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**Optimal Regulation of Quality in Developing Countries:  
A Model with Asymmetric Information**

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Master's Thesis presented to the Graduate Program in Economics of the University of Brasília as a partial requirement to obtain the degree of Master in Economics.

**Supervisor:** Prof. Rodrigo Andrés de Souza Peñaloza, Ph.D.

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## RESUMO

Este estudo apresenta um modelo principal-agente de regulação ótima de qualidade em países em desenvolvimento. Bastante trabalho tem sido feito sobre o tema de regulação de qualidade, mas nenhum que leve em consideração as especificidades dos países em desenvolvimento. Partindo de Laffont e N'Gbo (2000), nós consideramos uma economia com dois tipos de consumidores: aqueles que vivem na área rica e aqueles que vivem na área pobre. O bem ou serviço tem a natureza de um serviço público e é provido por uma firma monopolista através de uma rede, a qual já está instalada na área rica. O governo deseja expandir esta rede para a área pobre, enquanto também regula o nível de qualidade a ser provisionado pela firma. Baseando-nos no modelo de qualidade ótima de Laffont e Tirole (1993), nós assumimos que a qualidade é não-contratável, e que a demanda por qualidade depende de um parâmetro observado privadamente pela firma a partir de um contínuo de tipos. Usando *first-order approach*, resolvemos o modelo tanto para o caso com informação completa e incompleta. A seguir, nós consideramos como a regulação permite que o governo implemente o nível socialmente ótimo de qualidade, superando o resultado ineficiente obtido no mercado sem regulação, e como a presença de assimetria informacional gera distorções que afetam as decisões regulatórias. Nós damos atenção especial à relação entre qualidade, extensão da rede e o custo social dos fundos públicos. Nós também consideramos como a presença de *Universal Service Obligation* afeta o contrato regulatório. Finalmente, tiramos nossas conclusões e fazemos recomendações de política baseadas nos nossos resultados.

**Palavras-chave:** Modelo Principal-Agente. Seleção Adversa. First-Order Approach. Regulação de Qualidade. Desenvolvimento.

## ABSTRACT

This study presents a principal-agent model for optimal regulation of quality in developing countries. Much work has been done on the subject of regulation of quality, but none that takes into account the specificities of developing countries. Drawing from Laffont and N'Gbo (2000), we consider an economy with two types of consumers: those who live in the rich area and those who live in the poor area. The good or service has the nature of a public utility and is provided by a monopolistic firm through a network, which is already built in the rich area. The government wishes to expand this network to the poor area, while also regulating the level of quality to be provisioned by the firm. Basing ourselves on Laffont and Tirole's (1993) regulation of quality model, we assume that quality is non-contractible, and that the demand for quality depends on a parameter which is privately observed by the firm from a continuum of types. Using first-order approach, we solve the model for both the complete and incomplete information scenarios. Then, we consider how regulation allows the government to implement the socially optimal level of quality, overcoming the inefficient result obtained from unregulated markets, and how the presence of informational asymmetry generates distortions that affect regulatory decisions. We pay special attention to the relationship between quality, network extension and the social cost of public funds. We also consider how Universal Service Obligation affects the regulatory contract. Finally, we draw our conclusions and make policy recommendations based on the obtained results.

**Keywords:** Principal-Agent Model. Adverse Selection. First-Order Approach. Regulation of Quality. Development.

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## 1 INTRODUCTION

Poor quality of public utilities in developing countries is often the cause of complaints and even social unrest. Even though it's a well known problem that in every economy unregulated monopolies lack market incentives for providing socially optimal quality levels, the specificities faced by developing countries make regulation of quality a much harder task. Regulators often need to make their decisions despite the lack of proper funding and skilled professionals, dealing with distinct problems such as the high social cost of public funds, an inexperienced and cumbersome judiciary system, limited commitment from both government and firms, and the legal requirement of expanding access to the regulated service's network.

The lack of a theory of optimal regulation of quality that takes into consideration such specificities makes it harder to properly assess the trade-offs faced by these regulators. With this problem in mind, we wish to propose one such model. As is the case of all optimal regulation literature, we do not intend our model to be immediately applicable for real world scenarios. Rather, we want to provide regulators with a better understanding of the social costs and benefits of their decisions, so that they can be taken into account in concrete regulatory cases.

In order to model some aspects of developing countries, we will base ourselves on the work of Laffont and N'Gbo (2000). This allows us to introduce two types of consumers, those who live in the rich area and those who live in the poor area. Just like in the original paper, the network infrastructure from which the service is provided is already implemented in the rich area, and the government wishes to extend this network to consumers living in the poor area. However, while in Laffont and N'Gbo the main focus is the analysis of the effect of the social cost of public funds in network extension, we expand their model in order to more thoroughly analyze the provision of service quality, a point that was only briefly mentioned by the authors in their original paper.

To accomplish this, we will use the framework given by Laffont and Tirole (1993) for quality regulation. Their model introduces an unknown parameter to the demand curve,  $\theta$ , which is related to the demand for quality. However, we make some adaptations to this framework so that it fits the specificities of developing countries. We assume for simplicity that, since consumers in the poor area don't have close substitutes for the regulated service, their demand does not vary with respect to quality, which allows us to focus the problem of adverse selection on the demand of the rich area. Moreover, since all consumers are connected to the same network, we assume that there is some degree of indivisibility between the level of quality provided in both areas.

Before this undertaking, however, we briefly review the literature on the subject. We begin section 2 by revisiting the controversy over natural monopoly pricing. Then, we revisit the application of the principal-agent framework to regulatory problems. Further,

we see how regulation of quality was incorporated into these models. Finally, we examine why and how specific aspects of regulation in developing countries were brought into the optimal regulation literature, specially in the later work of the french economist Jean-Jacques Laffont.

Once concluded our review, we present our basic model in section 3. For benchmarking purposes, as is common in optimal regulation literature, we develop in section 4 a model considering complete information. The results we get in this section will help us understand how asymmetry of information affects regulatory decisions and its economic consequences.

In section 5 we introduce asymmetric information into our model. We consider that informational asymmetry exists with respect to a demand parameter  $\theta$ , which is known by the firm, but not by the government. Therefore, in order to extract the true information from the firm, the government implements a truthful direct revelation mechanism, which, of course, requires the payment of an informational rent to the firm.

The results are discussed in section 6, where we formulate some of the trade-offs faced by regulators in their decision making. We consider how regulation allows the government to implement the socially optimal level of quality, overcoming the inefficient result obtained from unregulated markets, and how the presence of informational asymmetry generates distortions that affect the optimal provision of quality and the optimal extension of the network. We also discuss how the magnitude of the social cost of public funds affects the implemented level of quality, and how it changes the use of tariffs and taxation to finance quality provision.

In this same section we explore the relationship of quality and network extension. If quality is left unregulated, we demonstrate that it decreases as the network extension increases. With regulation, however, quality and network extension might be substitutes or complements. Later, we consider how Universal Service Obligation, that is, the obligation to implement universal access to the public utility, might negatively affect service quality.

Finally, in the conclusion, we will make brief public policy recommendations based on our results. First, we stress the importance of regulation for quality provision. Then, we argue that, since regulation loses importance as the social cost of public funds increase, decreasing this cost is an important condition to improve the provision of quality in public utilities. Lastly, we comment on the effects of Universal Service Obligation, and on how our study might provide a legal justification for regulators to refrain from this obligation and to implement a network extension that, while not universal, is the one that optimally balances the negative effect that such policy might have on quality provision.

## 2 LITERATURE REVIEW

### 2.1 Natural monopolies and marginal pricing

Natural monopolies have caught the attention of early political economy with the debate on pricing. In competitive markets, it is a well known result that Pareto efficiency is achieved by equating prices to marginal costs. However, in natural monopolies, fixed costs are large with respect to marginal costs, and pricing at marginal cost would result in a negative profit for the firm, which, evidently, is an unsustainable situation (VARIAN, 2014, p. 469).

According to Laffont (2000), this debate can be traced back to Adam Smith (1776), who suggested that tariffs on roads should be priced by inflating marginal costs, charging from each consumer a tariff proportional to the wear and tear this consumer would cause to the infrastructure. This inflating factor should be large enough so as to just cover the costs of the firm and thus prevent negative profits.

Adam Smith was particularly wary of subsidizing natural monopolies, as he believed this would cause projects with low social value to be implemented. His reasoning is the follow:

A magnificent high road cannot be made through a desert country where there is little or no commerce, or merely because it happens to lead to the country villa of the intendant of the province, or that of some great lord to whom the intendant finds it convenient to make his court. A great bridge cannot be thrown over a river at a place where nobody passes, or merely to embellish the view from the windows of a neighbouring palace.

Somewhat similarly, Dupuit (1849) argues for an ideal tariff, proportional to the utility perceived by the user. He believed that such a tariff would cause no social loss, since the amount of users would be the same as in a free-tariff scenario, being at the same time necessary to ensure that the project was useful and sustainable<sup>1</sup>:

The best of all tariffs would be the one which the road user pays a toll proportional to the utility he obtains from its use. [...] It's evident that the effect of such tariff would be: at first, it would allow the passage of the same amount of users than free tariff; and therefore no utility loss to the society; and then it would provide an income sufficient enough so that a useful work could be done<sup>2</sup>. (our translation).

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<sup>1</sup>According to Laffont (2000), Hotelling misunderstood Dupuit's proposal as a defense of marginal pricing. In fact, his ideas were much closer to Smith's, since he proposes a tariff according to consumers' utility and not to the firm's marginal costs. Laffont also points out that Dupuit, just as Smith, was wary of subsidizing such projects with public money.

<sup>2</sup>Original french: Le meilleur de tous les tarifs serait celui qui ferait payer a ceux qui passent sur une voie de communication un peage proportionnel a l'utilité qu'ils retirent du passage. [...] Il est evident que l'effet d'un tel tarif serait : d'abord de laisser passer autant de monde que si le passage etait gratuit ; ainsi point d'utilité perdue pour la societe ; ensuite de donner une recette toujours suffisante pour qu'un travail utile put se faire.

Say (1852), even though generally agreeing with Smith and Dupuit, argued that in some special cases marginal cost pricing would be justifiable for public goods, since social gains could be high enough so as to justify public subsidies, specially in cases where even inflated prices would be lower than average costs, thus making the project unsustainable.

A dissenting voice in this debate was Marshall (1890), who proposed in his analysis that industries with decreasing returns to scale should be taxed and industries with increasing returns to scale should be subsidized. This proposition was later defended by Hotelling (1938), who advocated marginal prices and subsidies for the latter:

The efficient way to operate a bridge, and the same applies to a railroad or a factory, if we neglect the small cost of an additional unit of product or of transportation, is to make it free to the public, so long at least as the use of it does not increase to a state of overcrowding. A free bridge costs no more to construct than a toll bridge, and costs less to operate; but society, which must pay the cost in some way or other, gets far more benefit from the bridge if it is free, since in this case it will be more used. Charging a toll, however small, causes some people to waste time and money in going around by longer but cheaper ways, and prevents others from crossing. The higher the toll, the greater is the damage done in this way; [...] There is no such damage if the bridge is paid for by income, inheritance, and land taxes, or for example by a tax on the real estate benefited, with exemption of new improvements from taxation, so as not to interfere with the use of the land.

Hotelling's idea is that marginal pricing would keep incentives at the margin, thus allowing the Pareto efficient level of production and consumption. He understood tariffs on natural monopolies as a form of taxation, which generated social loss. Hotelling also criticized how different tariff rates were used for purposes other than financing the infrastructure:

A railway rate is of essentially the same nature as a tax. Authorized and enforced by the government, it shares with taxes a considerable degree of arbitrariness. Rate differentials have, like protective tariffs and other taxes, been used for purposes other than to raise revenue. Indeed, the difference between rail freight rates between the same points, according as the commodity is or is not moving in international transport, has been used in effect to nullify the protective tariff.

There are three main problems with Hotelling's analysis, though. First, it ignores the distortions caused by taxation on income, specially with respect to the labor and leisure trade-off. Thus, the taxation proposed by Hotelling in order to finance infrastructure projects also generates deadweight losses, and this social cost of efficiency, given by the shadow cost of public funds, should be taken into account by the government.

This problem was pointed out by many authors, like Frisch (1939) and Meade (1944). Hotelling (1939) believed he could circumvent this problem by using taxes he claimed were non-distortionary, like taxes on inheritance and on rents of land. Vickrey

(1948) pointed out that, even if these taxes really worked like lump-sum taxes, income and other distortionary taxation would still be needed in the margin.

Thus, the second-best result should be obtained by maximizing social utility considering the social cost of public funds used to finance transfers to the firm, restricted by the condition of non-negative profit. This result is similar to the Boiteux-Ramsey pricing, where price is obtained by inflating marginal costs by a factor proportional to both the Lagrangian multiplier, in this case, the social cost of public funds, and by the inverse of the price-elasticity of demand. Interestingly though, Boiteux (1956) himself did not advocate for the use of public transfers, and his paper considered an endogenous Lagrangian multiplier obtained by the necessity to inflate prices in order to cover the firm's costs without transfers.

It might be tempting to think that this second-best solution is a defeat for the marginalists, since price is inflated with respect to marginal cost. However, As Laffont and Tirole (1993) point out, this result is closer to the marginal cost pricing, because “even though price exceeds marginal cost, pricing is unrelated to average cost pricing, since price is independent of the fixed cost”. Therefore, this form of Boiteux-Ramsey pricing is the socially optimal solution for the trade-off between using public funds to finance the project, which causes deadweight loss through distortionary taxation, and charging prices above marginal costs, which creates production inefficiency for the natural monopoly.

The second problem with Hotelling's analysis is that it does not consider the existence of asymmetric information with respect to the demand curve and to the firm's fixed and marginal costs. Efficiently pricing and subsidizing the firm requires knowledge of demand and costs, which more frequently than not are at least partly unknown to regulators.

Finally, the third problem is that this asymmetry of information opens the door for government capture, since the firm and politicians, responsible for regulating the firm, could agree to report a false value for demand and cost parameters, which would not only allow rent extraction by the firm, but also the implementation of projects with little to no social value, a very frequent problem in developing countries.

This latter points have inspired the study of principal-agent relationship, upon which our model was based. We will therefore briefly review it below.

## 2.2 Principal-agent models and optimal regulation

The principal-agent literature emerged as a way to model relationships where one party, the principal, needs to delegate a task to another party, the agent. The principal wishes the agent to act as to maximize the principal's utility, but the agent seeks to maximize his own.

In a complete information scenario, the principal can solve this problem by offering a contract that binds the agent to act as the principal wishes, making a transfer in order to

cover the agent's participation constraint, that is, the remuneration for the task required by the agent, and imposing an out-of-equilibrium penalty should the agent deviate.

However, the problem arises when there is either hidden information or hidden action, and the principal has to act without full knowledge of reality. In these situations, the principal needs to give up some rent to the agent in order to either obtain a truthful report of the unknown parameter or to make the agent implement the desired level of effort (LAFFONT; MARTIMORT, 2002).

Principal-agent models have their origin in the literature of organizations, championed by the works of Barnard (1938), Arrow (1963), Ross (1973) and Williamson (1975). With respect to regulation, Loeb and Magat (1979) were the first to apply a principal-agent model to come up with a solution for the problem of asymmetric information on marginal costs.

Assuming that the regulator has no knowledge about the marginal cost curve, Loeb and Magat proposed a decentralized regulation implemented by a Vickrey-Clarke-Groves mechanism. In this mechanism, the monopolistic firm would decide how to price its product, and receive a subsidy equivalent to the consumers' total surplus. Therefore, since marginal pricing maximizes social welfare, the firm would lose revenue if it chose anything other than to price the product at the marginal cost, making it a dominating strategy to report its true value.

An advantage of this incentive-compatible regulatory scheme is that the firm would automatically adjust prices if costs change, either because of inflationary pressures, changes in supply conditions, or technological improvements. Not only that, but it would have incentives to be efficient and to lower its costs, so as to increase total surplus and, thus, its own profits.

This rather ingenious mechanism has two main interrelated problems. First, it considers that transfers to the firm are costless. Second, and in part due to the first problem, transfers to the firm are very large. Loeb and Magat anticipate some of this criticism in their paper, claiming that a franchising system, where different companies would bid for the monopoly, would greatly reduce subsidies. They also proposed a lump-sum tax on consumers, so as to at least partly finance the monopoly and decrease the need for transfers.

Baron and Myerson (1982) also propose an incentive-compatible mechanism for regulating a natural monopoly with unknown costs, but, diverging from Loeb and Magat, they assume that there is only one company capable of operating the monopoly, so that bidding for the franchise is not possible. They also criticize Loeb and Magat's proposal of a lump-sum tax on the ground that, with unknown costs, there is the risk that the regulator could set the lump-sum tax too high and the firm would decline to supply the good.

The key idea from their paper, however, is to take into consideration distributive

issues by weighting the welfare of consumers and firm, so that transfers to the firm are costly in the regulator's point of view. They then demonstrate that costly transfers create a trade-off between efficiency and rent extraction, generating second-best results. By penalizing transfers to the firm, Baron and Myerson show that, in the eyes of consumers, letting the firm operate the monopoly without regulation is better than implementing the Vickrey-Clarke-Groves mechanism proposed by Loeb and Magat, even if total social surplus decreases.

Finally, Laffont and Tirole (1986) identify Baron and Myerson's distributive weights as the social cost of public funds, since transfers are financed by distortionary taxation, and the social cost of public funds is precisely the social sacrifice of levying an additional dollar from consumers in order to transfer an additional dollar to the firm. They would later generalize this model in Laffont and Tirole (1993), which can be seen as foremost example of the so-called optimal regulation literature.

Optimal regulation is the modeling of regulation as a principal-agent relationship, with the aim of analyzing how the regulator optimally pursues his goals in the presence of information asymmetry. As Armstrong and Sappington (2007, p. 1606) point out, rather than been a practical guide for regulation, it is an important normative approach that intends to extract insights on the economic trade-offs faced by regulators. These insights can then be used to evaluate current regulatory practices and to propose advances in more practical and simple regulatory designs.

Two examples of these practical and relatively more simple regulatory designs are the rate-of-return regulation and price caps.

As the name indicates, rate-of-return regulation is the practice of setting prices with the goal of granting the firm a certain rate of return on the invested capital. Usually, this rate is set so as to cover the firm's opportunity cost, ensuring its participation in the industry, but, at the same time, limiting the profit it would make as an unregulated monopoly. Consequently, rate-of-return regulation helps keeping prices at a level that is, at least ideally, seen as fair by costumers.

There are two main problems with rate-of-return regulation: first, it creates perverse incentives for the firm not to cut costs. It is easy to see why: since the firm has a guaranteed rate of return on its investment, cutting costs will generate a decrease in prices to keep the same rate, and thus won't increase its profits. In the long run, the firm will tend towards inefficiency by adopting production methods that are expensive, demanding more transfers and higher prices. The second problem is that, since the rate-of-return is settled with respect to the invested capital, the firm has incentives to over-invest capital in order to increase its overall profits. This is called the Averch-Johnson effect, and can be achieved by either substituting labor for capital or by procuring more capital goods than necessary for production, making this process inefficient.

A price cap, on the other hand, is a regulatory practice that fixes the maximum price

that the monopolist firm can charge. Contrary to rate-of-return regulation, this mechanism creates strong incentives for the firm to cut costs in order to increase profits. For this reason, price caps are often referred to as “high-powered regulation” and rate-of-return regulation as “low-powered regulation”.

Implementing a price cap policy requires a good knowledge of the firm’s costs, or else price levels can be set at a level that does not cover the firm’s participation constraint. Price caps can also be difficult to implement in places where commitment to a contract is difficult to enforce. This creates incentives for the firm to lobby for higher prices, to judicialize regulatory decisions or to engage in typical hold-up strategies in order to renegotiate the contract in better terms. Since asymmetries of information and commitment problems are more severe in developing countries, it’s easy to understand why price caps have been more frequently implemented in developed countries. We will go back to the specificities of regulation in developing countries in subsection 2.4, *infra*.

### 2.3 Regulation of service quality

Goods can be classified as search goods or experience goods. Search goods are goods whose quality are known by the consumer before purchase, while experience goods are goods whose quality is only discovered after acquisition (LAFFONT; TIROLE, 1993). Evidently, quality only affects experience goods in repeated purchases. Since our model is based on a single-time decision made by consumers, we will for simplicity consider only search goods from now on.

Spence (1975) demonstrates that unregulated monopolies do not have the proper market incentives to provide the socially optimal level of quality. In his model, he assumes that quality increases the consumers’ willingness to pay and that the firm cannot price discriminate. If the firm is considering an incremental increase in quality, it will verify if the incremental revenue generated by this incremental increase compensates the incremental costs. However, the problem is that the preferences of this marginal consumer with respect to quality might not be representative of the average consumer. Spence states that:

The average is the relevant quantity for welfare, but the firm responds to the marginal individual. (For the firm to do otherwise would require the ability to price discriminate.) When the average valuation of quality increments exceeds the marginal valuation, the firm sets quality too low; it stops increasing quality too soon.

Mussa and Rosen (1978) developed their own model where the assumption that the firm cannot price discriminate is relaxed, and demonstrated that the result is also inefficient. They assume different types of consumers with different demands for quality. In order to increase sales for consumers demanding the highest quality, the monopolist will decrease quality for all other consumers, thus creating a separating equilibrium. In some cases, consumers demanding very low quality, who would purchase the good in



a perfect competition scenario, might be priced out of the market by the monopolist. This happens because these consumers “impose the largest negative externalities on the monopolist’s power to extract consumer’s surplus from others” (Idem, p. 315). Overall, quality is undersupplied for consumers that are not the highest quality demanding ones. As Armstrong and Sappington (2007) explain it:

An unregulated monopolist that sells its products to consumers with heterogeneous valuations of quality will tend to deliver less than the welfare-maximizing level of quality to consumers who have relatively low valuations of quality. This under-supply of quality to low-valuation customers enables the monopolist to extract more surplus from high-valuation customers. It does so by making particularly unattractive to high-valuation customers the variant of the firm’s product that low-valuation consumers purchase. Faced with a particularly unattractive alternative, high-valuation customers are willing to pay more for a higher-quality variant of the firm’s product.

In fact, Mussa and Rosen’s results can be seen as the formalization of an early insight by french economist Dupuit. He realized that the poor conditions faced by train passengers in the third class, such as the absence of roof on top of the wagon, could not be explained by a simple problem of cost. Rather, Dupuit believed the railway company would deliberately implement a low quality of service in order to scare passengers who could afford second-class tickets from buying third class tickets. He is quoted saying:

It is not because of the few thousand francs which would have to be spent to put a roof over the third-class carriages or to upholster the third-class seats that some company or other has open carriages with wooden benches. [...] What the company is trying to do is to prevent the passengers who can pay the second-class fare from traveling third class; it hits the poor, not because it wants to hurt them, but to frighten the rich. And it is again for the same reason that the companies, having proved almost cruel to third-class passengers and mean to second-class ones, become lavish in dealing with first-class passengers. Having refused the poor what is necessary, they give the rich what is superfluous. (DUPUIT *apud* EKELUND, 1970)

The conclusion that markets fail to provide optimal quality opens the doors for regulation as an alternative. However, introducing a quality parameter also makes regulation a trickier task, and several additional problems can arise: first, it is difficult for the government to assess the consumers’ quality preferences, since those are their private information. Second, even if consumers’ preferences can be extracted through market surveys, for example, the quality of the good or service effectively provided by the firm can be hard to measure. Third, it might not be possible to contract a specific level of quality if the Court of Law doesn’t have regulatory expertise, making it difficult to produce evidence showing that the firm has deviated from the contract. In this last case we say that quality is *non-contractible*.

On the other hand, if there is some degree of verifiability and contractibility, regulation of quality might dispose of instruments other than price setting. For example, regulators could survey consumers, either to have a better knowledge of their preferences or to have a better knowledge of the level of quality offered by the firm. Another possible instruments are the imposition of a minimum quality standard or of service quality bonuses and penalties (SAPPINGTON, 2005, p. 132).

The model we will develop *infra* does not presuppose verifiability or contractibility, but it could be extended in order to include the kind of regulatory policies seen above. For example, a survey of consumers' opinion on the quality provided by the firm could be modeled as an informative sign that alters the regulator's probability distribution of the quality parameters in a more favorable fashion<sup>3</sup>.

As we've seen, the analysis of quality provision in unregulated markets paved the way for studies on quality regulation. Besanko *et al.* (1987) is one of the first papers to deal with this subject. The authors analyze two regulatory options for Mussa and Rosen's model: price ceilings for products on the high end of the quality array and minimum quality standards. Since price ceilings make providing high quality products with high prices less attractive, Besanko *et al.* conclude that it decreases the incentives for distorting quality downward on the low end of the quality array. On the other hand, a minimum quality standard also increases the quality on the low end of the quality array, but the liquid effects are ambiguous because it might also exclude part of these consumers from the market.

One of the first works to deal with the regulation of quality in the framework of optimal regulation is Lewis and Sappington (1988). Rather than assuming that the firm has costs that are unknown to regulators, the authors assume that it is the consumers' demand curve that have an unknown parameter. This preference parameter can be understood as a demand for quality. Lewis and Sappington show that when the firm has non-decreasing marginal costs, the regulator is able to costlessly extract the true value of the parameter from the firm, and consequently to implement the first-best policy. This is due to the fact that, in their model, the first-best price is equal to the the marginal cost, and the rest of the cost is covered by transfers. So, if the firm misrepresents the demand in order for the regulator to set a higher price, it will lose revenue from transfers. Conversely, if it misrepresents the demand in order for the regulator to set a lower price, it will lose revenue from sales. Indeed, this kind of mechanism is somewhat similar to Loeb and Magat (1979), already seen *supra*, but here no rent is given to the firm, since costs are known to the regulator.

However, if the firm has decreasing marginal costs, then its private information cannot be used for social benefit, and regulators should set a price that is invariant to the actual value of the unknown demand parameter. According to Laffont and Tirole (1993, p.

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<sup>3</sup>See Laffont and Martimort (2002), chapter 2.14 for a presentation of this modeling technique.

161), this is an “instance of the phenomenon of *nonresponsiveness* of the allocation with respect to private information”.

Based upon Baron and Myerson’s work, that we saw before, Lewis and Sappington’s paper also attributes different weights to consumers and the firm’s welfare. Laffont and Tirole (1993) show that, if we consider these weights as the shadow cost of public funds, the results that the government can costlessly extract the firm’s private information when marginal costs are non-decreasing no longer holds. This happens because, in this case, price is settled according to Ramsey-Boiteux’s method, so that price exceeds the marginal cost. Therefore, differently from Lewis and Sappington’s model, the firm could have incentives to expand demand by asking for a lower price.

In this same work, Laffont and Tirole would reformulate Lewis and Sappington’s model to their own framework of optimal regulation, i.e., the framework that considers the social cost of public funds. The authors show that, in order to implement the socially optimal level of quality, the government needs to give up some rent to the firm. Moreover, since providing quality becomes more expensive because of this informational rent, it is socially optimal to decrease it with respect to the first-best outcome. They also show that prices could or could not be distorted, depending on how the unknown parameter affects the demand.

Our own model of optimal regulation of quality, to be presented *infra*, will be based on the model above. However, we will also introduce some specific elements from developing countries.

## 2.4 Regulation in developing countries

Further developments in the literature of optimal regulation started to take into consideration specific aspects of regulation in developing countries. The interest in this approach begun with the realization that some parts of the agenda for development prescribed in developed countries had failed to achieve success in developing nations, a failure that can be partially explained by the different way that regulatory institutions work in the latter. As Bourguignon puts it, “today, it is increasingly recognized that, in many instances, the problem was that reformers disregarded the functioning of regulatory institutions, assuming implicitly they would work as in developed countries” (*apud* LAFFONT, 2005, p. xi).

In fact, realizing the importance of this issue, Laffont dedicated the last years of his life to the study and modeling of these specific aspects. His analysis would focus on what he called *institutional weakness* of developing countries. According to Laffont, the modeling and subsequent policy recommendations for these countries should take their specificities into account. As he states:

Developing economies are often described as “economies with missing markets.” In the contractual world of regulation, this translates into in-

complete contracts. Contracts are incomplete because of players' bounded rationality as in any economy, but also because of institutional weaknesses in enforcement, commitment, auditing, etc. [...] Policy recommendations for such a country should be based on a model which incorporates all these features. (LAFFONT, 2005, p. 245-246)

For our own review, we follow Estache and Wren-Lewis' (2009) summary of Laffont's proposed adjustments of regulation modeling to the reality of developing countries. They identify four main characteristics, or institutional weaknesses, that were pointed out by Laffont: limited regulatory capacity, limited accountability, limited commitment and limited fiscal efficiency.

Limited regulatory capacity comes from both the scarcity of financial resources to employ skilled professionals and the scarcity of skilled professionals themselves in the market. Good regulators are highly educated and specialized employees, which are harder to recruit in developing countries. An underdeveloped auditing system and inexperienced judiciary are also important factors that contribute to this limited regulatory capacity.

There are several ways to introduce this limitation in a regulatory model. For example, it could be modeled as a less favorable distribution of probability over the unknown parameter, or, if we assume the regulator as someone who obtains an informative signal about this parameter with a certain probability  $\xi$ , we can model this limitation as a lower value of  $\xi$ .

Moreover, an inexperienced judiciary can be modeled as a restriction on contractable variables. If a Court of Law does not have the necessary expertise to assert whether the firm has fulfilled the terms of the contract, then it becomes ineffective for the regulator to impose a certain value to be implemented by the firm. Indeed, for this very same reason, we will assume in section 3 that quality is a non-contractable variable in our own model.

Limited accountability is the idea that it is easier for government officials in developing countries to pursue their own interests, rather than maximizing social welfare. One way it can be modeled is through a parameter that measures the easiness to bribe a regulator, which has the nature of a transactional cost. However, since this greater easiness to bribe a public official might increase the required bribe, the government would have to make larger transfers to the regulators in order to keep incentive compatibility, and this isn't compatible with data obtained from developing countries. According to Estache and Wren-Lewis, "if anything, regulators in developing countries have fewer financial incentives to produce information than in developed countries, contrary to the model".

One way to explain this is by assuming that, with a certain probability, a regulator can be honest or dishonest. For this reason, it would be a waste of money to pay large incentives to regulators, since honest regulators would report the true observed signal anyway. Therefore, transfers would decrease according to the expected state of nature and it would be socially optimal to allow for the possibility of capture. In developed countries, on the other hand, transfers are already small, so that it is socially optimal

for the government to pay these incentives and to eliminate completely the possibility of capture.

Another interesting point is that a greater easiness to bribe regulators favors low powered incentives, so that the firm has less incentive to capture regulators. Moreover, it might be a good strategy for the government to have more than one agency responsible for observing the same signal, so that competition among regulators decreases the incentives for capture.

Limited accountability can also be accounted for by adopting the framework of a non-benevolent government, so that the government itself could be captured by the firm. One way to model this is to assume that the government puts a greater weight into the firm's utility. This is similar to Baron and Myerson's approach, although here the weights represent not the social preferences, but the firm's own interests. Our model will not approach this possibility, and we will for simplicity assume the government's benevolence.

Limited commitment is another problem faced by developing countries, and it might be understood both as government noncommitment or firm-led renegotiation. According to Estache and Wren-Lewis, government noncommitment is most prominent in developing countries, meaning that the government cannot promise a sufficient return to investors. Therefore, when the firm makes an investment decision, it looks not to the government's promised rate of return or to the government's promised transfers, but rather to the expected value given the possibility of a government-caused breach of contract. The clear consequence is underinvestment in these sectors, which is indeed a characteristic of developing countries. In repeated games, government noncommitment also makes it more attractive for the firm not to disclosure private information.

However, firm-led renegotiation is also an important issue in developing countries. For example, Guasch, Laffont, and Straub (2008) show that one third of infrastructure renegotiations in the Latin America are initiated by the firms. Whenever there is possibility for renegotiation, social welfare is splitted according to each party's bargaining power. Also, renegotiation makes it more difficult to implement social optimal results when the firm faces uncertainty and a bad state of nature is realized. According to Greenwald (1984), it is also possible, as we have anticipated before, that rate-of-return regulation generates less incentives for firm-led renegotiation, which can be an important feature in the presence limited commitment.

Limited fiscal efficiency means that it is more socially costly for the government to levy taxes from society. This is due to a conjunction of several factors found in developing countries: inefficiency on government expending, leading to the need for greater taxation, inefficiency of the tax system, leading to a higher efficiency cost, or a greater level of corruption. Limited fiscal efficiency can be modeled by adjusting the social cost of public funds parameter. Indeed, Laffont (2005) states that, while the social cost of public funds is around 0.3 in developed countries, it can be "well beyond 1" in developing countries.

One interesting consequence of a greater social cost of public funds is that, differently from the classical result that public monopolies should be necessarily subsidized, sometimes it is optimal to charge higher prices and to tax the firm in order to raise funds for the government. In other words, if the social cost of public funds is sufficiently high, it is best to make negative transfers to the firm. This is easy to understand: in these cases, raising money to the public treasury generates a greater incremental social welfare than expanding the services of the public monopoly by subsidizing it. This trade-off between expansion of network and fund-raising was first identified in Laffont and N'Gbo (2000), and it helps understand why in so many developing countries infrastructure is often offered in small networks with high prices.

Indeed, Laffont and N'Gbo also introduce another important modeling tool for regulation in developing countries, that is, the division of consumers in a rich and a poor area. Their work deals with the issue of Universal Service Obligation in the face of limited fiscal efficiency. Even though this is not the language used by Laffont himself, Universal Service Obligation can be understood as an incomplete contract given by the Constitution to the government, stating that the government should seek to implement universal access to the service, even though it does not specify how politicians should achieve this goal, leaving them discretionary power, as is typical of incomplete contracts.

In their paper, Laffont and N'Gbo argue that cross subsidies between rich and poor areas could be an interesting approach in order to overcome the problems of limited fiscal efficiency. They also make a few brief remarks with respect to quality, even though they don't completely develop the idea. This is precisely the aim of our own model: to expand on Laffont and N'Gbo's work in order to analyze the regulation of quality in developing countries. As we will show, the idea of limited fiscal efficiency has important consequences for quality provision.

### 3 BASIC SETTING

In this model, we consider a principal-agent relationship between the government and the monopolistic firm that it regulates.

A good or a service is provided by the firm to consumers. This good or service has the nature of a public utility, like electricity, water, gas, public transportation, etc. Consumers can either live in the rich area or in the poor area, which we call areas 1 and 2, respectively. As is often the case in developing countries, the rich area is already entirely connected to the network infrastructure, and the government seeks its expansion to the poor area.

The firm will expand the infrastructure in order to connect consumers in the poor area to the network. It then sells  $q_1$  units to each representative consumer in the rich area at a price  $p_1$ . Similarly, it sells  $q_2$  units to each connected representative consumer in the poor area at a price  $p_2$ . We assume that consumers in the poor area spend a fixed part  $R$  of their income in acquiring the good or service. This allows us to take into account both the income effect and the lack of substitute goods in this area.

The firm chooses the public utility's level of quality. Since both areas are connected in a network, quality is not entirely divisible between rich and poor consumers. The level of quality provisioned to the rich area is given by  $s$ , and  $\delta s$  is the quality provisioned to the poor area, where  $0 < \delta \leq 1$ . Quality is observable, but non-contractible, that is, it cannot be ascertained by a benevolent Court of Law, so that the contract cannot be dependent on the variable  $s$ . The demand for quality depends upon a variable  $\theta$ , which is privately observed by the firm.

#### 3.1 Consumers

There is a mass  $[0, 1]$  of consumers in the rich area. All consumers in this area are already connected to the network. The network will be expended to the poor area in order to connect  $\nu$  poor consumers. We define  $\nu$  as the proportion of poor consumers to be connected with respect to the number of rich consumers.

Consumers in rich and poor areas observe prices  $p_1$ ,  $p_2$ , quality  $s$ ,  $\delta s$  and then purchase  $q_1$ ,  $q_2$  units of the good or service, respectively. Since consumers observe quality before buying, the public utility is a search good.

The representative consumer's gross surplus is given by a strictly concave function  $S_1(q_1, s, \theta)$  in the rich area and  $S_2(q_2, \delta s)$  in the poor area, with  $\frac{\partial S_i}{\partial q_i} > 0$ ,  $\frac{\partial^2 S_i}{\partial q_i^2} < 0$  and  $\frac{\partial S_i}{\partial s} > 0$ ,  $\frac{\partial^2 S_i}{\partial s^2} < 0$  for  $i = 1, 2$ .

Consumers in the rich area observes  $p_1$  and  $s$  and choose  $q_1$  according to the utility maximization problem, given below:

$$\max_{q_1} S_1(q_1, s, \theta) - p_1 q_1$$

The solution in  $q_1$  satisfies the first-order condition:

$$\frac{\partial S_1(q_1, s, \theta)}{\partial q_1} = p_1 \quad (1)$$

The rich consumer's utility maximization problem gives us the inverse demand in the rich area, that we define below.

**Definition 1.** Inverse demand in the rich area is given by the following function, which is continuously differentiable of class  $C^1$  in the interior of its domain:

$$P_1(q_1, s, \theta) \quad (2)$$

By convention, we assume that  $\frac{\partial P_1}{\partial \theta} < 0$ . We also assume that  $\frac{\partial P_1}{\partial s} > 0$ .

Consumers in both areas value quality, but poor area consumers always spend a fixed amount  $R$  of their income in the purchase. One possible explanation for this is that consumers in this area have no close substitutes in the market. For example, a consumer in the rich area might use his car instead of public transportation if its quality is too low, or buy bottled water for consumption instead of tap water, while in the poor area the public utility is the only option. Therefore, demand in the poor area depends only on  $p_2$ , and not on quality  $\delta s$ , and is given by  $q_2 = R/p_2$ .

Besides paying for the good or service, consumers also pay taxes to the government, which will finance transfers to the firm.

### 3.2 Firm

The monopolistic firm provides the public utility to both areas, charging a tariff from each consumer and receiving a transfer  $T$  from the government. We define below the firm's *per capita* variable cost for both areas:

**Definition 2.** *Per capita* variable cost is given by the function

$$C_i(q_i), \text{ for } i = 1, 2$$

with  $\frac{dC_i(q_i)}{dq_i} > 0$  and  $\frac{d^2C_i(q_i)}{dq_i^2} \geq 0$ .

The firm also incurs in fixed costs in order to expand and maintain the infrastructure. In the rich area, since all consumers are already connected to the network, *per capita* fixed cost is simply given by the variable  $K_1$ . In the poor area, fixed cost is a function of the proportion of poor consumers to be connected to the network, given by  $\nu$ . This function is defined below. We assume that areas with smaller fixed cost are equipped first, so that the function is convex.



**Definition 3.** *Per capita* fixed cost in the rich area is given by  $K_1$ . *Per capita* fixed cost in the poor area is given by the function  $K_2(\nu)$ , with  $\frac{dK_2(\nu)}{d\nu} > 0$  and  $\frac{d^2K_2(\nu)}{d\nu^2} \geq 0$ .

Therefore, total fixed cost is  $K_1$  in the rich area and  $\nu K_2(\nu)$  in the poor area.

After observing the parameter  $\theta$  of the rich area's demand function, the firm provides quality levels  $s$  and  $\delta s$ . The provision of quality requires a non-monetary effort by the firm, modeled by the function defined below.

**Definition 4.** *Per capita* non-monetary effort for quality provision is given by the function  $\psi(s)$ , with  $\frac{d\psi(s)}{ds} > 0$ ,  $\frac{d^2\psi(s)}{ds^2} > 0$  and  $\frac{d^3\psi(s)}{ds^3} \geq 0$ .

Even though  $\psi(s)$  is observable by the government, because  $s$  is non-contractible, contracts cannot depend on the value of  $\psi(s)$ .

**Definition 5.** The firm's utility is given by the piecewise differentiable<sup>4</sup> function:

$$U = \underbrace{T}_{\text{Transfers}} + \underbrace{p_1 q_1 + \nu p_2 \frac{R}{p_2}}_{\text{Income from sales}} - \underbrace{K_1 - \nu K_2(\nu)}_{\text{Fixed costs}} - \underbrace{C_1(q_1) - \nu C_2(q_2)}_{\text{Variable costs}} - \underbrace{\psi(s) - \nu \psi(\delta s)}_{\text{Non-monetary effort}} \quad (3)$$

### 3.3 Government

The government regulates the firm providing the public utility. It settles prices and quantities for each area, the extension of the network to be implemented in the poor area and makes transfers to the firm.

In order to finance these transfers, it levies a distortionary tax on consumers. The social cost of public funds is  $(1 + \lambda)$ , with  $\lambda > 0$ . Following Laffont and Tirole (1993), we make the accounting convention that the government receives the firm's income from sales and reimburses it for its fixed and variable costs. Any residual transfer is given by  $t$ , such that we can redefine the firm's utility as below.

**Definition 6.** The firm's utility is given by the piecewise differentiable function

$$U = t - \psi(s) - \nu \psi(\delta s) \quad (4)$$

where

$$t = T + p_1 q_1 + \nu p_2 \frac{R}{p_2} - K_1 - \nu K_2(\nu) - C_1(q_1) - \nu C_2(q_2)$$

The firm's outside option is normalized to zero, so that a contract is accepted only if it gives the firm a non-negative utility.

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<sup>4</sup>We define a piecewise differentiable function as a function that is differentiable everywhere but on a finite set of points, at which, nevertheless, the side derivatives are well defined.

The government maximizes an utilitarian social welfare function  $W$ , given by the sum of the utilities of consumers in the rich and poor areas and the utility of the firm. This function is defined below.

**Definition 7.** The social welfare function is given by:

$$W = \underbrace{S_1(q_1, s, \theta) - p_1 q_1}_{\substack{\text{Rich Consumers'} \\ \text{Liquid Surplus}}} + \underbrace{\nu S_2(q_2, \delta s) - \nu \frac{R}{p_2} p_2}_{\substack{\text{Poor Consumers'} \\ \text{Liquid Surplus}}} - \underbrace{(1 + \lambda)T}_{\substack{\text{Opportunity} \\ \text{Cost of} \\ \text{Public Funds}}} + \underbrace{U}_{\substack{\text{Firm's} \\ \text{Profit}}} \quad (5)$$

#### 4 OPTIMAL REGULATION UNDER COMPLETE INFORMATION

For benchmarking purposes, we consider the case where the rich area's parameter  $\theta$  is *common knowledge*, that is, the government is capable of completely observing the demand curve.

The government's problem is to maximize the social welfare function  $W$  restricted to the firm's participation constraint:

$$\begin{aligned} \max_{q_1, q_2, \nu, s, T} \quad & S_1(q_1, s, \theta) - P_1 q_1 + \nu S_2(q_2, \delta s) - \nu \frac{R}{p_2} p_2 - (1 + \lambda)T + U \\ \text{s.t.} \quad & U \geq 0 \end{aligned} \quad (6)$$

Remember that the firm's utility is given by:

$$U = T + P_1 q_1 + \nu R - K_1 - \nu K_2(\nu) - C_1(q_1) - \nu C_2(q_2) - \psi(s) - \nu \psi(\delta s)$$

So that the problem can be rewritten as:

$$\begin{aligned} \max_{q_1, q_2, \nu, s, T} \quad & S_1(q_1, s, \theta) - P_1 q_1 + \nu S_2(q_2, \delta s) - \nu \frac{R}{p_2} p_2 - (1 + \lambda)T + U \\ \text{s.t.} \quad & T + P_1 q_1 + \nu R - K_1 - \nu K_2(\nu) - C_1(q_1) - \nu C_2(q_2) - \psi(s) - \nu \psi(\delta s) \geq 0 \end{aligned} \quad (7)$$

We can write transfers as:

$$T = -P_1 q_1 - \nu R + K_1 + \nu K_2(\nu) + C_1(q_1) + \nu C_2(q_2) + \psi(s) + \nu \psi(\delta s) + U$$

We can now substitute  $T$  from the equation above into the objective function at (6), so that we have:

$$\begin{aligned} W = S_1(q_1, s, \theta) - P_1 q_1 + \nu S_2(q_2, \delta s) - \nu R - (1 + \lambda)[-P_1 q_1 - \nu R \\ + K_1 + \nu K_2(\nu) + C_1(q_1) + \nu C_2(q_2) + \psi(s) + \nu \psi(\delta s) + U] + U \end{aligned}$$

Rearranging the equation above yields:

$$\begin{aligned} W = S_1(q_1, s, \theta) + \lambda P_1 q_1 + \nu S_2(q_2, \delta s) + \lambda \nu R \\ - (1 + \lambda)[K_1 + \nu K_2(\nu) + C_1(q_1) + \nu C_2(q_2) + \psi(s) + \nu \psi(\delta s)] - \lambda U \end{aligned}$$

Now, since making transfers to the firm is socially costly, optimality requires that  $U = 0$ . Therefore, we can rewrite the government's problem as the unrestricted

optimization program below, with just the following control variables:  $q_1$ ,  $q_2$ ,  $\nu$  and  $s$ .

$$\begin{aligned} \max_{q_1, q_2, \nu, s} \quad & S_1(q_1, s, \theta) + \lambda P_1 q_1 + \nu S_2(q_2, \delta s) + \lambda \nu R \\ & - (1 + \lambda)[K_1 + \nu K_2(\nu) + C_1(q_1) + \nu C_2(q_2) + \psi(s) + \nu \psi(\delta s)] \end{aligned} \quad (8)$$

The theorem below summarizes the conditions for the first-best optimal regulation under complete information.

**Theorem 1.** *Optimal regulation under complete information is defined by the first-order conditions given below:*

$$\begin{aligned} (i) \quad & \frac{p_1 - \frac{dC_1(q_1)}{dq_1}}{p_1} = \frac{\lambda}{1 + \lambda} \frac{1}{|\eta|} \\ (ii) \quad & \frac{1}{1 + \lambda} \frac{\partial S_2(q_2)}{\partial q_2} = (1 + \lambda) \frac{dC_2(q_2)}{dq_2} \\ (iii) \quad & K_2(\nu) + \nu \frac{dK_2(\nu)}{d\nu} = \frac{1}{1 + \lambda} [S_2(q_2, \delta s) + \lambda R] - C_2(q_2) - \psi(\delta s) \\ (iv) \quad & \frac{1}{1 + \lambda} \left[ \frac{\partial S_1(q_1, s, \theta)}{\partial s} + \nu \delta \frac{\partial S_2(q_2, s)}{\partial s} \right] + \frac{\lambda}{1 + \lambda} \frac{\partial P_1}{\partial s} q_1 = \frac{d\psi(s)}{ds} + \nu \delta \frac{d\psi(s)}{ds} \end{aligned}$$

*Proof.* The first-order condition with respect to  $q_1$  is:

$$\frac{\partial S_1}{\partial q_1} + \lambda \frac{\partial P_1}{\partial q_1} q_1 + \lambda P_1 - (1 + \lambda) \frac{dC_1(q_1)}{dq_1} = 0 \quad (9)$$

Following (1), we can substitute  $\partial S_1/\partial q_1$  by  $p_1$ . Then, we can divide both sides by  $p_1$ :

$$1 + \lambda \frac{\partial P_1}{\partial q_1} \frac{q_1}{p_1} + \lambda - (1 + \lambda) \frac{dC_1(q_1)}{dq_1} \frac{1}{p_1} = 0$$

Since  $\frac{\partial P_1}{\partial q_1} \frac{q_1}{p_1} = \frac{1}{\eta}$ :

$$1 + \lambda \frac{1}{\eta} + \lambda - (1 + \lambda) \frac{dC_1(q_1)}{dq_1} \frac{1}{p_1} = 0$$

or

$$(1 + \lambda) \left( 1 - \frac{dC_1(q_1)}{dq_1} \frac{1}{p_1} \right) = -\frac{\lambda}{\eta}$$

Which yields:

$$\frac{p_1 - \frac{dC_1(q_1)}{dq_1}}{p_1} = -\frac{\lambda}{(1 + \lambda)\eta}$$

Since  $\eta < 0$ , we can rewrite it as:

$$\frac{p_1 - \frac{dC_1(q_1)}{dq_1}}{p_1} = \frac{\lambda}{(1 + \lambda) |\eta|} \quad (10)$$

Which is the Ramsey pricing rule.

The first-order condition with respect to  $q_2$  is:

$$\frac{\partial S_2(q_2)}{\partial q_2} - (1 + \lambda) \frac{\partial C_2(q_2)}{\partial q_2} = 0$$

or

$$\frac{\partial S_2(q_2)}{\partial q_2} = (1 + \lambda) \frac{\partial C_2(q_2)}{\partial q_2}$$

That is, the optimal value of  $q_2$  equates the social marginal benefit with the social marginal cost of an additional unit provided to the poor area.

With respect to  $\nu$  we have:

$$S_2(q_2, \delta s) + \lambda R - (1 + \lambda) \left[ \nu \frac{dK_2(\nu)}{d\nu} + K_2(\nu) + C_2(q_2) + \psi(\delta s) \right] = 0$$

or

$$K_2(\nu) + \nu \frac{dK_2(\nu)}{d\nu} = \frac{1}{1 + \lambda} [S_2(q_2, \delta s) + \lambda R] - C_2(q_2) - \psi(\delta s) \quad (11)$$

We can also calculate the socially optimal provision of quality by differentiating (8) with respect to  $s$  and equating it to zero, which yields:

$$\frac{\partial S_1}{\partial s} + \lambda \frac{\partial P_1}{\partial s} q_1 + \nu \delta \frac{\partial S_2}{\partial s} = (1 + \lambda) \left[ \frac{d\psi(s)}{ds} + \nu \delta \frac{d\psi(s)}{ds} \right]$$

or

$$\frac{1}{1 + \lambda} \left[ \frac{\partial S_1(q_1, s, \theta)}{\partial s} + \nu \delta \frac{\partial S_2(q_2, s)}{\partial s} \right] + \frac{\lambda}{1 + \lambda} \frac{\partial P_1}{\partial s} q_1 = \frac{d\psi(s)}{ds} + \nu \delta \frac{d\psi(s)}{ds} \quad (12)$$

□

## 5 MODEL WITH ASYMMETRIC INFORMATION

### 5.1 Adverse selection with a continuum of types

We now go back to our initial assumption that the firm privately observes a parameter  $\theta$  of the demand curve, which is related to the demand for quality. The government knows only the probability distribution of the continuous random variable  $\theta$ , which is drawn from a cumulative distribution  $F$  with strictly positive density on the closed interval  $[\underline{\theta}, \bar{\theta}]$ .

To simplify our analysis we assume the log-concavity of  $F$ , which is equivalent to assume that  $F$  has a *monotone hazard rate*. This assumption is satisfied by most of the usual distributions, and can be mathematically translated as:

$$\frac{d\left(\frac{F(\theta)}{f(\theta)}\right)}{d\theta} \geq 0 \quad (13)$$

As we have mentioned before, quality  $s$  is assumed to be observable by all players, but it is non-contractible. This means that quality level cannot be correctly inferred by a benevolent Court of Law, which makes it impossible for the government to offer a contract dependent on  $s$ . The non-contractibility assumption can be justified by the fact that the Court of Law lacks the government's regulatory expertise.

Since demand in the poor area is invariant with respect to the level of quality, we can look exclusively to the demand for quality in the rich area. Remember that the inverse demand in the rich area is given by equation (2), that is

$$P_1(q_1, s, \theta)$$

The firm chooses the level of quality,  $s$ , so as to maximize its utility, according to the definition given below.

**Definition 8.** Given  $p_1$ , let  $s$  be the implicit solution of the function  $P_1(q_1, s, \theta)$ . According to the Implicit Function Theorem,  $s$  can be expressed as:

$$s = X(p_1, q_1, \theta) \quad (14)$$

To simplify our calculations, let's consider the special case where  $p_1 = P_1(q_1, \zeta(s, \theta))$  and

$$s = \xi(p_1, q_1) + \theta \quad (15)$$

If the firm observes  $\theta$  and reports  $\tilde{\theta} > \theta$ , the government expects the firm to provide the quality level for type  $\tilde{\theta}$ , which is greater than the true demand for quality. This greater quality requires a greater effort in providing it, so the government makes an increased liquid transfer  $t$  in order to compensate the firm, according to equation (4). However, the

firm would actually provide a lower level of quality, which translates into a smaller effort. Consequently, the firm is able to extract a rent.

Moreover, inefficiency also arises from the fact that, without a truthful report of the value of  $\theta$  by the firm, the government cannot set the socially optimal levels of price and quantity. Consequently, the government cannot indirectly contract optimal quality by setting both price and quantity together. If it does set prices and quantity together, this means that the government is assuming a random value for  $\theta$ . See Laffont and Tirole (1993, p. 119) for a proof that this kind of stochastic contract cannot be optimal<sup>5</sup>.

We demonstrate below that it is possible to create a truthful direct revelation mechanism that increases allocative efficiency.

## 5.2 Truthful direct revelation mechanism

According to Laffont and Martimort (2002), the Revelation Principle states that, being  $\Theta$  the set of all possible types that the agent can announce, there is no need to take into consideration mechanisms more complex than the direct revelation mechanism where the agent simply reports  $\tilde{\theta} \in \Theta$ .

**Definition 9.** Let  $\phi(\theta, \tilde{\theta})$  be the firm's utility level as a function of the true value of  $\theta$  and the announced value  $\tilde{\theta}$ , so that, for each  $\tilde{\theta}$  announced by the firm, the government offers a contract with respect to the liquid transfer, the prices and quantities for both area, and the extension of the network in the poor area. Therefore, the firm's utility is:

$$\phi(\theta, \tilde{\theta}) \equiv t(\tilde{\theta}) - \psi(X(p_1(\tilde{\theta}), q_1(\tilde{\theta}), \theta)) - \nu(\tilde{\theta})\psi(\delta X(p_1(\tilde{\theta}), q_1(\tilde{\theta}), \theta))$$

We want to build a direct revelation mechanism which is truthful, i.e., it is a weakly dominating strategy for the agent to report the true value of  $\theta$ . In order to achieve this, we need to ensure that the mechanism is incentive-compatible. The truthful direct revelation mechanism is defined below.

**Definition 10.** A direct revelation mechanism  $g(\tilde{\theta}) = \{t(\tilde{\theta}), p_1(\tilde{\theta}), p_2(\tilde{\theta}), q_1(\tilde{\theta}), q_2(\tilde{\theta}), \nu(\tilde{\theta})\}$  is truthful if, for any pair  $(\theta', \hat{\theta}) \in [\underline{\theta}, \bar{\theta}]$ , the direct revelation mechanism satisfies the following incentive compatibility constraints:

$$\phi(\theta', \theta') \geq \phi(\hat{\theta}, \theta') \tag{16}$$

and

$$\phi(\hat{\theta}, \hat{\theta}) \geq \phi(\theta', \hat{\theta}) \tag{17}$$

---

<sup>5</sup>This is why we need the assumption that  $\frac{d^3\psi(s)}{ds^3} \geq 0$

or, equivalently,

$$\begin{aligned} & t(\theta') - \psi(X(p_1(\theta'), q_1(\theta'), \theta')) - \nu(\theta')\psi(\delta X(p_1(\theta'), q_1(\theta'), \theta')) \\ & \geq t(\hat{\theta}) - \psi(X(p_1(\hat{\theta}), q_1(\hat{\theta}), \theta')) - \nu(\hat{\theta})\psi(\delta X(p_1(\hat{\theta}), q_1(\hat{\theta}), \theta')) \end{aligned} \quad (18)$$

and

$$\begin{aligned} & t(\hat{\theta}) - \psi(X(p_1(\hat{\theta}), q_1(\hat{\theta}), \hat{\theta})) - \nu(\hat{\theta})\psi(\delta X(p_1(\hat{\theta}), q_1(\hat{\theta}), \hat{\theta})) \\ & \geq t(\theta') - \psi(X(p_1(\theta'), q_1(\theta'), \hat{\theta})) - \nu(\theta')\psi(\delta X(p_1(\theta'), q_1(\theta'), \hat{\theta})) \end{aligned} \quad (19)$$

For the specific case of our model, the direct revelation mechanism is given by  $g(\tilde{\theta}) = \{t(\tilde{\theta}), p_1(\tilde{\theta}), p_2(\tilde{\theta}), q_1(\tilde{\theta}), q_2(\tilde{\theta}), \nu(\tilde{\theta})\}$ , so that, for each  $\tilde{\theta}$  announced by the firm, the government offers a contract with respect to the liquid transfer, the prices and quantities for both area, and the extension of the network in the poor area.

Notice that now the government can offer a contract with respect to both price and quantity, since the truthful direct revelation mechanism ensures that the firm announces the true value of  $\theta$ .

If the government wishes to increase the contracted quantity, it will adjust prices downward instead of adjusting quality upward in order to keep consumers indifferent. This happens because, according to equation (4), adjusting quality upward requires giving up higher rents to the firm, which requires higher funding by taxation and, consequently, a higher social cost. Price adjustments, on the other hand, are settled through the reimbursement rule, that is, through the convention that the government receives the firm's income from sales and reimburses it for its fixed and variable costs. Therefore, by adjusting prices alone, the government is able to keep allocative efficiency and incentives unchanged. This gives us the following Lemma:

**Lemma 1.** *Let  $s^* = X(p_1, q_1, \theta)$  be the solution in  $s$  of  $p_1 = P_1(q_1, s, \theta)$ . Then,*

$$\frac{\partial X}{\partial P_1} \frac{\partial P_1}{\partial q_1} + \frac{\partial X}{\partial q_1} = 0$$

*Proof.* From equation (14) we have:

$$s^* = X(p_1, q_1, \theta)$$

Since  $p_1 = P_1(q_1, s, \theta)$ , we can write it as:

$$s^* = X(P_1(q_1, s, \theta), q_1, \theta)$$



Differentiating both sides by  $q_1$  yields:

$$\frac{\partial s^*}{\partial q_1} = \frac{\partial X(P_1(q_1, s, \theta), q_1, \theta)}{\partial q_1}$$

Since  $\frac{ds^*}{dq_1} = 0$ , we have:

$$\frac{\partial X}{\partial P_1} \frac{\partial P_1}{\partial q_1} + \frac{\partial X}{\partial q_1} = 0$$

□

### 5.3 Incentive compatibility constraints

According to Laffont and Martimort (2002, p. 136), we can rewrite the infinite incentive compatibility constraints from Definition 10 in terms of only two constraints: a monotonicity condition and a differential equation.

The monotonicity condition is a requirement on how the value of each contractible variable changes according to the announced type  $\tilde{\theta}$ . Therefore, as expected, incentive compatibility reduces the set of possible allocations.

**Theorem 2.** *The monotonicity condition for incentive compatibility is*

$$\frac{d\nu}{d\theta} \leq 0$$

*Proof.* We can add inequalities (18) and (19) to obtain:

$$\begin{aligned} & t(\theta') - \psi(X(p_1(\theta'), q_1(\theta'), \theta')) - \nu(\theta')\psi(\delta X(p_1(\theta'), q_1(\theta'), \theta')) \\ & + t(\hat{\theta}) - \psi(X(p_1(\hat{\theta}), q_1(\hat{\theta}), \hat{\theta})) - \nu(\hat{\theta})\psi(\delta X(p_1(\hat{\theta}), q_1(\hat{\theta}), \hat{\theta})) \\ & \geq t(\hat{\theta}) - \psi(X(p_1(\hat{\theta}), q_1(\hat{\theta}), \theta')) - \nu(\hat{\theta})\psi(\delta X(p_1(\hat{\theta}), q_1(\hat{\theta}), \theta')) \\ & + t(\theta') - \psi(X(p_1(\theta'), q_1(\theta'), \hat{\theta})) - \nu(\theta')\psi(\delta X(p_1(\theta'), q_1(\theta'), \hat{\theta})) \end{aligned}$$

Rearranging the inequality yields:

$$\begin{aligned} & t(\theta') - \psi(X(p_1(\theta'), q_1(\theta'), \theta')) - \nu(\theta')\psi(\delta X(p_1(\theta'), q_1(\theta'), \theta')) \\ & - t(\theta') - \psi(X(p_1(\theta'), q_1(\theta'), \hat{\theta})) - \nu(\theta')\psi(\delta X(p_1(\theta'), q_1(\theta'), \hat{\theta})) \\ & \geq t(\hat{\theta}) - \psi(X(p_1(\hat{\theta}), q_1(\hat{\theta}), \theta')) - \nu(\hat{\theta})\psi(\delta X(p_1(\hat{\theta}), q_1(\hat{\theta}), \theta')) \\ & - t(\hat{\theta}) - \psi(X(p_1(\hat{\theta}), q_1(\hat{\theta}), \hat{\theta})) - \nu(\hat{\theta})\psi(\delta X(p_1(\hat{\theta}), q_1(\hat{\theta}), \hat{\theta})) \end{aligned}$$

According to the Fundamental Theorem of Calculus, the inequality above can be rewritten as:

$$\int_{\hat{\theta}}^{\theta'} \left\{ -\frac{d\psi(s)}{ds} \frac{\partial X(p_1(\theta'), q_1(\theta'), x)}{\partial x} - \nu(\theta') \delta \frac{d\psi(s)}{ds} \frac{\partial X(p_1(\theta'), q_1(\theta'), x)}{\partial x} \right\} dx$$

$$- \int_{\hat{\theta}}^{\theta'} \left\{ -\frac{d\psi(s)}{ds} \frac{\partial X(p_1(\hat{\theta}), q_1(\hat{\theta}), x)}{\partial x} - \nu(\hat{\theta}) \delta \frac{d\psi(s)}{ds} \frac{\partial X(p_1(\hat{\theta}), q_1(\hat{\theta}), x)}{\partial x} \right\} dx \geq 0$$

Since  $\frac{\partial X}{\partial \theta} = 1$ , we can simplify the inequality above to:

$$\int_{\hat{\theta}}^{\theta'} \left\{ -\frac{d\psi(X(p_1(\theta'), q_1(\theta'), x))}{ds} - \nu(\theta') \delta \frac{d\psi(X(p_1(\theta'), q_1(\theta'), x))}{ds} \right\} dx$$

$$- \int_{\hat{\theta}}^{\theta'} \left\{ -\frac{d\psi(X(p_1(\hat{\theta}), q_1(\hat{\theta}), x))}{ds} - \nu(\hat{\theta}) \delta \frac{d\psi(X(p_1(\hat{\theta}), q_1(\hat{\theta}), x))}{ds} \right\} dx \geq 0$$

Applying once again the Fundamental Theorem of Calculus we obtain:

$$\int_{\hat{\theta}}^{\theta'} \int_{\hat{\theta}}^{\theta'} \left\{ -\frac{d^2\psi(X(p_1(y), q_1(y), x))}{ds^2} \left( \frac{\partial X}{\partial P_1} \frac{\partial P_1}{\partial q_1} \frac{\partial q_1}{\partial y} + \frac{\partial X}{\partial q_1} \frac{\partial q_1}{\partial y} \right) \right.$$

$$- \frac{d\nu}{dy} \delta \frac{d\psi(X(p_1(y), q_1(y), x))}{ds}$$

$$\left. - \nu(y) \delta \frac{d^2\psi(X(p_1(y), q_1(y), x))}{ds^2} \left( \frac{\partial X}{\partial P_1} \frac{\partial P_1}{\partial q_1} \frac{\partial q_1}{\partial y} + \frac{\partial X}{\partial q_1} \frac{\partial q_1}{\partial y} \right) \right\} dx dy \geq 0$$

According to Lemma 1, the inequality above can be further simplified to:

$$\int_{\hat{\theta}}^{\theta'} \int_{\hat{\theta}}^{\theta'} -\frac{d\nu}{dy} \delta \frac{d\psi(X(p_1(y), q_1(y), x))}{ds} dx dy \geq 0$$

or

$$\int_{\hat{\theta}}^{\theta'} \int_{\hat{\theta}}^{\theta'} \frac{d\nu}{dy} \delta \frac{d\psi(s)}{ds} dx dy \leq 0$$

Since  $\delta \frac{d\psi(s)}{ds} > 0$ , the inequality above is equivalent to:

$$\frac{d\nu}{d\theta} \leq 0$$

□

Theorem 3, below, defines incentive compatibility in terms of the monotonicity condition and a differential equation for our specific model.

**Theorem 3.** *The direct revelation mechanism  $g(\tilde{\theta})$  is incentive compatible if and only if, for all  $\theta \in [\underline{\theta}, \bar{\theta}]$ :*

$$(i) \quad \frac{dU}{d\theta} = -\frac{d\psi(s)}{ds} - \nu\delta\frac{d\psi(s)}{ds} \quad (20)$$

$$(ii) \quad \frac{d\nu}{d\tilde{\theta}} \leq 0 \quad (21)$$

*Proof.* The firm will choose to announce the value  $\tilde{\theta}$  that maximizes its utility. Thus, the problem of the firm is given by:

$$\max_{\tilde{\theta}} \phi(\theta, \tilde{\theta}) \quad (22)$$

Therefore, the first order condition for truth-telling is:

$$\phi_2(\theta, \theta) = 0, \text{ almost everywhere.} \quad (23)$$

where  $\phi_2$  is the derivative of  $\phi(\theta, \tilde{\theta})$  with respect to the second argument.

That is,

$$\begin{aligned} \phi_2(\theta, \theta) = & \frac{dt(\theta)}{d\tilde{\theta}} - \frac{d\psi(s)}{ds} \left( \frac{\partial X}{\partial p_1} \frac{\partial P_1}{\partial q_1} \frac{\delta q_1}{\partial \tilde{\theta}} + \frac{\partial X}{\partial q_1} \frac{\delta q_1}{\partial \tilde{\theta}} \right) \\ & - \frac{d\nu(\theta)}{d\tilde{\theta}} \psi(\delta s) - \nu(\theta)\delta \frac{d\psi(s)}{ds} \left( \frac{\partial X}{\partial p_1} \frac{\partial P_1}{\partial q_1} \frac{\delta q_1}{\partial \tilde{\theta}} + \frac{\partial X}{\partial q_1} \frac{\delta q_1}{\partial \tilde{\theta}} \right) = 0 \end{aligned} \quad (24)$$

almost everywhere. The first-order condition assures that there is no gain in locally deviating from the strategy of announcing the true value  $\theta$ . Theorem 4, *infra*, shows that, if the monotonicity condition is obeyed, this first order condition is sufficient for a global maximum.

Now, the first-order condition (24) is written with respect to the liquid transfer, and we want to rewrite it with respect to the firm's utility.

Let  $\theta^*$  be the solution of (22), then the firm's rent,  $U(\theta)$ , can be redefined as:

$$U(\theta) \equiv \phi(\theta, \theta^*) = t(\theta^*) - \psi(X(p_1^*, q_1^*, \theta)) - \nu(\theta^*)\psi(\delta X(p_1^*, q_1^*, \theta))$$

If the first-order condition is obeyed, then  $\theta^* = \theta$ . Following Laffont and Martimort (2002, p. 136), we can apply the Envelope Theorem to obtain:

$$\begin{aligned} \frac{dU}{d\theta} = & \frac{dt(\theta)}{d\theta} - \frac{d\psi(s)}{ds} \left( \frac{\partial X}{\partial p_1} \frac{\partial P_1}{\partial q_1} \frac{\delta q_1}{\partial \theta} + \frac{\partial X}{\partial q_1} \frac{\delta q_1}{\partial \theta} + \frac{\partial X}{\partial \theta} \right) \\ & - \frac{d\nu(\theta)}{d\theta} \psi(\delta s) - \nu(\theta)\delta \frac{d\psi(s)}{ds} \left( \frac{\partial X}{\partial p_1} \frac{\partial P_1}{\partial q_1} \frac{\delta q_1}{\partial \theta} + \frac{\partial X}{\partial q_1} \frac{\delta q_1}{\partial \theta} + \frac{\partial X}{\partial \theta} \right) \end{aligned}$$

Because of the first-order condition (24) and because  $\frac{\partial X}{\partial \theta} = 1$ , we can rewrite the

equation above as:

$$\frac{dU}{d\theta} = -\frac{d\psi(s)}{ds} - \nu\delta\frac{d\psi(s)}{ds}$$

□

In the following theorem we will know show that, if the first-order condition described in Theorem 3 is obeyed, then the monotonicity condition is sufficient for a global maximum.

**Theorem 4.** (Sufficiency of the monotonicity condition). *Let  $\phi(\theta, \tilde{\theta})$  be the firm's utility as a function of the true value of  $\theta$  and the announced value  $\tilde{\theta}$ , so that the problem of the firm is:*

$$\max_{\tilde{\theta}} \phi(\theta, \tilde{\theta})$$

*Then conditions (20) and (21) are sufficient for a global maximum.*

*Proof.* Assume, for the sake of contradiction, that the firm strictly prefers to announce  $\tilde{\theta} \neq \theta$ . Therefore:

$$\begin{aligned} \phi(\theta, \tilde{\theta}) &> \phi(\theta, \theta), \text{ or} \\ \phi(\theta, \tilde{\theta}) - \phi(\theta, \theta) &> 0 \end{aligned}$$

According to the Fundamental Theorem of Calculus, the inequality above can be rewritten as:

$$\int_{\theta}^{\tilde{\theta}} \phi_2(\theta, y) dy > 0$$

where  $\phi_2$  is the derivative of  $\phi(\theta, \tilde{\theta})$  with respect to the second argument.

Following the first-order condition, we can write:

$$\int_{\theta}^{\tilde{\theta}} \{\phi_2(\theta, y) - \phi_2(y, y)\} dy > 0$$

Since  $\phi_2(y, y) = 0$ . Or, equivalently,

$$\int_{\theta}^{\tilde{\theta}} \int_y^{\theta} \phi_{12}(x, y) dx dy > 0 \tag{25}$$

where  $\phi_{12}$  is the cross derivative of  $\phi(\theta, \tilde{\theta})$  with respect to the first and second arguments.

Function  $\phi_{12}(y, x)$  exists almost everywhere and, because of the monotonicity

condition, it satisfies:

$$\begin{aligned}\phi_{12}(x, y) &= \frac{\partial}{\partial y} \left[ -\frac{d\psi(s)}{ds} - \nu(y)\delta\frac{d\psi(s)}{ds} \right] \\ &= -\frac{d\nu}{dy}\delta\frac{d\psi(s)}{ds} \geq 0\end{aligned}$$

Suppose that  $\tilde{\theta} > \theta$ , so that  $y \geq \theta$  for all  $y \in [\theta, \tilde{\theta}]$ . Therefore, integral (25) will be non-positive because of the innermost integral, a contradiction.

Suppose that  $\tilde{\theta} < \theta$ , so that  $y \leq \theta$  for all  $y \in [\theta, \tilde{\theta}]$ . Therefore, integral (25) will be non-positive because of the outermost integral, a contradiction.  $\square$

Together with the conditions for truth-telling that we just saw above, we must also make sure that, for whatever value of  $\theta$ , the firm's participation constraint is respected, i.e., that the firm will obtain a non-negative utility. The theorem below defines this restriction.

**Theorem 5.** *For any  $\theta \in [\underline{\theta}, \bar{\theta}]$ , the firm's participation constraint is satisfied if:*

$$U(\bar{\theta}) \geq 0 \quad (26)$$

*Proof.* This comes directly from the fact due to equation (20), the firm's utility is strictly decreasing, that is,  $\frac{dU}{d\theta} < 0$ . Ergo, the participation constraint with respect to type  $\bar{\theta}$  ensures participation for all possible values of  $\theta$ .  $\square$

#### 5.4 Social welfare optimization problem

The government's problem is to maximize the expected social welfare by implementing the truthful direct revelation mechanism we have defined *supra*.

Social welfare is given by:

$$\begin{aligned}W = \int_{\underline{\theta}}^{\bar{\theta}} \left\{ S_1(q_1, s, \theta) + \lambda P_1 q_1 + \nu S_2(q_2, \delta s) + \lambda \nu R - (1 + \lambda)[K_1 \right. \\ \left. + \nu K_2(\nu) + C_1(q_1) + \nu C_2(q_2) + \psi(s) + \nu \psi(\delta s)] - \lambda U \right\} dF(\theta) \quad (27)\end{aligned}$$

As discussed in subsection 5.2, in order to implement mechanism  $g(\tilde{\theta})$ , the government maximizes  $W$  with respect to the control variables, keeping the incentive compatibility and participation constraints.

We want to substitute the firm's utility  $U$  out of equation (27). From the incentive

compatibility constraint (20), we know that condition

$$\frac{dU(\theta)}{d\theta} = -\frac{d\psi(s)}{ds} - \nu\delta\frac{d\psi(s)}{ds}$$

must be satisfied. Thus, we can integrate both sides by  $\theta$ , obtaining:

$$\int_{\bar{\theta}}^{\theta} \frac{dU}{d\theta} d\theta = \int_{\bar{\theta}}^{\theta} \left\{ -\frac{d\psi(X(p_1, q_1, x))}{ds} - \nu\delta\frac{d\psi(X(p_1, q_1, x))}{ds} \right\} dx$$

Because  $s = X(p_1, q_1, \theta)$ . Solving the integral on the left-hand side yields:

$$U(\theta) - U(\bar{\theta}) = \int_{\bar{\theta}}^{\theta} \left\{ -\frac{d\psi(X(p_1, q_1, x))}{ds} - \nu\delta\frac{d\psi(X(p_1, q_1, x))}{ds} \right\} dx$$

Since it is socially costly to make transfers to the firm, the optimum requires the participation constraint (26) to be binding for type  $\bar{\theta}$ , that is,  $U(\bar{\theta}) = 0$ . So, we can write:

$$U(\theta) = \int_{\bar{\theta}}^{\theta} \left\{ -\frac{d\psi(X(p_1, q_1, x))}{ds} - \nu\delta\frac{d\psi(X(p_1, q_1, x))}{ds} \right\} dx$$

Integrating both sides yields:

$$\int_{\underline{\theta}}^{\bar{\theta}} U(\theta) dF(\theta) = \int_{\underline{\theta}}^{\bar{\theta}} \left\{ \int_{\underline{\theta}}^{\bar{\theta}} \left\{ -\frac{d\psi(X(p_1, q_1, x))}{ds} - \nu\delta\frac{d\psi(X(p_1, q_1, x))}{ds} \right\} dx \right\} dF(\theta)$$

Since  $dF(\theta) = f(\theta)d\theta$ , we can write:

$$\int_{\underline{\theta}}^{\bar{\theta}} U(\theta) dF(\theta) = \int_{\underline{\theta}}^{\bar{\theta}} \left\{ \int_{\underline{\theta}}^{\bar{\theta}} \left\{ -\frac{d\psi(X(p_1, q_1, x))}{ds} - \nu\delta\frac{d\psi(X(p_1, q_1, x))}{ds} \right\} dx f(\theta) \right\} d\theta$$

Integrating by parts, we obtain:

$$\begin{aligned} \int_{\underline{\theta}}^{\bar{\theta}} U(\theta) dF(\theta) &= \left[ \int_{\underline{\theta}}^{\bar{\theta}} \left\{ -\frac{d\psi(X(p_1, q_1, x))}{ds} - \nu\delta\frac{d\psi(X(p_1, q_1, x))}{ds} \right\} dx F(\theta) \right] \Big|_{\underline{\theta}}^{\bar{\theta}} \\ &\quad - \int_{\underline{\theta}}^{\bar{\theta}} \left\{ \frac{d \left[ \int_{\underline{\theta}}^{\bar{\theta}} \left\{ -\frac{d\psi(X(p_1, q_1, x))}{ds} - \nu\delta\frac{d\psi(X(p_1, q_1, x))}{ds} \right\} dx \right]}{d\theta} \right\} F(\theta) d\theta \end{aligned}$$

The first term is zero and, thus, we get:

$$\int_{\underline{\theta}}^{\bar{\theta}} U(\theta) dF(\theta) = - \int_{\underline{\theta}}^{\bar{\theta}} \left\{ \frac{d \left[ \int_{\underline{\theta}}^{\bar{\theta}} \left\{ -\frac{d\psi(X(p_1, q_1, x))}{ds} - \nu\delta\frac{d\psi(X(p_1, q_1, x))}{ds} \right\} dx \right]}{d\theta} \right\} F(\theta) d\theta$$

According to the Fundamental Theorem of Calculus, we obtain:

$$\int_{\underline{\theta}}^{\bar{\theta}} U(\theta) dF(\theta) = - \int_{\underline{\theta}}^{\bar{\theta}} \left\{ \left( -\frac{d\psi(X(p_1, q_1, \theta))}{ds} - \nu\delta \frac{d\psi(X(p_1, q_1, \theta))}{ds} \right) F(\theta) \right\} d\theta$$

or

$$\int_{\underline{\theta}}^{\bar{\theta}} U(\theta) dF(\theta) = \int_{\underline{\theta}}^{\bar{\theta}} \left\{ \left( \frac{d\psi(X(p_1, q_1, \theta))}{ds} + \nu\delta \frac{d\psi(X(p_1, q_1, \theta))}{ds} \right) F(\theta) \right\} d\theta$$

We can multiply the integrand by  $\frac{f(\theta)}{f(\theta)}$ :

$$\int_{\underline{\theta}}^{\bar{\theta}} U(\theta) dF(\theta) = \int_{\underline{\theta}}^{\bar{\theta}} \left\{ \left( \frac{d\psi(X(p_1, q_1, \theta))}{ds} + \nu\delta \frac{d\psi(X(p_1, q_1, \theta))}{ds} \right) \frac{F(\theta)}{f(\theta)} f(\theta) \right\} d\theta$$

Since  $dF(\theta) = f(\theta)d\theta$ , we can write:

$$\int_{\underline{\theta}}^{\bar{\theta}} U(\theta) dF(\theta) = \int_{\underline{\theta}}^{\bar{\theta}} \left\{ \left( \frac{d\psi(s)}{ds} + \nu\delta \frac{d\psi(s)}{ds} \right) \frac{F(\theta)}{f(\theta)} \right\} dF(\theta) \quad (28)$$

Which is the firm's expected rent.

Now, we can substitute (28) in the social welfare equation (27) and write the government's maximization problem:

$$\begin{aligned} \max_{q_1, q_2, \nu, s} \int_{\underline{\theta}}^{\bar{\theta}} \left\{ S_1(q_1, s, \theta) + \lambda P_1 q_1 + \nu S_2(q_2, \delta s) + \lambda \nu R - (1 + \lambda)[K_1 + \nu K_2(\nu) \right. \\ \left. + C_1(q_1) + \nu C_2(q_2) + \psi(s) + \nu\psi(\delta s)] - \lambda \frac{F(\theta)}{f(\theta)} \left( \frac{d\psi(s)}{ds} + \nu\delta \frac{d\psi(s)}{ds} \right) \right\} dF(\theta) \\ s.t. \quad \frac{d\nu}{d\theta} \leq 0 \end{aligned} \quad (29)$$

## 5.5 Optimal regulation under asymmetric information

Let's first analyze the case where the monotonicity condition is satisfied for all possible types. We will then see what happens if this condition is not satisfied.

**Theorem 6.** *If  $\frac{d\nu}{d\theta} < 0$ ,  $\forall \theta \in [\underline{\theta}, \bar{\theta}]$ , then optimal regulation under asymmetric information*

is defined by the first-order conditions given below:

$$(i) \frac{p_1 - \frac{dC_1(q_1)}{dq_1}}{p_1} = \frac{\lambda}{1 + \lambda} \frac{1}{|\eta|}$$

$$(ii) \frac{\partial S_2(q_2)}{\partial q_2} = (1 + \lambda) \frac{dC_2(q_2)}{dq_2}$$

$$(iii) K_2(\nu) + \nu \frac{dK_2(\nu)}{d\nu} = \frac{1}{1 + \lambda} [S_2(q_2, \delta s) + \lambda R] - C_2(q_2) - \psi(\delta s) - \frac{\lambda}{1 + \lambda} \frac{F(\theta)}{f(\theta)} \delta \frac{d\psi(s)}{ds}$$

$$(iv) \frac{1}{1 + \lambda} \left[ \frac{\partial S_1(q_1, s, \theta)}{\partial s} + \nu \delta \frac{\partial S_2(q_2, s)}{\partial s} \right] + \frac{\lambda}{1 + \lambda} \frac{\partial P_1}{\partial s} q_1 \\ = \frac{d\psi(s)}{ds} + \nu \delta \frac{d\psi(s)}{ds} + \frac{\lambda}{1 + \lambda} \frac{F(\theta)}{f(\theta)} \left[ \frac{d^2\psi(s)}{ds^2} + \nu \delta \frac{d^2\psi(s)}{ds^2} \right]$$

*Proof.* Differentiating (29) with respect to  $q_1$  yields:

$$\frac{\partial S_1(q_1, s, \theta)}{\partial q_1} + \lambda \frac{\partial P_1}{\partial q_1} q_1 + \lambda P_1 - (1 + \lambda) \frac{dC_1}{dq_1} = 0 \quad (30)$$

Which is the same condition we found on the model with complete information, thus yielding the same result:

$$\frac{p_1 - \frac{dC_1(q_1)}{dq_1}}{p_1} = \frac{\lambda}{1 + \lambda} \frac{1}{|\eta|} \quad (31)$$

Regarding  $q_2$ , it is easy to see that no distortion with respect to the first-best will be made. This is due to the fact that, in our model, consumers in the poor area do not change their consumption because of quality, and the effort to provide quality increases with network extension, but not with quantity. Therefore, rents are not affected by  $q_2$ .

With respect to  $\nu$ , the first-order condition implies:

$$S_2(q_2, \delta s) + \lambda R - (1 + \lambda)[K_2(\nu) + \nu K_2'(\nu) + C_2(q_2) + \psi(\delta s)] - \lambda \frac{F(\theta)}{f(\theta)} \delta \psi_s(s) = 0$$

Rearranging the equation, we get:

$$(1 + \lambda)[K_2(\nu) + \nu \frac{dK_2(\nu)}{d\nu}] = S_2(q_2, \delta s) + \lambda R - (1 + \lambda)[C_2(q_2) + \psi(\delta s)] - \lambda \frac{F(\theta)}{f(\theta)} \delta \psi_s(s)$$

and, therefore,

$$K_2(\nu) + \nu \frac{dK_2(\nu)}{d\nu} = \frac{1}{1 + \lambda} [S_2(q_2, \delta s) + \lambda R] - C_2(q_2) - \psi(\delta s) - \frac{\lambda}{1 + \lambda} \frac{F(\theta)}{f(\theta)} \delta \psi_s(s) \quad (32)$$



Differentiating with respect to  $s$ , we obtain:

$$\begin{aligned} \frac{\partial S_1(q_1, s, \theta)}{\partial s} + \lambda \frac{\partial P_1}{\partial s} q_1 + \nu \delta \frac{\partial S_2(q_2, s)}{\partial s} - (1 + \lambda) \left[ \frac{d\psi(s)}{ds} + \nu \delta \frac{d\psi(s)}{ds} \right] \\ - \lambda \frac{F(\theta)}{f(\theta)} \left[ \frac{d^2\psi(s)}{ds^2} + \nu \delta \frac{d^2\psi(s)}{ds^2} \right] = 0 \end{aligned}$$

or

$$\begin{aligned} \frac{1}{1 + \lambda} \left[ \frac{\partial S_1(q_1, s, \theta)}{\partial s} + \nu \delta \frac{\partial S_2(q_2, s)}{\partial s} \right] + \frac{\lambda}{1 + \lambda} \frac{\partial P_1}{\partial s} q_1 \\ = \frac{d\psi(s)}{ds} + \nu \delta \frac{d\psi(s)}{ds} + \frac{\lambda}{(1 + \lambda)} \frac{F(\theta)}{f(\theta)} \left[ \frac{d^2\psi(s)}{ds^2} + \nu \delta \frac{d^2\psi(s)}{ds^2} \right] \quad (33) \end{aligned}$$

□

If the solution above does not satisfies the monotonicity condition for all possible types, then we are faced with a situation called *nonresponsiveness* by Laffont and Guesnerie (1984). According to Laffont and Martimort (2002, p. 54), nonresponsiveness occurs when there is a conflict between the Principal's desire to increase efficiency and the monotonicity condition imposed by asymmetry of information. In such cases, the Principal – in our model, the government – is forced to use a pooling allocation.

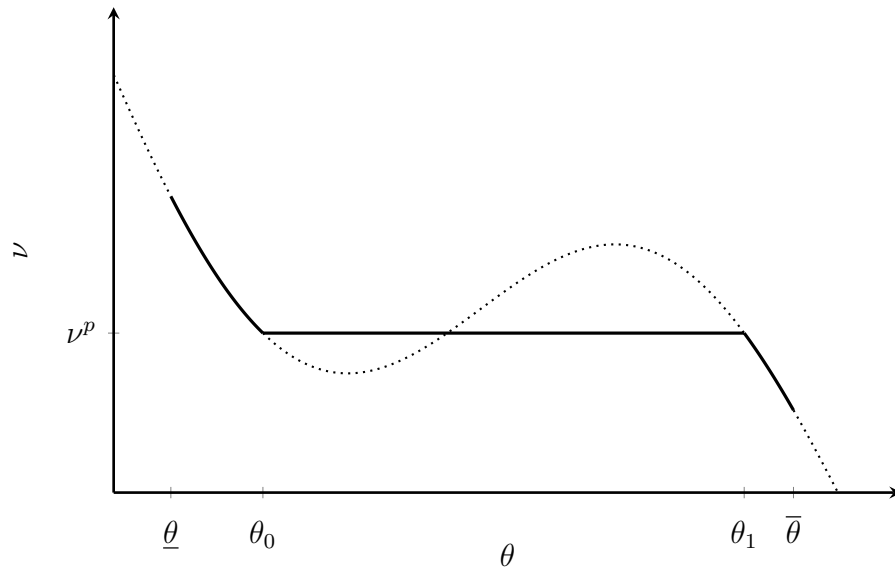
Pooling, also called bunching, happens when the Principal implements the same value for a contractable variable across several different types. In our specific model, pooling might occur exclusively with respect to the variable  $\nu$ , since the monotonicity condition is affected only by network extension, with the other variables following the first-order condition from Theorem 6.

In this case, the monotonicity condition is trivially satisfied, since constant  $\nu$  implies that  $\frac{d\nu}{d\theta} = 0$ . Notice that pooling will not necessarily occur for all types, but only when required by optimality. Also important is the fact that pooling might be extended beyond the interval where  $\frac{d\nu}{d\theta} \leq 0$  is not satisfied by the first-order conditions from Theorem 6, since the Pontryagin Maximum Principle, used to solve such cases, requires continuity.

We can see below a graphical example<sup>6</sup> of pooling for a single increasing interval on a arbitrary function of  $\nu$ . The dotted line represents the function given by the first-order condition for  $\nu$ , while the network extension actually implemented by regulation is represented by the continuous line. Pooling occurs on the interval  $[\theta_0, \theta_1] \subset [\underline{\theta}, \bar{\theta}]$ . Notice how continuity might require pooling even in subintervals where the function  $\nu$  is non-increasing.

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<sup>6</sup>Our graphical example is based on Laffont and Martimort (2002, p. 142)



The monotonicity condition might be violated in one or more intervals. See Laffont and Guesnerie (1984) for a moral general solution with several intervals.

## 6 RESULTS

We now analyze the results obtained from our model, both in the complete and incomplete information scenarios.

First, we show the distortions created in optimal regulation by the presence of asymmetric information. Next, we study how regulation affects the provision of quality by the firm. We then analyze the effect of the social cost of public funds in quality provision and network extension and the relationship between these two variables. Finally, we consider how the presence of Universal Service Obligation affects quality provision.

### 6.1 Distortions with respect to the first-best

#### 6.1.1 Distortion with respect to quality

**Corollary 1.** *Quality is distorted downward in the presence of asymmetric information.*

*Proof.* Second-best quality level is defined by equation (33):

$$\begin{aligned} \frac{1}{(1+\lambda)} \frac{\partial S_1(q_1, s, \theta)}{\partial s} + \frac{\lambda}{(1+\lambda)} \frac{\partial P_1}{\partial s} q_1 + \frac{1}{(1+\lambda)} \nu \delta \frac{\partial S_2(q_2, s)}{\partial s} \\ = \frac{d\psi(s)}{ds} + \nu \delta \frac{d\psi(s)}{ds} + \frac{\lambda}{(1+\lambda)} \frac{F(\theta)}{f(\theta)} \left[ \frac{d^2\psi(s)}{ds^2} + \nu \delta \frac{d^2\psi(s)}{ds^2} \right] \end{aligned}$$

Comparing this equation with equation (12), we can conclude that distortion with respect to the first-best scenario is given by the distortionary term

$$\frac{\lambda}{1+\lambda} \frac{F(\theta)}{f(\theta)} \left[ \frac{d^2\psi(s)}{ds^2} + \nu \delta \frac{d^2\psi(s)}{ds^2} \right] \quad (34)$$

which is clearly positive.

Since the right-hand side of equation (33) represents the non-monetary cost of providing quality, the distortionary term given by (34) increases this cost. To balance the equality, quality needs to be distorted downward.  $\square$

This downward distortion can be explained by the fact that, since implementing higher quality requires giving higher rents, it is socially optimal for the regulator to decrease the implemented level of quality. Indeed, the effect of quality on the firm's expected rent can be seen in equation (28):

$$\int_{\underline{\theta}}^{\bar{\theta}} U(\theta) dF(\theta) = \int_{\underline{\theta}}^{\bar{\theta}} \left\{ \left( \frac{d\psi(s)}{ds} + \nu \delta \frac{d\psi(s)}{ds} \right) \frac{F(\theta)}{f(\theta)} \right\} dF(\theta)$$

which increases with  $s$ .

### 6.1.2 Distortion with respect to price and quantity

**Corollary 2.** *The Ramsey pricing rule is unaffected by the presence of asymmetric information.*

Corollary 2 comes directly by comparing equations (10) and (31).

This result is explained by the fact that, in our model, pricing is not used to extract part of the firm's informational rent. Incentives for truth-telling are provided entirely by transfers and the reimbursement rule<sup>7</sup>, so that it is not necessary to adjust the pricing rule according to the revealed type. Laffont and Tirole (1993) call this phenomenon the incentive-pricing dichotomy.

**Corollary 3.** *Quantity is distorted downward in the presence of asymmetric information.*

*Proof.* This corollary comes directly from the fact the the government uses quantity and prices to indirectly contract quality. Since quality is distorted downward and pricing is unaffected, we can conclude that quantity and prices also decrease with respect to the first-best. Indeed, from equations (30) we have:

$$\frac{\partial S_1(q_1, s, \theta)}{\partial q_1} + \lambda \frac{\partial P_1(q_1, s, \theta)}{\partial q_1} q_1 + \lambda P_1(q_1, s, \theta) = (1 + \lambda) \frac{dC_1(q_1)}{dq_1} \quad (35)$$

For both first and second-best scenarios. However, since  $s^{sb} < s^{fb}$ , we know that:

$$\begin{aligned} \frac{\partial S_1(q_1, s^{sb}, \theta)}{\partial q_1} + \lambda \frac{\partial P_1(q_1, s^{sb}, \theta)}{\partial q_1} q_1 + \lambda P_1(q_1, s^{sb}, \theta) \\ < \frac{\partial S_1(q_1, s^{fb}, \theta)}{\partial q_1} + \lambda \frac{\partial P_1(q_1, s^{fb}, \theta)}{\partial q_1} q_1 + \lambda P_1(q_1, s^{fb}, \theta) \end{aligned}$$

Therefore, in order to balance equation (35),  $q_1^{sb}$  needs to be distorted downward, so that  $q_1^{sb} < q_1^{fb}$ .  $\square$

Notice that there is no contradiction between Corollaries 2 and 3, because the Ramsey pricing rule is a relationship between price and quantity. So, if quantity is decreased, prices are adjusted accordingly to keep the Ramsey pricing rule.

### 6.1.3 Distortion with respect to $\theta$ network extension

**Corollary 4.** *Network extension is distorted downward in the presence of asymmetric information.*

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<sup>7</sup>That is, the rule where the government receives the firm's income from sales and reimburses it for its fixed and variable costs.

*Proof.* Second-best network extension is defined by equation (32):

$$K_2(\nu) + \nu \frac{dK_2(\nu)}{d\nu} = \frac{1}{1+\lambda} [S_2(q_2, \delta s) + \lambda R] - C_2(q_2) - \psi(\delta s) - \frac{\lambda}{1+\lambda} \frac{F(\theta)}{f(\theta)} \delta \psi_s(s)$$

Since  $K_2(\nu) > 0$  and  $\frac{dK_2(\nu)}{d\nu} > 0$ , then  $\nu$  increases as the right-hand side of the equation above increases, and decreases as the right-hand side of the equation above decreases.

Comparing the equation above with equation (11), we can conclude that distortion with respect to the first-best scenario is given by the term

$$-\frac{\lambda}{1+\lambda} \frac{F(\theta)}{f(\theta)} \delta \psi_s(s)$$

which is clearly negative. Therefore,  $\nu^{sb} < \nu^{fb}$ .  $\square$

This distortion can be explained by the same reasoning used in subsection 6.1.1. According to equation (28), the firm's expected rent is increasing in  $\nu$ . Therefore, asymmetry of information makes the expansion of the network more expensive, and, consequently, it is socially optimal to decrease its extension.

## 6.2 Effects of regulation on quality provision

Regulation allows the implementation of the socially optimal level of quality because the government doesn't take into consideration exclusively the preferences of the marginal consumer, but also the preferences of all consumers. That is to say, contrary to an unregulated firm, the government takes into consideration the social gains of quality provision that are not expressed in prices.

To better understand this, let's suppose that the government implements a market solution with respect to  $s$ . In other words, the government proposes a contract determining price or quantity and the network extension, but it does not regulate quality. Evidently, this situation cannot be optimal from the point of view of social welfare, but is a frequent practice in regulation<sup>8</sup>.

In this market solution there is no asymmetry of information, so that conditions (i) to (iii) from Theorem 1 are still applicable, but quality is implemented according to the Corollary below.

**Corollary 5.** *In the market solution with respect to quality,  $s^m$  is given by*

$$\frac{\partial P_1(q_1^m, s^m, \theta)}{\partial s} q_1^m = \frac{d\psi(s^m)}{ds} + \nu^m \delta \frac{d\psi(s^m)}{ds} \quad (36)$$

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<sup>8</sup>We can assume that this is due to lack of regulatory expertise in developing countries.

Proof comes directly from the fact that the firm maximizes its profit by equating the marginal revenue from quality and the marginal effort in providing it.

**Corollary 6.** *The regulated firm implements the socially optimal level of quality.*

Proof comes directly from the government's maximization program.

Indeed, without regulation, the firm implements quality according to Corollary 5. In this case, the firm looks exclusively to the marginal consumer, whose preferences are not a good indicator of the average quality demand from all consumers. Therefore, if the marginal consumer values quality less than the average quality demand from consumers, the firm will undersupply quality in the market solution. Conversely, if the marginal consumer values quality more than the average quality demand from consumers, the firm will oversupply quality in the market solution. Thus, in this aspect, our model reaches the same conclusion from Spence (1975).

If transfers to the firm were not socially costly, then the government would choose quality by simply equating the marginal social benefit with the marginal effort of providing it. However, since quality is financed by transfers, the government needs to take into consideration the social cost of public funds. In fact, from equations (12) and (33) we know that the marginal benefit from quality is given by:

$$\frac{1}{1 + \lambda} \left[ \frac{\partial S_1(q_1, s, \theta)}{\partial s} + \nu \delta \frac{\partial S_2(q_2, s)}{\partial s} \right] + \frac{\lambda}{1 + \lambda} \frac{\partial P_1}{\partial s} q_1$$

which is clearly an average between the marginal social benefit and the firm's marginal revenue, weighted by the inverse of the social cost of public funds. Therefore, the existence of social cost of public funds makes the socially optimal solution second-best in its nature, even with complete information.

**Corollary 7.** *Regulation takes into consideration the marginal social benefit of quality in the poor area, which has a positive effect on quality.*

*Proof.* In the market solution, the firm considers only the preference of the marginal consumer in the rich area. However, since it cannot discriminate quality between areas, marginal effort in the poor area, given by  $\nu \delta \frac{d\psi(s)}{ds}$ , is taken into account by the firm. Hence, the inclusion, through regulation, of the marginal social benefit in the poor area has a positive effect on quality with respect to the market solution.  $\square$

Notice that the analysis carried out in this subsection is valid for both complete and incomplete information scenarios.

We now investigate how quality varies according to the value of  $\theta$ . Despite the fact that demand for quality increases with  $\theta$ , optimal quality implemented through regulation might actually decrease with  $\theta$  in specific intervals. This is due to the fact that two other

factors affect the optimal level of quality: the effect of quality in the poor area and, in the case with asymmetry of information, the effect of quality on informational rents.

Let's start our analysis considering the case with complete information. Differentiating both sides of equation (12) by  $\theta$  yields:

$$\begin{aligned} \frac{1}{1+\lambda} \left[ \frac{\partial^2 S_1}{\partial s \partial q_1} \frac{\partial q_1}{\partial \theta} + \frac{\partial^2 S_1}{\partial s^2} \frac{\partial s}{\partial \theta} + \frac{\partial^2 S_1}{\partial s \partial \theta} \right] \\ + \frac{1}{1+\lambda} \left[ \frac{\partial \nu}{\partial \theta} \delta \frac{\partial S_2}{\partial s} + \nu \delta \frac{\partial^2 S_2}{\partial s^2} \frac{\partial s}{\partial \theta} \right] + \frac{\lambda}{1+\lambda} \left[ \frac{\partial P_1}{\partial s} \frac{\partial q_1}{\partial \theta} \right] \\ - \frac{d^2 \psi(s)}{ds^2} \frac{\partial s}{\partial \theta} - \frac{\partial \nu}{\partial \theta} \delta \frac{d\psi(s)}{ds} - \nu \delta \frac{d^2 \psi(s)}{ds^2} \frac{\partial s}{\partial \theta} = 0 \end{aligned}$$

Factoring  $\frac{\partial s}{\partial \theta}$  and isolating the rest yields:

$$\begin{aligned} \frac{\partial s}{\partial \theta} \left[ \frac{1}{1+\lambda} \left( \frac{\partial^2 S_1}{\partial s^2} + \nu \delta \frac{\partial^2 S_2}{\partial s^2} \right) - \frac{d^2 \psi(s)}{ds^2} - \nu \delta \frac{d^2 \psi(s)}{ds^2} \right] \\ = - \frac{1}{1+\lambda} \left[ \frac{\partial^2 S_1}{\partial s \partial q_1} \frac{\partial q_1}{\partial \theta} + \frac{\partial^2 S_1}{\partial s \partial \theta} + \lambda \frac{\partial P_1}{\partial s} \frac{\partial q_1}{\partial \theta} \right] - \frac{\partial \nu}{\partial \theta} \delta \left[ \frac{1}{1+\lambda} \frac{\partial S_2}{\partial s} - \frac{d\psi(s)}{ds} \right] \end{aligned}$$

Since  $\frac{\partial^2 S_1}{\partial s \partial q_1} = \frac{\partial^2 S_1}{\partial q_1 \partial s} = \frac{\partial P_1}{\partial s}$ , we can write:

$$\begin{aligned} \frac{\partial s}{\partial \theta} \left[ \frac{1}{1+\lambda} \left( \frac{\partial^2 S_1}{\partial s^2} + \nu \delta \frac{\partial^2 S_2}{\partial s^2} \right) - \frac{d^2 \psi(s)}{ds^2} - \nu \delta \frac{d^2 \psi(s)}{ds^2} \right] \\ = - \frac{1}{1+\lambda} \frac{\partial^2 S_1}{\partial s \partial \theta} - \frac{\partial P_1}{\partial s} \frac{\partial q_1}{\partial \theta} - \frac{\partial \nu}{\partial \theta} \delta \left[ \frac{1}{1+\lambda} \frac{\partial S_2}{\partial s} - \frac{d\psi(s)}{ds} \right] \end{aligned}$$

or, reversing the sign,

$$\begin{aligned} \frac{\partial s}{\partial \theta} \left[ - \frac{1}{1+\lambda} \left( \frac{\partial^2 S_1}{\partial s^2} + \nu \delta \frac{\partial^2 S_2}{\partial s^2} \right) + \frac{d^2 \psi(s)}{ds^2} + \nu \delta \frac{d^2 \psi(s)}{ds^2} \right] \\ = \frac{1}{1+\lambda} \frac{\partial^2 S_1}{\partial s \partial \theta} + \frac{\partial P_1}{\partial s} \frac{\partial q_1}{\partial \theta} + \frac{\partial \nu}{\partial \theta} \delta \left[ \frac{1}{1+\lambda} \frac{\partial S_2}{\partial s} - \frac{d\psi(s)}{ds} \right] \quad (37) \end{aligned}$$

Let  $\phi = - \frac{1}{1+\lambda} \left( \frac{\partial^2 S_1}{\partial s^2} + \nu \delta \frac{\partial^2 S_2}{\partial s^2} \right) + \frac{d^2 \psi(s)}{ds^2} + \nu \delta \frac{d^2 \psi(s)}{ds^2}$  so that:

$$\frac{\partial s}{\partial \theta} = \frac{\frac{1}{1+\lambda} \frac{\partial^2 S_1}{\partial s \partial \theta} + \frac{\partial P_1}{\partial s} \frac{\partial q_1}{\partial \theta} + \frac{\partial \nu}{\partial \theta} \delta \left[ \frac{1}{1+\lambda} \frac{\partial S_2}{\partial s} - \frac{d\psi(s)}{ds} \right]}{\phi}$$

Now,  $\phi$  is clearly positive, because  $\frac{\partial^2 S_1}{\partial s^2} < 0$ . So  $\frac{\partial s}{\partial \theta}$  will have the same sign of the numerator in the fraction above, that is:

$$\frac{1}{1+\lambda} \frac{\partial^2 S_1}{\partial s \partial \theta} + \frac{\partial P_1}{\partial s} \frac{\partial q_1}{\partial \theta} + \frac{\partial \nu}{\partial \theta} \delta \left[ \frac{1}{1+\lambda} \frac{\partial S_2}{\partial s} - \frac{d\psi(s)}{ds} \right] \quad (38)$$

The first term is always positive. The second term is ambiguous because of  $\frac{\partial q_1}{\partial \theta}$ . The third term is also ambiguous: it measures the welfare effect that an increasing  $s$  have on the poor area. This term will have a negative effect on  $\frac{\partial s}{\partial \theta}$  if the incremental welfare gain from quality,  $\frac{\partial S_2}{\partial s} - \frac{d\psi(s)}{ds}$ , is negative, and network extension is increasing in  $\theta$ ; or if the incremental welfare gain from quality,  $\frac{\partial S_2}{\partial s} - \frac{d\psi(s)}{ds}$ , is positive, but network extension is decreasing in  $\theta$ .

If the sum of the negative effects is sufficiently large on a certain interval of  $\theta$ , then  $\frac{\partial s}{\partial \theta}$  will be negative on this interval. Otherwise,  $\frac{\partial s}{\partial \theta}$  will be positive.

Now, let's analyze the case with asymmetric information. We can define:

$$\phi^{sb} = -\frac{1}{1+\lambda} \left( \frac{\partial^2 S_1}{\partial s^2} + \nu \delta \frac{\partial^2 S_2}{\partial s^2} \right) + \frac{d^2 \psi(s)}{ds^2} + \nu \delta \frac{d^2 \psi(s)}{ds^2} + \frac{\lambda}{1+\lambda} \frac{F(\theta)}{f(\theta)} \left( \frac{d^3 \psi(s)}{ds^3} + \nu \delta \frac{d^3 \psi(s)}{ds^3} \right)$$

The term above, again, is always positive. We can thus focus our attention exclusively on the numerator, which for this case is:

$$\begin{aligned} \frac{1}{1+\lambda} \frac{\partial^2 S_1}{\partial s \partial \theta} + \frac{\partial P_1}{\partial s} \frac{\partial q_1}{\partial \theta} - \left| \frac{\partial \nu}{\partial \theta} \right| \delta \left[ \frac{1}{1+\lambda} \frac{\partial S_2}{\partial s} - \frac{d\psi(s)}{ds} - \frac{\lambda}{1+\lambda} \frac{F(\theta)}{f(\theta)} \frac{d^2 \psi(s)}{ds^2} \right] \\ - \frac{\lambda}{1+\lambda} \frac{d \left( \frac{F(\theta)}{f(\theta)} \right)}{d\theta} \left[ \frac{d^2 \psi(s)}{ds^2} + \nu \delta \frac{d^2 \psi(s)}{ds^2} \right] \quad (39) \end{aligned}$$

The analysis is similar to the complete information scenario, but with two differences: first, because of the monotonicity condition,  $\frac{\partial \nu}{\partial \theta} \leq 0$  for all  $\theta$ , that is, network extension is non-increasing in  $\theta$ .

Therefore, the sign of the second term depends exclusively on the social marginal welfare effect from quality in the poor area, given by  $\frac{\partial S_2}{\partial s} - \frac{d\psi(s)}{ds} - \frac{\lambda}{1+\lambda} \frac{F(\theta)}{f(\theta)} \frac{d^2 \psi(s)}{ds^2}$ . Second, we now need to take into account the fact that rents are increasing in  $s$ , which has a negative effect on optimal quality. Therefore, once again,  $\frac{\partial s}{\partial \theta}$  might be positive or negative.

### 6.3 Effects of the social cost of public funds on quality provision

Since quality is partly financed through taxation, then a higher social cost of public funds has effects on quality provision.

Let's take the extreme cases to illustrate. First, consider the case with complete information. If the social cost of public funds diverges to infinity, that is,  $\lambda \rightarrow \infty$ , we have:

$$\lim_{\lambda \rightarrow \infty} \left\{ \frac{1}{1+\lambda} \left[ \frac{\partial S_1(q_1, s, \theta)}{\partial s} + \nu \delta \frac{\partial S_2(q_2, s)}{\partial s} \right] + \frac{\lambda}{1+\lambda} \frac{\partial P_1}{\partial s} q_1 = \frac{d\psi(s)}{ds} + \nu \delta \frac{d\psi(s)}{ds} \right\}$$

and, therefore,

$$\frac{\partial P_1}{\partial s} q_1 = \frac{d\psi(s)}{ds} + \nu \delta \frac{d\psi(s)}{ds}$$



Which is precisely the market solution, given by (36). Now, in the incomplete information scenario, we can evaluate the distortion caused by asymmetry of regulation:

$$\begin{aligned} \lim_{\lambda \rightarrow \infty} \frac{1}{1 + \lambda} \left[ \frac{\partial S_1(q_1, s, \theta)}{\partial s} + \nu \delta \frac{\partial S_2(q_2, s)}{\partial s} \right] + \frac{\lambda}{1 + \lambda} \frac{\partial P_1}{\partial s} q_1 \\ = \frac{d\psi(s)}{ds} + \nu \delta \frac{d\psi(s)}{ds} + \frac{\lambda}{1 + \lambda} \frac{F(\theta)}{f(\theta)} \left[ \frac{d^2\psi(s)}{ds^2} + \nu \delta \frac{d^2\psi(s)}{ds^2} \right] \end{aligned} \quad (40)$$

so that we have:

$$\frac{\partial P_1}{\partial s} q_1 = \frac{d\psi(s)}{ds} + \nu \delta \frac{d\psi(s)}{ds} + \frac{F(\theta)}{f(\theta)} \left[ \frac{d^2\psi(s)}{ds^2} + \nu \delta \frac{d^2\psi(s)}{ds^2} \right]$$

As  $\lambda$  goes to infinity, the optimal level of quality approaches the market solution discounted by a distortionary term. This distortionary term increases as  $\lambda$  goes to infinity. Again, this distortion decreases quality, and, consequently, the amount of informational rent obtained by the firm.

In this extreme case where the social cost of public funds diverges to infinity, the Ramsey pricing rule, given by equations (10) and (31), approaches the monopoly pricing rule. In other words, allocative inefficiency is maximized in order to maximize revenue through tariffs, which allows transfers and taxation to be minimized. Therefore, in this scenario, tariffs have maximal importance in financing quality.

Conversely, if the social cost of public funds tends to zero, that is,  $\lambda \rightarrow 0$ , then, with complete information, we have

$$\lim_{\lambda \rightarrow 0} \left\{ \frac{1}{1 + \lambda} \left[ \frac{\partial S_1(q_1, s, \theta)}{\partial s} + \nu \delta \frac{\partial S_2(q_2, s)}{\partial s} \right] + \frac{\lambda}{1 + \lambda} \frac{\partial P_1}{\partial s} q_1 = \frac{d\psi(s)}{ds} + \nu \delta \frac{d\psi(s)}{ds} \right\}$$

and, therefore,

$$\frac{\partial S_1(q_1, s, \theta)}{\partial s} + \nu \delta \frac{\partial S_2(q_2, s)}{\partial s} = \frac{d\psi(s)}{ds} + \nu \delta \frac{d\psi(s)}{ds}$$

Now the Ramsey pricing rule approaches competitive pricing, and tariffs are used not to raise revenue, but to keep incentives at the margin in order to achieve full allocative efficiency. Therefore, in this scenario, transfers and taxation have maximal importance in financing quality.

Notice also that no distortion is needed in quality, so that the first-best socially optimal level of quality is achieved.

With asymmetric information, it's easy to see that the limit when  $\lambda$  approaches zero is the same as above. That is, informational rents become so cheap that they require no distortions, and the case with incomplete information converges to the case with complete information.

**Corollary 8.** *If in the market solution the firm undersupplies quality, then the level of quality implemented by regulation decreases as the social cost of public funds increases.*

*Proof.* Proof comes immediately from the fact that expression

$$\left(\frac{1}{1+\lambda}\right) \frac{\partial S_1(q_1, s, \theta)}{\partial s} + \left(\frac{\lambda}{1+\lambda}\right) \frac{\partial P_1}{\partial s} q_1$$

is a weighted average. So, if  $\frac{\partial P_1}{\partial s} q_1 < \frac{\partial S_1(q_1, s, \theta)}{\partial s}$  then the expression above decreases in value as  $\lambda$  increases. Additionally, regulation also adds the term  $\frac{1}{1+\lambda} \nu \delta \frac{\partial S_2(q_2, s)}{\partial s}$  to this expression, which increases quality but also decreases in value as  $\lambda$  increases.  $\square$

The analysis above has important implications for understanding the provision of quality in developing countries, which is the focus of our present study. We can conclude that the fact that developing countries usually have a higher social cost of public funds is one of the many possible reasons why quality provision is lower in these countries than in developed nations. Also, the higher social cost of public funds makes regulation less capable of yielding social welfare gains when compared to the market solution.

#### 6.4 Effects of the social cost of public funds on network extension

We demonstrate in the Corollary below that there is a  $\lambda^*$  sufficiently large such that the optimal network extension is zero, that is, it is socially best to shutdown the infrastructure in the poor area. This is the same conclusion reached by Laffont and N'Gbo (2000) in their model.

**Corollary 9.** *There is a  $\lambda^*$  sufficiently large so that the optimal solution requires the shutdown of the network in the poor area.*

*Proof.* From equation (32) we know that  $\nu$  increases with the expression:

$$\frac{1}{1+\lambda} [S_2(q_2, \delta s) + \lambda R] - C_2(q_2) - \psi(\delta s) - \frac{\lambda}{1+\lambda} \frac{F(\theta)}{f(\theta)} \delta \frac{d\psi(s)}{ds} \quad (41)$$

As  $\lambda$  goes to infinity, expression (41) decreases to a negative number, requiring the corner solution where  $\nu = 0$ . Therefore, by continuity, there is a  $\lambda^*$  sufficiently large so that expression (41) is zero.  $\square$

We can conclude that optimality might require a decrease or even the shutdown of the network in the poor area if the social cost of public funds is large enough, a common situation in developing countries. However, as we will see in subsection 6.6, this optimal solution might not be implementable due to legal constraints.

## 6.5 Relationship between quality and network extension

Let's begin our analysis of the relationship between quality and network extension by considering the market solution with unregulated quality.

**Corollary 10.** *In the market solution, quality decreases as network extension increases.*

Proof comes immediately from equation (36), which is:

$$\frac{\partial P_1(q_1^m, s^m, \theta)}{\partial s} q_1^m = \frac{d\psi(s^m)}{ds} + \nu^m \delta \frac{d\psi(s^m)}{ds}$$

Notice that, since the Ramsey pricing rule is unaffected, the lower quality also decreases the quantity purchased by consumer in the market solution, i.e.,  $q_1^m$ .

The Corollary above also help us understand the low provision of quality in developing countries, where quality is often unregulated. Even though expanding the infrastructure network is necessary for economic growth and development, and even, as we will discuss in the subsection about Universal Service Obligation, a legal obligation of the government, this expansion has a negative effect on quality. Therefore, regulating quality can be an important tool for improving the welfare of consumers.

Now, let's examine the relationship between quality and network extension in the regulation solution, with complete information and incomplete information. To better describe this relationship, we first define substitutability and complementarity between two control variables.

**Definition 11.** Two control variables are said to be *substitutes* if an increase in one variable negatively affects the other, or a decrease in one variable positively affects the other. Two control variables are said to be *complements* if an increase in one variable positively affects the other, or a decrease in one variable negatively affects the other.

We can now describe the relationship between these variables in the Corollary below.

**Corollary 11.** *Without asymmetric information, quality and network extension are complements if the social marginal benefit from quality in the poor area is positive, and substitutes if the social marginal benefit from quality in the poor area is negative.*

*Proof.* Differentiating both sides of equation (11) by  $\theta$  yields:

$$\left( \frac{dK_2(\nu)}{d\nu} + \frac{dK_2(\nu)}{d\nu} + \nu \frac{d^2 K_2(\nu)}{d\nu^2} \right) \frac{\partial \nu}{\partial \theta} = \frac{1}{1 + \lambda} \delta \frac{\partial S_2(q_2, s)}{\partial s} \frac{\partial s}{\partial \theta} - \delta \frac{d\psi(s)}{ds} \frac{\partial s}{\partial \theta}$$

We want to determine the sign of  $\frac{\partial \nu}{\partial \theta}$ . Since  $\frac{dK_2(\nu)}{d\nu} + \frac{dK_2(\nu)}{d\nu} + \nu \frac{d^2 K_2(\nu)}{d\nu^2} > 0$ , we can pay attention exclusively to the right-hand term of the equation above, which can be rewritten as:

$$\frac{\partial s}{\partial \theta} \delta \left[ \frac{1}{1 + \lambda} \frac{\partial S_2(q_2, s)}{\partial s} - \frac{d\psi(s)}{ds} \right] \quad (42)$$

Compare the equation above with equation (38), which gives the sign of  $\frac{\partial s}{\partial \theta}$ :

$$\frac{1}{1+\lambda} \frac{\partial^2 S_1}{\partial s \partial \theta} + \frac{\partial P_1}{\partial s} \frac{\partial q_1}{\partial \theta} + \frac{\partial \nu}{\partial \theta} \delta \left[ \frac{1}{1+\lambda} \frac{\partial S_2}{\partial s} - \frac{d\psi(s)}{ds} \right]$$

Notice that the social marginal benefit from quality in the poor area is given by  $\frac{1}{1+\lambda} \frac{\partial S_2}{\partial s} - \frac{d\psi(s)}{ds}$ . Therefore, if  $\frac{1}{1+\lambda} \frac{\partial S_2}{\partial s} - \frac{d\psi(s)}{ds} > 0$ , an increase in  $\nu$  has a positive effect on  $s$  and vice-versa. Otherwise, if  $\frac{1}{1+\lambda} \frac{\partial S_2}{\partial s} - \frac{d\psi(s)}{ds} < 0$  an increase in  $\nu$  will have a negative effect on  $s$ , and vice-versa.  $\square$

We will now show that, in the presence of asymmetric information, the condition for quality and network extension to be substitutes from Corollary 11 is no longer sufficient. This is due to fact that rents are increasing in both  $\nu$  and  $s$ , so that the negative effect of increasing rents must also be taken into account.

**Corollary 12.** *In the presence of asymmetric information, the condition for quality and network extension to be substitutes from Corollary 11 is no longer sufficient.*

*Proof.* From the monotonicity condition we know that  $\frac{\partial \nu}{\partial \theta} \leq 0$ . We can rewrite equation (39), which gives us the sign for  $\frac{\partial s}{\partial \theta}$ :

$$\begin{aligned} \frac{1}{1+\lambda} \frac{\partial^2 S_1}{\partial s \partial \theta} + \frac{\partial P_1}{\partial s} \frac{\partial q_1}{\partial \theta} - \left| \frac{\partial \nu}{\partial \theta} \right| \delta \left[ \frac{1}{1+\lambda} \frac{\partial S_2}{\partial s} - \frac{d\psi(s)}{ds} - \frac{\lambda}{1+\lambda} \frac{F(\theta)}{f(\theta)} \frac{d^2 \psi(s)}{ds^2} \right] \\ - \frac{\lambda}{1+\lambda} \frac{d\left(\frac{F(\theta)}{f(\theta)}\right)}{d\theta} \frac{d^2 \psi(s)}{ds^2} (1 + \nu \delta) \end{aligned}$$

Now, if the social marginal benefit from quality in the poor area is positive, i.e.,  $\frac{1}{1+\lambda} \frac{\partial S_2}{\partial s} - \frac{d\psi(s)}{ds} - \frac{\lambda}{1+\lambda} \frac{F(\theta)}{f(\theta)} \frac{d^2 \psi(s)}{ds^2} > 0$ , then the effect on  $\frac{\partial s}{\partial \theta}$  of decreasing  $\nu$  with respect to  $\theta$  will continue to be negative. However, if  $\frac{1}{1+\lambda} \frac{\partial S_2}{\partial s} - \frac{d\psi(s)}{ds} - \frac{\lambda}{1+\lambda} \frac{F(\theta)}{f(\theta)} \frac{d^2 \psi(s)}{ds^2} < 0$ , we still need to check the magnitude of the term  $\frac{\lambda}{1+\lambda} \frac{d\left(\frac{F(\theta)}{f(\theta)}\right)}{d\theta} \frac{d^2 \psi(s)}{ds^2} (1 + \nu \delta)$ , which is also related to  $\nu$ . Therefore, the effect on  $s$  of decreasing  $\nu$  with respect to  $\theta$  might be positive or negative.  $\square$

In case with asymmetry of information, if  $s$  is increasing, then rents are also increasing. It is precisely the negative effect that these incremental rents have on welfare that must be taken into account if decreasing  $\nu$  has a positive effect on  $s$ .

Finally, notice that our monotone hazard rate assumption does not exclude the possibility that  $\frac{d\left(\frac{F(\theta)}{f(\theta)}\right)}{d\theta} = 0$ . In this very specific case, the condition for quality and network extension to be substitutes from Corollary 11 is still sufficient.

## 6.6 Universal Service Obligation

Frequently, in developing countries, the Constitution imposes on the government a certain level of network extension, which usually is the universal access to the service.

This is known as Universal Service Obligation (USO), already discussed in our literature review. Since regulators must abide to the Constitution, they must impose this USO to the regulated firm, even if it is not optimal to do so otherwise<sup>9</sup>. In mathematical terms, this is equivalent to adding the following restriction in our maximization problem:

$$\nu^{uso} \equiv \max(\nu)$$

Notice that, in such case, the monotonicity condition given by Theorem 2 is trivially satisfied. We will attain to the more interesting case where the restriction above is binding, that is,  $\nu^{uso} > \nu^{fb}$  and  $\nu^{uso} > \nu^m$ .

Let's first analyze the market solution with USO, that is, when the government contracts the  $\nu^{uso}$  network extension, as required by the Constitution, but does not regulate quality.

**Corollary 13.** *In the market solution, the presence of Universal Service Obligation decreases quality.*

*Proof.* As we've seen, the market solution with respect to  $s$  is given by:

$$\frac{\partial P_1(q_1^m, s^m, \theta)}{\partial s} q_1^m = \frac{d\psi(s^m)}{ds} + \nu^m \delta \frac{d\psi(s^m)}{ds}$$

If, however, the government is obligated to implement  $\nu^{uso} > \nu^m$ , the condition is:

$$\frac{\partial P_1(q_1^{uso}, s^{uso}, \theta)}{\partial s} q_1^{uso} = \frac{d\psi(s^{uso})}{ds} + \nu^{uso} \delta \frac{d\psi(s^{uso})}{ds}$$

Since  $\nu^{uso} > \nu^m$ , and since the pricing rule is unaffected, then  $s^{uso} < s^m$  and also  $q_1^{uso} < q_1^m$ .  $\square$

Therefore, we can conclude that USO decreases quality in the market solution.

The Corollary above also have consequences for the regulation solution. As we have seen on subsection 6.3, the regulation solution for quality approaches the market solution as  $\lambda$  increases. In developing countries, where  $\lambda$  is large, and with the presence of Universal Service Obligation, we can conclude that regulation loses its power to increase the provision of quality in the public utility. This negative effect on quality caused by a large  $\lambda$  would otherwise be attenuated by the decrease or even shutdown of the poor area, as we have seen in Corollary 9. However, Universal Service Obligation prevents the regulator from decreasing network extension, and as a result, quality must be decreased.

Another consequence for the regulation solution is that, since the firm's expected rent is increasing in both network extension and quality, and rents are socially costly, then

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<sup>9</sup>In legal terms, we say that the Constitutional norm implementing the USO is hierarchically superior *vis-à-vis* the regulatory contract.

Universal Service Obligation requires a greater downward distortion in quality with respect to the first-best:

**Corollary 14.** *The presence of Universal Service Obligation creates a greater downward distortion in quality with respect to the first-best solution.*

*Proof.* From Corollary 1 we know that second-best quality is distorted downward by the term

$$\frac{\lambda}{1 + \lambda} \frac{F(\theta)}{f(\theta)} \left[ \frac{d^2\psi(s)}{ds^2} + \nu\delta \frac{d^2\psi(s)}{ds^2} \right]$$

Since  $\nu^{uso} > \nu^{sb}$ , we can conclude that:

$$\frac{\lambda}{1 + \lambda} \frac{F(\theta)}{f(\theta)} \left[ \frac{d^2\psi(s)}{ds^2} + \nu^{uso}\delta \frac{d^2\psi(s)}{ds^2} \right] > \frac{\lambda}{1 + \lambda} \frac{F(\theta)}{f(\theta)} \left[ \frac{d^2\psi(s)}{ds^2} + \nu^{sb}\delta \frac{d^2\psi(s)}{ds^2} \right]$$

Such that the downward distortion is greater in the presence of Universal Service Obligation.  $\square$

Since USO requires that the government extends the network, quality suffers a greater downward distortion in order to pay for the higher rents required by the larger-than-optimal network.

We can therefore conclude in this subsection that USO has a negative effect on quality provision, both in the market solution and in the regulation solution.

## 7 CONCLUSION

We have presented a model of optimal regulation of quality that tries to incorporate some of the specific difficulties encountered by regulators in developing countries. This model allowed us to demonstrate the effects of regulation on quality provision, some of the trade-offs faced by the government, and the distortions with respect to the first-best outcome introduced by asymmetric information.

In the presence of asymmetric information, we have shown that, since implementing higher quality requires giving up higher rents to the firm, it is optimal to distort quality downward. We have also shown that the implementation of a truthful direct revelation mechanism allows for no distortion regarding the Ramsey pricing rule, although the need to decrease quality does create a downward distortion in quantity. Then, we saw that, since the firm's rent also increases with a greater network extension, it is therefore socially optimal to decrease access to the network with respect to the complete information scenario.

We were also able to extract more general results with respect to quality regulation, results that are valid independently of the existence of informational asymmetry. In this respect, we have seen that, in our model, regulation allows the government to implement the socially optimal level of quality, overcoming the inefficient result obtained from unregulated markets. This inefficiency is due to the fact that, in the market solution, the firm looks exclusively to the preferences of the marginal consumer. Therefore, the socially optimal level of quality implemented by regulation could be greater or smaller than the market solution, depending on how the marginal social gains of quality compare to the marginal consumer's valuation of quality. Whatever the case might be, we saw that regulation takes into account the marginal social gains in the poor area as well, which has a positive effect on quality. Finally, we have also seen that, even with regulation and complete information, the socially optimal level of quality has a second-best nature, since transfers to the firm are socially costly.

Moreover, the magnitude of the social cost of public funds was also shown to have important consequences to the provision of quality. If in the market solution the firm undersupplies quality, then the level of quality implemented by regulation decreases as the social cost of public funds increases. Not only that, but as the social cost of public funds diverges to infinity, the regulation solution approaches the market solution for quality. On the other hand, if the social cost of public funds tends to zero, the first-best optimal level of quality is implemented through regulation, whether or not there is asymmetry of information.

We have also shown that, if the social cost of public funds diverges to infinity, then the Ramsey pricing rules converges to the monopoly pricing rule, and allocative inefficiency is maximized in order to maximize revenues from tariffs and, therefore, to minimize the

need for taxation and transfers to the firm in order to finance quality. In contrast, if the social cost of public funds tends to zero, then the Ramsey pricing rules converges to the competitive pricing, maximizing allocative efficiency. In this scenario, taxation and transfers to the firm have maximal importance in financing quality provision.

Other results concern the relationship between quality and network extension. In the so-called market solution, where the regulator contracts prices (or quantity) and network extension, but does not regulate quality provision, we showed that increasing the network extension decreases the provision of quality by the firm. This is an important conclusion that also helps understand low quality of public utilities in developing nations, since this kind of solution is very commonly implemented in these countries, where the lack of regulatory expertise makes regulating quality a very hard task.

If quality is regulated, on the other hand, we've seen that quality and network extension can either be substitutes or complements. In the case with complete information, this relationship will depend on the social marginal effect of quality in the poor area. If this effect is positive, then quality and network extension are complements. If it is negative, then quality and network extension are substitutes.

In the case with asymmetric information, however, the condition above is no longer sufficient, since we also need to take into consideration the effect that these variables have on informational rents. Therefore, in this case, a negative social marginal effect of quality in the poor area is no longer a sufficient condition for quality and network extension to be substitutes.

Finally, we analyzed how legal provisions imposing universal access to the service – the so-called Universal Service Obligation – might affect quality provision. In the market solution, we saw that the presence of Universal Service Obligation decreases quality. Since the regulatory solution approaches the market solution as the social cost of public funds increase, we can expect that in developing countries Universal Service Obligation will have a negative effect on quality even with regulation. Furthermore, in the presence of asymmetric information, Universal Service Obligation creates the need for a higher distortion in quality in order to pay for the higher rents required by the larger-than-optimal network. Therefore, according to our model, the presence of Universal Service Obligation not only decreases quality in the market solution, but also decreases the power that regulation has to improve its provision.

We can summarize the results obtained through our model in three main conclusions relevant for developing countries: first, regulation of quality is an important tool that can be used by governments in order to improve quality provision and general social welfare. The limited regulatory capacity faced by developing countries, often due to lack of resources or institutional constraints<sup>10</sup>, undermines the government's ability to implement

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<sup>10</sup>Such as the requirement to use civil service pay scales, which makes working for the government unattractive for highly skilled professionals (ESTACHE, A.; WREN-LEWIS, L., p. 733).



a proper regulation of quality. Our model shows that investing in regulatory capacity might bring social welfare gains.

Second, the importance of regulation for quality provision is negatively affected by the presence of a large social cost of public funds. To put it differently, implementing quality regulation alone might not be sufficient to significantly improve quality provision and bring social welfare gains in developing countries. In this regard, adopting policies that aim to decrease the country's social cost of public funds can be as important as regulation to achieve this aim. Examples of such policies include the adoption of a balanced government budget, which allows for lower interest rates, the reform of the tax system in order to promote its efficiency, and the implementation of measures to reduce government corruption.

Third, the presence of Universal Service Obligation is another factor that might decrease quality and the power of regulation as a tool for increasing social welfare. Even though granting universal access to public utilities is an important goal to promote human development, governments should focus on the economic factors that negatively affect optimal network extension, such as the social cost of public funds and the income of consumers in the poor area, rather than relying on legal obligations. Because of the negative effect that Universal Service Obligation might have on quality, we believe that our model provides an important legal argument for regulators to refrain from implementing universal access if it is not socially optimal to do so, since the provision of public services with good quality is usually also a legal right protected by the legal system. Therefore, the implementation of the socially optimal network extension, rather than universal access, seems to be a better solution to this conflict of rights<sup>11</sup>.

We hope that the results obtained here can better elucidate the challenges of quality regulation in developing countries, and that this rather theoretical approach can bring practical results to the regulation of public utilities in these countries.

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<sup>11</sup>We are assuming that both rights – the right to universal access and the right to a good public service quality – are granted by norms of equal hierarchical value.

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