting (6) to (8) in (5), we have the HSSM, thus:

$$Y = \frac{A}{\alpha} \tag{9}$$

Where $A = X + Z$ and $\alpha = 1 - c - h + m$. Approaching the Napierian logarithm and deriving concerning time, we have the income growth rate, which is:

$$g_Y = g_A + \frac{\dot{h}}{\alpha} \tag{10}$$

For us, does not matter how A behaves, since our intention is to observe the labor performance. Substituting (10) in (3.1), we have:

$$\dot{u} = \left(2g_A + \frac{2\dot{h}}{\alpha} - g_L - \frac{hu}{v}\right)u \tag{3.2}$$

At this point, we can determine g_L . Thereafter, we have to consider that the average wage of the economy (w) is equal to its gross value (W) divided by the activity workforce (L):

$$w = \frac{W}{L} \tag{11}$$

Pasinetti (1962) raised an interesting consideration in Kaldor's (1956) paper; he argues that the economy is divided into two classes (workers and rentiers). His central point was to demonstrate the income discrepancy between them, so, he proposed both profit- and wage-shares. Since we are interested to understand how the labor activity affects the model, we proceed with the second share, thus: $\omega = \frac{W}{Y}$, so:

$$w = \frac{\omega Y}{L} \tag{11.1}$$

Rearranging (11.1), applying the logarithm, and deriving concerning time, we have:

$$g_L = \frac{\dot{Y}}{Y} - \frac{\dot{w}}{w} \tag{12}$$

Equation (12) demonstrates that labor demand growth depends on the interaction between the growth rates of output (positively) and average wages (negatively), which is a plausible solution since entrepreneurs will always have the objective of increasing profit. Therefore, from this formula, we can deliver the implications of the wage elasticity of labor demand. For us, such a concept reflects the variation of the quantity of labor with respect to the variation of the nominal wages: $e_L = \frac{dL}{L}\left(\frac{W}{dW}\right)$. Thus, approaching the following mathematical manipulations, we have g_L expressed by (12.1):

$$g_L = \frac{\dot{Y}}{Y} - \frac{\dot{w}}{w}$$

$$g_L = \left(\frac{\dot{L}}{L} - \frac{\dot{l}}{l}\right) - \frac{\dot{w}}{w}$$

$$g_L = \frac{dL}{dtL}\left(\frac{dW}{dW}\right)\left(\frac{W}{W}\right) - g_L + g_Y - \frac{\dot{w}}{w}$$

$$g_L = \frac{1}{2}\left[\frac{dW}{dt}\left(\frac{e_L}{W}\right) + g_Y - g_w\right] \qquad (12.1)$$

e_L presents how much a slight change in nominal wages affects the quantity of the needed labor force. In this vein, the wage elasticity of labor demand demonstrates the relevance of workers to capitalists, since if $W \to 0$, so also $e_L \to 0$ means that machines can provide the same action or increase the productivity harming the distribution of income and, thus, workers will have to accept lower levels of wages. On the other hand, is $W \to \infty$, so also $e_L \to \infty$ and, in this case, machines or AI cannot provide the actions and capitalists depend on the workforce. Since we already presented the wage-share concept, it is possible to define $\frac{dW}{dt}$, hence:

$$W = \omega Y$$

$$\frac{\dot{W}}{W} = \frac{\omega\dot{Y}}{W}$$

$$\frac{\dot{W}}{W} = \frac{\omega\dot{Y}}{\omega Y} \to \dot{W} = \frac{\dot{Y}}{Y}W = g_Y W \qquad (13)$$

The wage-share is not a dynamic variable since, according to Lavoie (2016, p. 176), in this kind of model the "income distribution does not change and is proxied by the share of profits". Therefore the gross wage and output do change, resulting in (13). Substituting (13) in (12.1), we have:

$$g_L = \frac{g_Y}{2}(1 + e_L) - \frac{g_w}{2} \tag{12.2}$$

Substituting (12.2) in (3.2), and rearranging the expression, we obtain:

$$\dot{u} = \left\{\left[g_A + \frac{\dot{h}}{\alpha}\right]\left(\frac{3-e_L}{2}\right) + \frac{g_w}{2} - \frac{hu}{v}\right\}u \tag{14}$$

Equation (14) is the capacity utilization variation in time concerning the elasticity and growth rate of the average wages. The HSSM, according to Serrano and Freiras (2017), admits the following dynamic equation to the investment (marginal propensity) accelerator mechanism:

$$\dot{h} = \gamma(u - \mu)h \tag{15}$$

(15) shows that investment is restricted by the level of capacity utilization and its normal level (μ) and, in a long-run perspective, u converges to μ. Substituting (15) in (14) we have our 2D dynamic system:

$$\begin{cases} \dot{h} = \gamma(u - \mu)h \\ \dot{u} = \left\{\left[g_A + \frac{\gamma(u-\mu)h}{\alpha}\right]\left(\frac{3-e_L}{2}\right) + \frac{g_w}{2} - \frac{hu}{v}\right\}u \end{cases} \tag{16}$$

The central discussion here is that the dynamics of u will define the trajectory of the investment, and, in a long-run perspective, the actual capacity utilization converges to its normal level, resulting in the steady-state. Considering these values:[48] $u^* = \mu$ and $h^* = \left[g_A\left(\frac{3-e_L}{2}\right) + \frac{g_w}{2}\right]\left(\frac{v}{\mu}\right)$, we have the following Jacobian Matrix (J):

$$J(P^*) = \begin{bmatrix} 0 & \gamma\left[g_A\left(\frac{3-e_L}{2}\right) + \frac{g_w}{2}\right]\left(\frac{v}{\mu}\right) \\ -\frac{u^2}{v} & \left[g_A\left(\frac{3-e_L}{2}\right) + \frac{g_w}{2}\right]\left(\frac{v}{\mu}\right)\left[\left(\frac{3-e_L}{2}\right)\frac{\gamma\mu}{\alpha} - \frac{2\mu}{v}\right] + \left(\frac{3-e_L}{2}\right)g_A + \frac{g_w}{2} \end{bmatrix} \tag{17}$$

[48] The steady-state values are presented in Appendix B - a.

According to Gandolfo (1996), to prove the stability, we need to bear in mind that the Routh-Hurwitz criterion states that $P^* = (h^*,\ u^*)$ is asymptotically stable if and only if $S_1(P^*) = Tr(J) < 0$ and $S_2(P^*) = Det(J) > 0$, where: $S_1(P^*)$ and $S_2(P^*)$ are the coefficients of the characteristic polynomial of matrix J evaluated at P^* which is given by:

$$\lambda^2 + \lambda(a_{11} + a_{22}) + (a_{11}a_{22} - a_{12}a_{21}) = 0 \tag{18}$$

Thus, we can prove that the following proposition is stable:

PROPOSITION 1: [49] Considering the impact of the labor market in the HSSM type of models and assuming that the elasticity is an important factor to determine the convergence of the capacity utilization to its normal level, so, we can guarantee the stability of the approach if, and only if:

(i) $e_L > 3 - \frac{2\alpha}{v\gamma}$

(ii) $e_L < 3 + \frac{g_w}{g_A}$

In this case, we ensure that the model is stable and that convergence is reached. It is also possible to prove that it admits a bifurcation (next theoretical result). Here we have a limit space where e_L is allocated, being: $0 < 3 - \frac{2\alpha}{v\gamma} < e_L < 3 + \frac{g_w}{g_A}$. Thus, if we consider $g_A \to 0$ or $g_w \to 1$, *coeteris paribus*; higher e_L can be. And if $g_A \to 1$ or $g_w \to 0$; more restricted the interval will be. In this vein, our approach incorporates the wage growth rate and the limits of the labor elasticity to the model. We can assume higher or lower levels of elasticity, even perfect elastic if the estipulate limits are respected.

PROPOSITION 2:[50] Considering the system above, the stable limit cycle stands if, and only if:

(i) $e_L > 3 - \frac{2\alpha}{v\gamma}$

[49] The mathematical proof is demonstrated in Appendix B - b.
[50] The mathematical proof is demonstrated in Appendix B - c.

The limit cycle suggests that the economy can be in a boom or in a bust situation through periods and this is analyzed by the trajectory of the phase space. In our framework, we proved that there is a convergence of the capacity utilization to its normal level and this is stable. Guaranteeing that the model is mathematically efficient. Thus, here, we attested the importance of the elasticity and growth rate of the average wages to the HSSM, which agrees with the Hicks quote at the beginning of the present chapter, presenting the importance of the labor market to this kind of approach. We also demonstrated the importance to look at these variables which were already demonstrated by Keynes in his General theory and, on some level, in Marglin's (2021) new book. Besides the mathematics, it is important to analyze how the framework behaves computationally, the next section presents a numerical simulation, considering plausible real values to an uneven economy.

4. Numerical Simulation and Graphical Interpretation

Now, we approach the numerical simulation in Equation (16) by using Software R and the package deSolve. As we already demonstrated, this is a 2D system that reflects the variation of both u and h. Our intention here is only to ensure that it is possible to design an empirical experiment (in our near future) using econometric analysis, including government and possible economic policies. Recently, Dvoskin and Landau (2022), Nikiforos, Santetti and Armin (2023) and Summa, Petrini and Teixeira (2023) opened the discussion of cycles in the HSSM demonstrating first their stability and then approaching an econometric analysis to the US economy. However, they looked to the simplest model, which, in part, we expanded, in this paper. We will only use computational numerical simulation, once the most important contribution of this section is to ensure the validity of the above results.

Here, it is admitted three different time variances of t,[51] where: $t = 200$ units for $e_L = 0$; $t = 2000$ for $e_L = 0.2$; and $t = 1000$ for both $e_L = 1$ and 4.[52] For us, each unit can represent one day in real life. Thus, Table 3 shows the values used to approximate the model and they are close to real observed economic values.

[51] We approach different times perspectives because we are interest to observe when each case converge the capacity utilization to its normal level. So, if we consider a large amount of time, it will be unnecessary hard to understand some of the graphics.

[52] The different scale is only to demonstrate how fast the convergence is. In this vein, when the normal capacity utilization is reached we stopped the experiment. Skott (2019) affirms that the Harrodian Instability take place when the model assume a more rapid convergence.

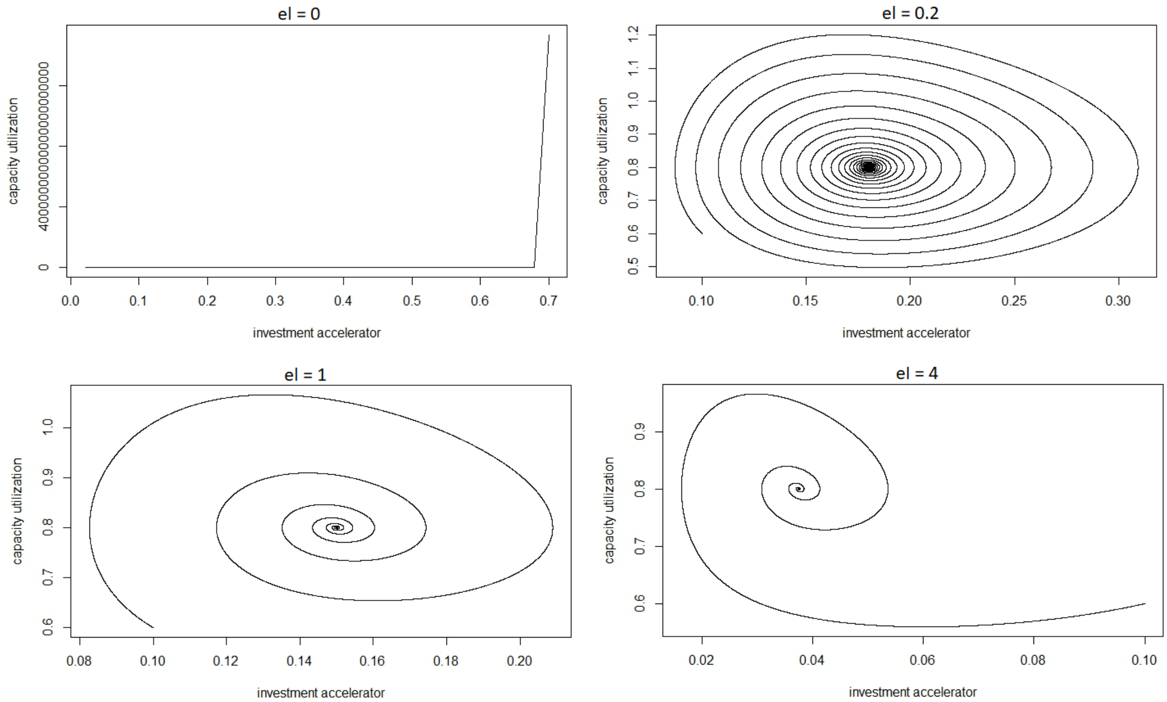
Table 3 - Values for Numerical Simulation

Variable	Value
u (initial)	0.6
h (initial)	0.1
γ	0.167
g_w	0.06
g_A	0.03
V	2
μ	0.8
c	0.7
m	0.4
e_L	0; 0.2; 1; 4

Elaborated by the author

Approaching the algorithm, the software presents Figure 10 as a solution. This graphic demonstrate the relation between the dynamics of the capacity utilization and accelerator investment.

Figure 10 - Endogenous Cycle

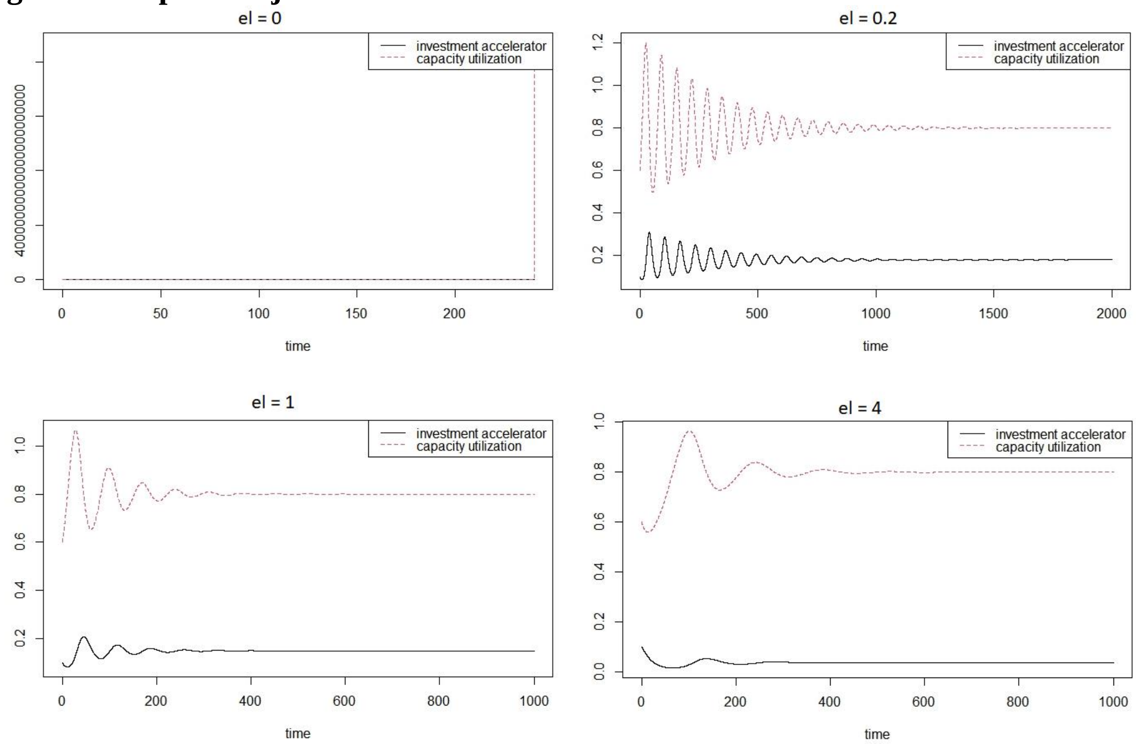


Elaborated by the author

Figure 10 evidence the mathematical proof in section 3. As we can see, we have four graphics. The first one is completely unstable, since we tend to not have salaries in this case, so W is near to zero expressing exploitation by capitalists; the main reason is that e_L is perfectly inelastic and is out of the limits of Proposition 1. The second shows the case where the model is inelastic; however, to the asymptotically convergence sustain, there are two problems here; first, the level of investment needs to be really higher, in some parts more the 30% to support the needs of cover the depreciation, since machines and IA will provide most of the productivity and workers will have to accept lower levels of salaries harming the income distribution and only providing inequality between class. Such a level was only observed in wars and for a small period. Second, we observed an over capacity utilization level, higher then 1.2, which is illogical. The third case reflects the same concerns; and the fourth corrects each one, establishing that the marginal propensity to invest will orientate the convergence, but at plausible levels, and the capacity utilization will not exceed 1. These behaviors represent the chosen values in Table 3 and will vary if the reader modifies them. Therefore, the conduct will be the same, if the limits of Propositions 1 and 2 are respected.

Indeed, we can see that the model converges the capacity utilization to its normal level and also generates a stable limit cycle, which means that we have an endogenous relationship. It is important to know that capacity utilization has been affected by elasticity, which is the proposed task here. This result is similar to the one presented by Nikiforos, Santetti and Armin (2023) when they demonstrate but do not prove the cycle, in the simplest HSSM without a labor market. These authors and Skott (2019) discuss the speed adjustment to steady growth. According to them, if the adjustment is "too fast", the response of accumulation will lead the model to be unstable, like in Harrod. In this vein, the next Figure presents the speed adjustment of the capacity utilization and investment accelerator mechanism.

Figure 11 – Speed Adjustment of u and h



Elaborated by the author

Here, we also have four graphics. The first shows the case of a perfectly wage inelastic labor demand assumption. In this case, the model is not efficient, exploding the capacity utilization to non-real levels. The second presents the convergence when a slight elasticity is considered. In this situation, we have a much sensitivity model, since their ups and downs in both capacity utilization and investment accelerator are huge. In fact, the first dynamic variable, exceeds the economic logic, and seems to provide an over-used capacity utilization, which is not accepted, like in Figure 10. Besides, both variables need a much longer time to converge to the steady state value. The third is the consideration of unitary elasticity. The two results are similar to the previous one, but a lot more smoothie and the convergence is faster (but reasonable). The last graphic presents a model when the elasticity is large, but not perfect. It seems that if we consider the higher cost to hire or fire the workers less oscillation is resulting. In this case, the model adjusts the capacity utilization to its normal level in a reasonable amount of time as well as the investment accelerator. This approximates the post-Keynesian vision but is not equal to it.

As we can see, this section demonstrates, graphically, the mathematical results of the previous one. Besides the first case, where a perfectly inelastic assumption is considered, it seems that the model converges to their steady-state values and maintains its stable conditions.

5. Concluding Remarks

Our new framework contributes, at least, in four contributions:

1st – We introduced the labor market perspective to the HSSM model looking for break of a *tabu* in this kind of framework, which usually only take into account for the capital restriction. We incorporated the assumption of labor demand and average wage to the approach. For future studies, it is possible to expand the idea by including the government and economic policies, especially imposing a minimum level of wage to observe the behavior of the effective demand.

2nd - We developed a new formulation considering the implications of wage elasticity of labor demand and the growth rate of average wage. These new assumptions make it possible the study of how the interaction between the labor and capital, in a Leontieff productivity function, affects the determination of capital accumulation and the convergence of capacity utilization to its normal level. These proposals, in this kind of models, reinforces the originality of the framework.

3rd – Our approach shows that labor is a restrictive statement since the equilibrium and limit cycle depend on the level of its elasticity. Furthermore, as we proved using computational simulation, it also affects the speed of the adjustment and the dynamics of the cycle. So, we cannot simply ignore the existence and effects of such a hypothesis, assuming, that the labor adjusts itself to the capital.

4th – Our chapter raises additional questions about this kind of restriction, such as: what happens if we restrict the labor supply? Is it possible to consider a minimum level of wage in our model, to understand the implications of a universal basic income in this kind of approach? How much the advancements in technology will affect the demand for workers? What do we have to do to avoid higher levels of unemployment if, as an extreme result, we suppose that technology substitutes all the workforce? And so on.

Despite Dvoskin and Landau (2023), Nikiforos, Santetti and Armin (2023) also Summa, Petrini and Teixeira (2023) having dealt with cycles, they do not presented a mathematical proof of the bifurcation, which is a necessary condition to ensure it. Their interest is to discuss, among other issues, a new interpretation of the HSSM considering different types of autonomous component, which is central for then, but is not for us. Our desire is to open new windows to the present theory, by creating a new environment where the labor is central to the discussion. Furthermore, our new extension also generates new opportunities to design economic policies that ensure dignity to the workers class. Therefore, the advance in the post-Keynesian theory will have to pass by the labor market, or else, central questions like the ones presented above, especially related to the 21st century, will continue without answers creating a hole without a bridge between the heterodox approaches, and the mainstream.

CHAPTER 5 - CONCLUDING REMARKS

The main conclusions of this Thesis were presented at the end of each chapter. The first one presents a historical overview of the development of the SSM. Such a framework is the origin of an international debate about the possibility or not the model to guarantee the capacity utilization convergence to its normal level. This is not the only contribution, at that part, we had the opportunity to raise some questions about the fragilities of the model, especially the ones that deal with the labor market and autonomous components. This chapter presents the basis for the next ones, justifying all our research agenda proposed three years ago.

Besides, is it possible to believe that this kind of framework has the potential to be the closure to the role of post-Keynesian perspective? In our view is not. The model presents a number of fragilities and the consensus between economists does not exist. In this vein, hardly this approach, in its current form, will be strong enough to compete with the traditional economic theory or even to the classical long period post-Keynesian perspective, such as the Cambridge Theory, neo-Kaleckian or post-Kaleckian frameworks. Therefore, the SSM contributes as a special alternative result to the present literature.

The second chapter came to answer a question raised before: is it possible to propose an alternative autonomous component to the SSM? In this vein, we recommend that the exports can be a more efficient one since this variable depends on the international income and this is autonomous to the national product. Thus, we design a new dynamic system, in a 3D perspective, and proved the stable conditions to two of three propositions, guaranteeing the efficiency of our new case. Furthermore, we attested that an endogenous cycle exists for each case, which shows that a single point exists and is the limit of the stable condition. If this value exceeds, the model falls to the Harodian Instability. After exhaustive research enquire, we did not found another work that proves such a condition, which is our central contribution here. Besides, we also provide a numerical analysis to demonstrate how the model behaves.

Still, about this chapter, our approach shows the possibility to link the SSM with the Thirlwall's Law. This perspective can be expanded by analyzing the effects of different prices in uneven and developed economies, since, our major point is to consider exports autonomous. In this vein, we demonstrated that there exist a plausible alternative assumption for AC, and we also provided the necessary tools to a new research agenda which improve the original framework and probably will satisfy both critics and defenders of the SSM approach.

The third chapter reinforces another fragility presented in the beginning of the Thesis concerning the labor market. Here, we show how the model behaves when a labor restriction is imposed. The model will generate a labor shortage and increase the excess demand. According to the introduction and the conclusion of that chapter, our results agree with the ones presented by the IMF and World Bank empirical evidence. This is in line with (although not equal) Robinson's (1962) "restrained golden age", where the growth is restricted by the population growth rate.

We are here presenting a gap, not only for the SSM, but to all post-Keynesian theory. Usually it assumes that Lewis's perspective of Labor supply is convenient to this type of framework. Actually, empirical evidences demonstrates that this solution does not match to the real world. Thus, in this chapter we are restricted to the original Supermultiplier, which can be also extend to the neo-Kaleckian, post-Kaleckian and Classical Cambridge Theory. The major problem is related to a non-modeled investment function like in Tobin's q. In this vein, it is necessary to propose a new function incorporating the effects of interaction between labor and capital to correct such a gap. Thus, this is the justification and theoretical basis for the fourth chapter, which demonstrates, more accurately how the labor market should behave.

In chapter four we design three different situations in the labor market. The first one shows how, usually, the post-Keynesians consider the labor supply perfectly elastic. The second characterizes the perception of neo-classical researchers when a perfectly inelastic assumption is assumed. The third, our contribution, deals with a more flexible elasticity. We believe that this is a more realistic solution and fits real economies. So, we improved the model presented in chapter three by introducing the elasticity of labor to the capacity utilization which directly affect the dynamic system. Furthermore, our model generates an endogenous cycle, converging the capacity utilization to its normal level, if and only if the elasticity is bigger than one. It provides the necessary and sufficient condition to ensure that both cases, neo-classical and post-Keynesians, should demonstrate more interest about the elasticity degree and the implications of a labor market to their models. This result, was not investigated by any other author and is a great topic to a new agenda.

Moreover, we demonstrated an enormous fragility of the workers force. If, the capitalists continue to improve their technology the political power of the worker's class will be reduced to zero. This will occur since capitalists will have the power to choose between guarantee high level of investments in capital (ensuring such advancements in technology or AI) or to pay higher levels of wages to workers. Thus, if the labor union manifest criticisms to the new empowered capitalist they will have the power to eliminate competition by dismissing workers and substituting then by machines. However, as presented in that chapter, the convergence of the capacity utilization to its normal level in the case of lower wages is slow. This is a huge social price to pay if capitalists solely dominate the system.

With these results, this Thesis accomplishes the current literature by demonstrating the fragilities of the SSM, but presenting alternatives to the debate. For us, both sides, critics and defenders of the model, present important points about the framework. It seems that our alternative approach is plausible and closer to the post-Keynesian long-period line of thinking. Naturally, the framework still has to be improved, eliminating the fragilities in the current debate.

REFERENCES

ACEMOGLU, D.; RESPRETO, P. Automation and New Tasks: How Technology Changes Labor Demand. **Journal of Economic Perspectives,** v. 33, n. 2, p. 3–30, 2019.

ACEMOGLU, D. Remaking the Post-COVID World. **IMF – Finance & Development,** Spring 2021. Acessado em 27 de Fevereiro de 2023. Disponível em: < https://www.imf.org/external/pubs/ft/fandd/2021/03/COVID-inequality-and-automation-acemoglu.htm>

ALLAIN, O. Talcking the Instability of Growth: a Kaleckian-Harrodian model with an Autonomous Expenditure Component. **Cambridge Journal of Economic**, Cambridge v. 39, n. 5, p. 1351-1371, 2014.

ALLAIN, O. Demographic growth, Harrodian (in)stability and the Supermultiplier. **Cambridge Journal of Economics,** v. 43, n. 1, 85-106, 2019

AMADEO, E. Notes on Capacity Utilization, Distribution and Accumulation. **Contributions to Political Economy**, v. 5, p. 83-94, 1986.

ARROW, K. J. The Economic Implications of Learning by Doing. **Review of Economic Studies**, v. 29, p. 155-173, 1962.

BHADURI, A.; MARGLIN, S. Unemployment and the Real Wage: the Economic Basis for Contesting Political Ideologies. **Cambridge Journal of Economics.** v. 14, n. 4, p. 375-393, 1990.

BLECKER, R. A.; SETTERFIELD, M. **Heterodox Macroeconomics: Models of Demand, Distribution and Growth**. Cheltenham: Edward Elgar Publishing Limited, 2019.

BORTIS, H. **Institutions, Behaviour and Economic Theory - A Contribution to Classical-Keynesian Political Economy**. Cambridge: Cambridge University Press, 1997.

CAMPOS, A. Futuro é preocupante e não vamos ter como competir com ia, diz vencedor do nobel de economia. **Valor Econômico**, 2023

CESARATTO, S. Neo-Kaleckian and Sraffian Controversies on the Theory of Accumulation. **Review of Political Economy**, v. 27, n. 2, p. 154-182, 2015.

DÁVILA-FERNANDEZ, M. J.; OREIRO, J. L.; PUNZO, L. F. Inconsistency and Over-determination in neo-Kaleckian Growth Models: A note. **Metroeconomica**, v. 70, n. 2, 313-319, 2019.

DE-JUAN, Ó. Paths of Accumulation and Growth: Towards a Keynesian Long-Period Theory of Output. **Review of Political Economy**, v. 17, n. 2, p. 231-252, 2005.

DIAMOND, P.A. National Debt in a Neoclassical Growth Model. **American Economic Review**, v. 55, p. 1126-1150, 1965.

DOMAR, E. Capital Expansion, Rate Growth, and Employment. **Econometrica.** v. 14, n. 2, p. 137-147, 1947.

DUTT, A. K. Stagnation, Income Distribution and Monopoly Power. **Cambridge Journal of Economics** v. 8, n. 1, p. 25-40, 1984.

DUTT, A. K. Autonomous demand growth, distribution, and fiscal and monetary policy in the short and long-runs. In Bougrine, H. and Rochon, L-P., **Economic Growth and Macroeconomic Stabilization Policies in Post Keynesian Economics,** 2020.

DUVAL, R.; OIKONOMOU, M.; TAVARES, M. M. Tight Jobs Market Is a Boon for Workers But Could Add To Inflation Risks. **IMF Working Series**, 2022.

DVOSKIN, A.; LANDAU, M. T. Income Distribution and Economic Cycle in an Open Economy Supermultiplier model. **Economic Research Working Paper,** n. 102, 2022.

DYER-WITHEFORD, N.; KJØSEN, A. M.; STEINHOFF, J. **Inhuman power: artificial intelligence and the future of capitalism.** London: Pluto Press (UK), 2019.

ENGELS, F. **Principles of communism**. Pattern Books, 2020

FEU, A. Evolution of the Capital/Product Ratio in Brazil and in the OECD Countries. **Economy & Energy,** n. 28, 2001.

GANDOLFO, G. **Economic Dynamics: Methods and Models.** New York: Elsevier Science Publishing Company Inc, 1996.

GAREGNANI, P. **Il Problema Della Demanda Effetiva Nello Svillupo Economico Italiano.** Roma: SVIMEZ, 1962.

GRAHAM, C. Robot Performs First Laparoscopic Surgery Without Human Help. **HUB – John Hopkins University**, 26 de Janeiro de 2022. Acessado em 27 de Fevereiro de 2023. Disponível em: < https://hub.jhu.edu/2022/01/26/star-robot-performs-intestinal-surgery/>

HARROD, R. An Essay in Dynamic Theory. **The Economic Journal**, v. 49, n. 193, p. 14-33, 1939.

HARROD, R. Notes on trade cycle theory. **The Economic Journal**, v. 61, n. 242, p. 261-275, 1951.

HARROD, R. **The Trade Cycle.** New York: Augustus M. Kelley Publisher, 1965.

HEIN, E. **Distribution and growth after Keynes:** a post-Keynesian guide. Cheltenha: Edward Elgar, 2014.

HICKS, J. Keynes' Theory of Employment. **The Economic Journal**, v. 46, n. 182, p. 238-253, 1936.

HICKS, J. **A Contribution to the Trade Cycle**. Oxford: Clarendon, 1950.

HUERGAS, E. F-.; ARIAS, J. G-. The post-keynesian view on labour demand in micro-and macroeconomic fields. **Economics & Sociology,** v. 12, n. 2, p. 109-128, 2019.

KALDOR, N. A Model of Trade Cycle. **The Economic Journal**, v. 50, n. 197, p. 78-92, 1940.

KALDOR, N. Alternatives Theories of Distribution. **The Review of Economic Studies.** v. 49, n. 193, p. 14-33, 1956.

KALECKI, M. **A theory of commodity, income and capital taxation.** Oxford: Oxford University Press, 1937.

KEYNES, J. M. **A Tract on Monetary Reform.** Cambridge: Cambridge University Press, 1923.

KEYNES, J. M. **The General Theory of Employment, Interest and Money.** London: Macmilian, 1936.

LAVOIE, M. Convergence Towards the Normal Rate of Capacity Utilisation in Neo Kaleckian Models: The Role of Non-Capacity Creating Autonomous Expenditures. **Metroeconomica**, v. 67, n. 1, p. 172-201, 2015.

LAVOIE, M. Prototypes, Reality and the Growth Rate of Autonomous Consumption Expenditure: a Rejoinder. **Metroeconomica**, n. 68, n. 1, p. 194-199, 2016.

LAVOIE, M. Inconsistencies in the note of Dávila-Fernández, Oreiro and Punzo. **Metroeocnomica.** n. 70, v. 2, p. 320-324, 2019.

LEWIS, W. A. Economic Development with Unlimited Supplies of Labour. **The Manchester School,** vol. 22, 139-191, 1954.

MARGLIN, S. Growth, Distribution, and Inflation: a Centennial Synthesis. **Cambridge Journal of Economic,** v. 8, p. 115-144, 1984.

MARGLIN, S. **Raising Keynes: A Twenty-First Century General Theory**. Cambridge: Harvard Press, 2021.

MCCOMBIE, J. Economic Growth, the Harrod Foreign Trade Multipler and the Hicks' Supermultiplier. **Applied Economics**, v. 17, n. 1, 1985.

NAH, W. J.; LAVOIE, M. Long-run Convergence in a neo-Kaleckian Open-Economy Model with Autonomous Export Growth. **Journal of Post Keynesian Economics**, v. 40, n. 2, p. 223-238, 2017.

NIKIFOROS, M. Some comments on the Sraffian Supermultiplier approach to growth and distribution. **Journal of Post-Keynesian Economics,** v. 41, n. 4, p. 659-675, 2018.

NIKIFOROS, M.; SANTETTI, M.; VON ARMIN, R. The Sraffian Supermultiplier and Cycles: Theory and Empirics. **Review of Political Economy**, v. 00, n. 00, p. 1-20, 2023.

OBRINSKY, M. **Profit Theory and Capitalism.** Berlin: De Gruyter, 1983.

OREIRO, J. L. C.; SILVA, G. J. C.; SANTOS, J. F. C. The Debate about Sraffian Supermultiplier Model and the Future of Heterodox Growth Models. **Brazilian Journal of Political Economy.** v. 40, n. 3, p. 510-531, 2019.

OREIRO, J. L. C.; SANTOS, J. F. C. The Impossible Quartet in a Demand Led Growth Supermultiplier Model for a Small Open Economy. **Unpublished paper,** 2021**.**

PALLEY, T. I. The Contradictions of Export-Led Growth. **Public Policy Brief.** New York: Levy Economics Institute, 2012.

PALLEY, T. I. Unemployment and Growth: Putting Unemployment into Post-Keynesian Growth Theory. **Review of Political Economy**, v. 31, p. 194-215, 2019.

PASINETTI, Luigi. Rate of Profit and Income Distribution to the Rate of Economic Growth. **The Review of Economic Studies.** v. 29, n. 4, p. 267-279, Oct. 1962.

PRETTNER, K.; BLOOM, D. The Macroeconomic Effects of Automation and the Role of COVID-19 in Reinforcing Their Dynamics. **Centre for Economic Policy Research,** 25 de Junho de 2020. Acessado em: 27 de Fevereiro de 2023. Disponível em: < https://cepr.org/voxeu/columns/macroeconomic-effects-automation-and-role-covid-19-reinforcing-their-dynamics>

ROBINSON, J. The Production Function and the Theory of Capital. **Review of Economic Studies**, v. 21, n. 2, p. 81-106, 1953.

ROBINSON, J. **Essay in the Theory of Economic Growth**. London: The Macmillan Press LTD, 1962.

ROSE, H. 2018. Aggregate Demand and Supply Analysis. In **The New Palgrave Dictionary of Economics.**

ROWTHORN, R. Demand, Real Wage and Economic Growth. **Thames Paper in Political Economy**, p. 1-39, 1981.

SCHWABE, H.; CASTELLACCI, F. Automation, Workers' Skills and Job Satisfaction. **PLOS ONE,** v. 15, n. 11, p. 1-26, 2020.

SEMUELS, A. Millions of Americans Have Lost Jobs in the Pandemic—And Robots and AI Are Replacing Them Faster Than Ever. **TIME,** 06 de Agosto de 2020. Acessado em: 27 de Fevereiro de 2023. Disponível em: < https://time.com/5876604/machines-jobs-coronavirus/>

SERRANO, F. Long Period Effective Demand and the Sraffian Supermultiplier. **Contributions to Political Economy**, v. 14, n. 0, p. 67-90, 1995a.

SERRANO, F. **The Sraffian Multiplier.** PhD Thesis, Cambridge: University of Cambridge, 1995b.

SERRANO, F.; FREITAS, F. The Sraffian Supermultiplier as an Alternative Closure for Heterodox Growth Theory. **Working Paper**, 2015.

SERRANO, F.; FREITAS, F. The Sraffian Supermultiplier as an Alternative Closure for Heterodox Growth Theory. **European Journal of Economics and Economic Policies: Intervention**, v. 14, n. 1, p. 70-91, 2017.

SERRANO, F.; FREITAS, F.; BHERING, G. The Trouble with Harrod: The Fundamental Instability of the Warranted Rate in the Light of the Sraffian Supermultiplier. **Metroeconomica**, v. 70, n. 2, p. 263-287, 2019.

SERRANO, F.; SUMMA, R.; MOREIRA, V. G. Stagnation and Unnaturally Low Interest Rate: a Simple Critique of the Amended New Consensus and the Sraffian Supermultiplier Alternative. **Review of Keynesian Economics.** v. 8, n. 3, p. 365-384, 2020.

SIMONSEN, M. H. **Macroeconomia.** São Paulo: ANPEC, 1988.

SKOTT, P. Growth, instability and cycles: Harrodian and Kaleckian models of accumulation and income distribution. In Setterfield, M. **Handbook of Alternative Theories of Economic Growth,** Cheltenham: Edward Elgar, 2010.

SKOTT, P. Theoretical and empirical shortcomings of the Kaleckian investment function. **Metroeconomica**, v. 63, n. 1, p. 109-138, 2012.

SKOTT, P. Autonomous Demand and the Harrodian Criticism of the Kaleckian Model. **Metroeconomica**, v. 68, n. 1, p. 185-193, 2017.

SKOTT, P. Autonomous Demand, Harrodian Instability and the Supply Side. **Metroeconomica**, v. 70, n. 2, p. 233-246, 2019.

SKOTT, P.; SANTOS, J. F. C.; OREIRO, J. L. Supermultipliers, Endogenous Autonomous Demand and Functional Finance. **Metroeconomica**, v. 73, n. 1, p. 220-244, 2022.

SOLOW, R. A Contribution to the Theory of Economic Growth. **The Quarterly Journal of Economics.** v. 70, n. 1, p. 65-94, 1956.

SRAFFA, P. **Production of Commodities by Means of Commodities**. Cambridge: Cambridge University Press, 1960.

SUMMA, R. An Alternative Model to Open-Economy “New Consensus” for Analysis of Inflation Targeting. **EconomiA.** v. 17, n. 3, p. 310-323, 2016.

SUMMA, R.; PETRINI, G.; TEIXEIRA, L. Cycles: Empirics and the Supermultiplier. **Review of Political Economy,** v. 00, n. 00, p. 1-23, 2023.

THIRLWALL, A. **The nature of economic growth.** Aldershot: Edward Elgar, 2002.

TUDELA, C. C.-; CLYMO, A.; MUNRO, D. Z.-. Over-50s are resigning en masse – new research explains who and why. **The Conversation**, 2022.

VERCHERAND, J. The neoclassical model of labor market. In **Labor: A heterodox approach.** New York: Palgrave MacMillan, 2014.

WOOD, A. Kaldor, Nicholas (1908-1986). In STEVEN, N. D.; LAWRENCE, E. B. The **New Palgrave Dictionary of Economics (3rd ed.)**. Palgrave, 2018. p. 7207-7216.

APPENDIX A: Chapter 2

a) First case $g_A = g_Z = g_X$

1st case: $0 < \alpha^* < 1$, if and only if, $g_Z = g_X$; $u^* = \mu$; $h^* = \frac{g_A v}{\mu}$, thus (16) will have the following Jacobian:

$$\begin{bmatrix} \dot{h} \\ \dot{u} \\ \dot{\alpha} \end{bmatrix} = \begin{bmatrix} 0 & \gamma \frac{g_A v}{\mu} & 0 \\ -\frac{\mu^2}{v} & g_A(2v\gamma\sigma - 1) & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad (16')$$

To prove the stability, we need to bear in mind that the Routh-Hurwitz criterion states that $P^* = (h^*,\ u^*,\ \alpha^*)$ is asymptotically stable if and only if $S_1(P^*) > 0$, $S_2(P^*) > 0$, $S_3(P^*) > 0$, and $E(P^*) = S_1(P^*)\, S_2(P^*) - \ S_3(P^*) > 0$, where: $S_1(P^*)$, $S_2(P^*)$, and $S_3(P^*)$ are the coefficients of the characteristic polynomial of matrix *J* evaluated at P^* which is given by: $\lambda^3 + \lambda^2 S_1 + \lambda S_2 + S_3 = 0$

Where $S_1(P^*) = -trJ(P^*)$, $S_2(P^*) = \det(J_1)\,(P^*) + \det(J_2)\,(P^*) + \det(J_3)\,(P^*)$ and $S_3(P^*) = -detJ(P^*)$.

Since we have a column and/or a line of the matrix null, we have the $S_3 = -|J| = 0$, which cannot guarantee the stability condition and will be discarded by us.

b) Second case: $g_Z < g_X$

Considering $\alpha^* = 0$; $u^* = \mu$; $h^* = \frac{g_A v}{\mu}$, thus (16) will have the following configuration:

$$\begin{bmatrix} \dot{h} \\ \dot{u} \\ \dot{\alpha} \end{bmatrix} = \begin{bmatrix} 0 & \gamma \frac{g_x v}{\mu} & 0 \\ -\frac{\mu^2}{v} & g_x(2v\gamma\sigma - 1) & 0 \\ 0 & 0 & g_Z - g_X \end{bmatrix}$$

Approaching the Routh-Hurvitz Theorem we determine the following characteristic polynomial, thus:

$$\lambda^3 + \lambda^2 S_1 + \lambda S_2 + S_3 = 0$$

Following the same criteria presented in Appendix A, we have the results of S_1, S_2, S_3, and E. Thus:

$S_1 = -Tr(J) = -g_x 2v\gamma\sigma - g_Z + 2g_X > 0$

If, and only if: $\gamma < \frac{1}{\sigma v} - \frac{g_Z}{2g_x\sigma v}$

Since the theory define $\gamma > 0$, it is necessary to admit: $g_x > \frac{g_Z}{2}$

Second: $S_2 = \sum|M_i| > 0$

$S_2 = \gamma\frac{g_x v}{\mu}\left(\frac{\mu^2}{v}\right) + [g_x(2v\gamma\sigma - 1)(g_Z - g_X)] > 0$

Being $\gamma > 0$, it is necessary to admit: $\frac{g_X - g_Z}{\mu + 2v\sigma + 2v\sigma g_Z} > 0$. If, and only if: $g_X > g_Z$

Third: $S_3 = -|J| > 0$

$S_3 = [\gamma g_X \mu(g_X - g_Z)] > 0$

Where $\gamma > 0$ and $g_X > g_Z$, which guarantee the stability so far. As a final condition we have:

$$E = S_1 S_2 - S_3 > 0$$
$$E = g_X^2(2v\sigma\gamma - 1)(g_Z - g_X) + g_X(g_Z - g_X)^2 - g_X^2\gamma\mu > 0 \qquad (*)$$

The positive result is possible if, and only if: $\gamma < \frac{(g_Z - g_X)(g_X^2 - g_Z + g_X)}{[g_X 2v\sigma(g_Z - g_X) - g_X^2\mu]}$

Since $\gamma > 0$, then: $\frac{(g_Z - g_X)(g_X^2 - g_Z + g_X)}{[g_X 2v\sigma(g_Z - g_X) - g_X^2\mu]} > 0$. From this result we need: $g_z < g_X$

To study the existence of limit cycles let us adopt the Hopf bifurcation theorem [see Gandolfo (1996)] for the system by using $\gamma > 0$ as the bifurcation parameter. We have to show first that there exists $\gamma > 0$ such that $E(P^*, \gamma) = 0$. By equalizing expression (*) to zero, and solving for γ, we obtain after some algebraic manipulation a quadratic expression, namely:

$$E = g_X^2(2v\sigma\gamma - 1)^2(g_Z - g_X) + g_X(2v\gamma\sigma - 1)(g_Z - g_X)^2 - g_X^2\gamma\mu(2v\gamma\sigma - 1) = 0$$
$$E = -\gamma^2\{4v\sigma g_X^2[v\sigma(g_Z - g_X) + 2\mu]\} + \gamma[2v\sigma g_X g_Z(g_Z - g_X) + g_X^2\mu] - [g_X^2(g_Z - g_X) + g_X(g_Z - g_X)^2] = 0$$

Defining:

$$\{2g_Z^2 v\sigma[2v\sigma(g_Z - g_X) - \mu]\} = a < 0\{2g_Z^2 v\sigma[2v\sigma(g_Z - g_X) - \mu]\} = a < 0$$
$$[2v\sigma g_X g_Z(g_Z - g_X) + g_X^2\mu] = b > 0[2v\sigma g_X g_Z(g_Z - g_X) + g_X^2\mu] = b > 0$$
$$g_X g_Z(g_Z - g_X) = c < 0g_X g_Z(g_Z - g_X) = c < 0$$

We have:

$$E = -\gamma^2 a + \gamma b - c = 0E = -\gamma^2 a + \gamma b - c = 0$$

To this case, $\Delta = b^2 - 4ac > 0$, if and only if: $b^2 > 4ac$, thus, the bifurcation parameter is determine by: $\gamma^* = -\frac{b}{2a} \pm \frac{\sqrt{\Delta}}{2a} > 0$. Therefore, the only possible result of this case is: considering $\gamma^* = -\frac{b}{2a} + \frac{\sqrt{\Delta}}{2a}$, where $\frac{\sqrt{\Delta}}{2a} > \frac{b}{2a}$, we have the result of $E(P^*, \gamma) = 0$ when $\gamma > 0$. Proving the existence of a critical point to the model, we can force $\gamma = \frac{(g_X - g_Z)(g_Z^2 - g_X + g_Z)}{[g_Z 2v\sigma(g_X - g_Z) - g_Z^2\mu]}$, thus, we have $E = 0$. To find the limit cycle, we need to apply the derivation of E with respect to γ, obtaining:

$$\frac{\partial E}{\partial \gamma} = 2v\sigma g_X^2(g_Z - g_X) - g_X^2\mu$$

Evaluating $\frac{\partial E}{\partial \gamma}$ at $\gamma = \frac{(g_X - g_Z)(g_Z^2 - g_X + g_Z)}{[g_Z 2v\sigma(g_X - g_Z) - g_Z^2\mu]}$, it yields $\frac{\partial E}{\partial \gamma} = \frac{g_X^2\mu}{2vg_X^2(g_Z - g_X)} \neq 0$, which guarantees inequality and the limit cycle.

c) Third case: $g_Z > g_X$

Considering $\alpha^* = 1; u^* = \mu\,;\ h^* = \frac{g_A v}{\mu}$. This configuration impacts the result of (8), which: $g_A = g_Z$, affecting the investment accelerator in optimal $h^* = \frac{g_Z v}{\mu}$. Thus, reconfiguring the Jacobian Matrix, we have:

$$\begin{bmatrix} \dot{h} \\ \dot{u} \\ \dot{\alpha} \end{bmatrix} = \begin{bmatrix} 0 & \gamma \frac{g_Z v}{\mu} & 0 \\ -\frac{\mu^2}{v} & g_Z(2v\gamma\sigma - 1) & 0 \\ 0 & 0 & g_X - g_Z \end{bmatrix} \qquad (16''')$$

Since the Routh-Hurvitz Theorem is determined by:

$$\lambda^3 + \lambda^2 S_1 + \lambda S_2 + S_3 = 0$$

First, we have to obtain $S_1 = -Tr(J) > 0$. Thus:

$$S_1 = -Tr(J) = -g_Z 2v\gamma\sigma - g_X + 2g_Z > 0$$

If, and only if: $\gamma < \frac{g_X}{2g_Z\sigma v} - \frac{1}{\sigma v}$. Since we want $\gamma > 0$, it is necessary to admit: $g_Z > \frac{g_X}{2}$

Second: $S_2 = \sum |M_i| > 0$

$$S_2 = \gamma \frac{g_Z v}{\mu}\left(\frac{\mu^2}{v}\right) + [g_Z(2v\gamma\sigma - 1)(g_X - g_Z)] > 0$$

If, and only if: $g_Z > g_X$.

Third: $S_3 = -|J| > 0$

$$S_3 = \left[\gamma \frac{g_Z v}{\mu}\frac{\mu^2}{v}(g_Z - g_X)\right] > 0$$

Where $\gamma > 0$ and $g_Z > g_X$, which guarantee the stability so far. As a final condition we have:

$E = S_1 S_2 - S_3 > 0 E = S_1 S_2 - S_3 > 0$
$E = g_Z^2(2v\sigma\gamma - 1)(g_X - g_Z) + g_Z(g_X - g_Z)^2 - g_Z^2 \gamma\mu > 0$

The positive result is possible only if: $\gamma < \frac{(g_X - g_Z)(g_Z^2 - g_X + g_Z)}{[g_Z 2v\sigma(g_X - g_Z) - g_Z^2 \mu]}$. We have $\gamma > 0$, then: $\frac{(g_X - g_Z)(g_Z^2 - g_X + g_Z)}{[g_Z 2v\sigma(g_X - g_Z) - g_Z^2 \mu]} > 0$. Resulting in: $g_X < g_Z$.

Following the second case, now we have to prove the critical point. Firstly that there exists $\gamma > 0$ such that $E(P^*, \gamma) = 0$.

$E = \gamma^2\{2g_Z^2 v\sigma[2v\sigma(g_X - g_Z) - \mu]\} + \gamma\{2v\sigma g_Z(g_X - g_Z)[g_X - g_Z] + g_Z^2\mu\} + g_Z g_X^2(g_X - g_Z) = 0$

Defining:

$\{2g_Z^2 v\sigma[2v\sigma(g_X - g_Z) - \mu]\} = a < 0$
$g_Z(g_X - g_Z)(2g_Z - g_X) = c < 0$

Thus:

$E = \gamma^2 a + \gamma b + c$

Applying the same approach presented in case 2, we have: $\Delta = \mathrm{b}^2 - 4ac$, $4ac < 0$, and $\mathrm{b}^2 > 0$, thus: $\Delta > 0$ if, and only if $\mathrm{b}^2 > 4ac$. We obtain: $\gamma^* = -\frac{b}{2a} \pm \frac{\sqrt{\Delta}}{2a} > 0$ if, and only if, $\sqrt{\Delta} > b$. Proving the bifurcation to this case too. Now, we have to Force $= \frac{(g_X - g_Z)(g_Z^2 - g_X + g_Z)}{[g_Z 2v\sigma(g_X - g_Z) - g_Z^2 \mu]}$, thus, we have $E = 0$. To find the limit cycle, we need to apply the derivation of E with respect to γ, obtaining:

$\frac{\partial E}{\partial \gamma} = 2v\sigma g_Z^2(g_X - g_Z) - g_Z^2\mu \neq 0$

Being $\sigma \neq \frac{g_Z^2 \mu}{2v g_Z^2(g_X - g_Z)}$, we have the desired result, which guarantees inequality and the limit cycle.

APPENDIX B: Chapter 4

a) Steady-state values:

1st - $u^* = \mu$; assuming $\dot{h} = 0$, we have:

$$0 = \gamma(u-\mu)h \rightarrow u^* = \mu$$

2nd - $h^* = \left[g_A\left(2 - \frac{1}{e_W}\right) - g_w\right]\left(\frac{v}{\mu}\right)$; assuming $\dot{u} = 0$, thus:

$$0 = \left\{\left[g_A + \frac{\gamma(u-\mu)h}{\alpha}\right]\left(\frac{3-e_L}{2}\right) + \frac{g_w}{2} - \frac{hu}{v}\right\}u$$

$$0 = \left[g_A + \frac{\gamma(u-\mu)h}{\alpha}\right]\left(\frac{3-e_L}{2}\right) + \frac{g_w}{2} - \frac{hu}{v}$$

$$0 = g_A\left(\frac{3-e_L}{2}\right) + \frac{g_w}{2} - \frac{hu}{v}$$

$$h^* = \left[g_A\left(\frac{3-e_L}{2}\right) + \frac{g_w}{2}\right]\left(\frac{v}{\mu}\right)$$

b) Mathematical proof of Proposition 1:

Following Equation (15), we have the Jacobian Matrix below:

$$J(P^*) = \begin{bmatrix} 0 & \gamma\left[g_A\left(\frac{3-e_L}{2}\right) + \frac{g_w}{2}\right]\left(\frac{v}{\mu}\right) \\ -\frac{\mu^2}{v} & \left[g_A\left(\frac{3-e_L}{2}\right) + \frac{g_w}{2}\right]\left(\frac{v}{\mu}\right)\left[\left(\frac{3-e_L}{2}\right)\frac{\gamma\mu}{\alpha} - \frac{2\mu}{v}\right] + \left(\frac{3-e_L}{2}\right)g_A + \frac{g_w}{2} \end{bmatrix}$$

According to Gandolfo (1996), to prove the stability, we need to bear in mind that the Routh-Hurwitz criterion states that $P^* = (h^*,\ u^*,)$ is asymptotically stable if and only if $S_1(P^*) = Tr(J) > 0$ and $S_2(P^*) = Det(J) > 0$, where: $S_1(P^*)$ and $S_2(P^*)$ are the coefficients of the characteristic polynomial of matrix *J* evaluated at P^* which is given by:

$$\lambda^2 + \lambda S_2 + S_3 = 0$$

Thus, we can prove that the following proposition is stable if:

$$Tr(J) = \left[g_A\left(\frac{3-e_L}{2}\right) + \frac{g_W}{2}\right]\left(\frac{v}{\mu}\right)\left[\left(\frac{3-e_L}{2}\right)\frac{\gamma\mu}{\alpha} - \frac{2\mu}{v}\right] + \left(\frac{3-e_L}{2}\right)g_A + \frac{g_W}{2} < 0$$

$$\left[g_A\left(\frac{3-e_L}{2}\right) + \frac{g_W}{2}\right]\left(\frac{v}{\mu}\right)\left[\left(\frac{3-e_L}{2}\right)\frac{\gamma\mu}{\alpha} - \frac{2\mu}{v}\right] < -\left(\frac{3-e_L}{2}\right)g_A - \frac{g_W}{2}$$

$$\left(\frac{3-e_L}{2}\right)\frac{\gamma\mu}{\alpha} - \frac{2\mu}{v} < \frac{-\left(\frac{3-e_L}{2}\right)g_A - \frac{g_W}{2}}{\left[g_A\left(\frac{3-e_L}{2}\right) + \frac{g_W}{2}\right]\left(\frac{v}{\mu}\right)}$$

$$\left(\frac{3-e_L}{2}\right)\frac{\gamma\mu}{\alpha} < \frac{2\mu}{v} - \frac{\left[\left(\frac{3-e_L}{2}\right)g_A + \frac{g_W}{2}\right]}{\left[g_A\left(\frac{3-e_L}{2}\right) + \frac{g_W}{2}\right]\left(\frac{v}{\mu}\right)}$$

$$\left(\frac{3-e_L}{2}\right)\frac{\gamma\mu}{\alpha} < \frac{\mu}{v} \rightarrow \frac{3-e_L}{2} < \frac{\mu}{v}\left(\frac{\alpha}{\mu\gamma}\right) \rightarrow -e_L < \frac{2\alpha}{v\gamma} - 3$$

$$e_L > 3 - \frac{2\alpha}{v\gamma}$$

$$Det(J) = \gamma\left[g_A\left(\frac{3-e_L}{2}\right) + \frac{g_W}{2}\right]\left(\frac{v}{\mu}\right)\left(\frac{\mu^2}{v}\right) > 0$$

$$\gamma\left[g_A\left(\frac{3-e_L}{2}\right) + \frac{g_W}{2}\right]\left(\frac{v}{\mu}\right)\left(\frac{\mu^2}{v}\right) > 0$$

$$g_A\left(\frac{3-e_L}{2}\right) + \frac{g_W}{2} > 0 \rightarrow \frac{3-e_L}{2} > -\frac{g_W}{2g_A} \rightarrow -e_L > -\frac{g_W}{g_A} - 3$$

$$e_L < 3 + \frac{g_W}{g_A}$$

Since $-\gamma\left[g_A\left(\frac{3-e_L}{2}\right) + \frac{g_W}{2}\right]\left(\frac{v}{\mu}\right)\frac{\mu^2}{v} = -\gamma\left[g_A\left(\frac{3-e_L}{2}\right) + \frac{g_W}{2}\right]\mu \neq 0$, thus, our model is globally stable.

Q.E.D.

c) Mathematical proof of Proposition 2:

According to Gandolfo (1996), we have:

Let $y_1 = y_1(t)$, $y_2 = y_2(t)$ be a periodic motion (limit cycle) of system (3.33). This limit cycle is locally stable (unstable) if its characteristic exponent

$$h = \frac{1}{T}\int_0^T \left\{\frac{\partial\varphi_1[y_1(t), y_2(t)]}{\partial y_1} + \frac{\partial\varphi_2[y_1(t), y_2(t)]}{\partial y_2}\right\} dt$$

is respectively negative (positive) (GANDOLFO, p. 444-445, 1996)

So,

$\frac{\partial\varphi_1[y_1(t), y_2]}{\partial y_1} = 0$

And

$\frac{\partial\varphi_2[y_1(t), y_2]}{\partial y_2} = \left[g_A\left(\frac{3-e_L}{2}\right) + \frac{g_w}{2}\right]\left(\frac{v}{\mu}\right)\left[\left(\frac{3-e_L}{2}\right)\frac{\gamma\mu}{\alpha} - \frac{2\mu}{v}\right] + \left(\frac{3-e_L}{2}\right)g_A + \frac{g_w}{2}$

Thus:

$$h = \frac{1}{T}\int_0^T \left\{\left[g_A\left(\frac{3-e_L}{2}\right) + \frac{g_w}{2}\right]\left(\frac{v}{\mu}\right)\left[\left(\frac{3-e_L}{2}\right)\frac{\gamma\mu}{\alpha} - \frac{2\mu}{v}\right] + \left(\frac{3-e_L}{2}\right)g_A + \frac{g_w}{2}\right\} dt$$

Following the Trace presented above, $\left[g_A\left(\frac{3-e_L}{2}\right) + \frac{g_w}{2}\right]\left(\frac{v}{\mu}\right)\left[\left(\frac{3-e_L}{2}\right)\frac{\gamma\mu}{\alpha} - \frac{2\mu}{v}\right] + \left(\frac{3-e_L}{2}\right)g_A + \frac{g_w}{2} < 0$, if, and only if, $e_L > 3 - \frac{2\alpha}{v\gamma}$. In this vein, we have $h < 0$, resulting in a limit cycle locally stable, and agreeing with the result presented in the Appendix B – b; which is globally stable.

Q.E.D.