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Business Cycles and Informal Hiring with Misallocation of Capital

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Statement of Authorship

I hereby declare that the thesis submitted is my own work. All direct or indirect sources used are acknowledged as references. I further declare that I have not submitted this thesis at any other institution in order to obtain a degree.

Abstract

The present work presents a model that explores the effects and behavior of informality along the business cycle in an economy characterized by capital misallocation. The intensive margin of informality is modelled through a size-dependent cost function and the extensive margin is modelled by following a dualistic approach to informality. The main results indicate that the intensive margin of informality gives less productive firms a competitive advantage during recessions; the elimination of informality decreases output and, overall, leads to more volatility; and both VAT and payroll taxes lead to intersectorial misallocation, but only the payroll tax leads to misallocation within the formal sector.

Keywords: Business Cycles, Informality, Financial Frictions, Misallocation

List of Figures

List of Tables

Contents

1 Introduction

Informality is a very widespread phenomenon in Brazil: according to Erosa; Fuster; Martínez (2020), around 30% of workers are estimated to be informal, 70% of which work in formal firms. Schneider; Enste (2000) estimate that the shadow economy corresponded to around 37.8% of Brazilian GDP between 1990-93. In addition, the informal sector is far more flexible than the formal sector. This is illustrated by the fact that the separation rates are approximately three times as high in the informal sector as in the formal sector (Bosch; Esteban-Pretel, 2012). Besides, Bosch; Esteban-Pretel also indicate that, although the unemployment rate is strongly countercyclical, the share of formal employment is procyclical in Brazil, which indicates that informality reacts to fluctuations in a non-trivial manner. Therefore, given its size and the degree of its fluctuations, the informal sector might play a relevant role in Brazilian business cycles.

Moreover, some studies suggest that there is a relationship between informality and financial frictions (Erosa; Fuster; Martínez, 2020; D'Erasmo; Moscoso Boedo, 2012; Antunes; Cavalcanti, 2007) and some authors model informality as inducing a size-dependent distortion (Erosa; Fuster; Martínez, 2020; Ulyssea, 2018). Additionally, financial frictions lead to capital misallocation, which means that more unproductive firms - usually more intensive in informal labor - remain active than it would be optimal in the absence of such frictions. Given that financial frictions typically go up during recessions, informality, therefore, might have short-term effects over the degree of misallocation in the economy. Therefore, we build a tractable theoretical model to assess how informality distorts capital allocation between firms through the business cycle and thereby affects key macroeconomic variables. This emphasis on the misallocation effects of informality is, to the best of our knowledge, novel in the business cycles literature.

Finally, we study informality in two margins: 1) the extensive margin, that is, the entrepreneurial decision of setting up a business informally; and 2) the intensive margin, that is, the decision, by formal firms, to hire workers informally (Ulyssea, 2018). We concentrate on the latter for two main reasons. First, it is the most quantitatively relevant margin. This is well illustrated by the fact that it corresponds to approximately 70% of all informal labor in Brazil. Besides, using data by ECINF[1] and PME[2], Ulyssea (2018) shows that over 40% of workers in Brazilian formal firms with two and three employees are informal and, around 52% of all informal workers are employed in companies with at least 11 employees. Second, the intensive margin has a relevant interaction with financial frictions (Erosa; Fuster; Martínez, 2020).

[1] *Pesquisa de Economia Informal Urbana*, in Portuguese.
[2] Brazilian Monthly Employment Survey. *Pesquisa Mensal de Emprego*, in Portuguese.

Our model elaborates on the interaction between informal hiring by formal entrepreneurs and fluctuations in economies characterized by strong financial frictions in order to assess how informality behaves along the business cycle and whether it has any effects over the volatility of key variables. We assess how informality affects the model economy by increasing informality's costs to an arbitrarily high level so as to nearly shut it off. We keep it tractable so that we can acquire intuition on the transmission mechanisms, but we also report some quantitative results. The results indicate that the presence of the intensive margin gives less productive entrepreneurs a competitive advantage during recessions. It also increases profit rates for the entrepreneurs and capital accumulation Besides, this margin leads to higher GDP and employment, lower inflation and tax collection and less volatility.

We also add the extensive margin to the model in a way consistent with the dualistic view on informality in order to study its interactions with a formal sector characterized by binding credit constraints. Though we assume informal companies do not use capital and, therefore, are not directly affected by credit constraints, the existence of an informal sector does affect the dynamics of the formal economy. In this extended version of the model, shutting down the intensive margin has the same effects as before, but some of the laid off workers are absorbed by the informal sector. Eliminating the extensive margin, on the other hand, increases output in the formal sector, but reduces total GDP, with no effects on the intensive margin. The effects over volatility are ambiguous.

Finally, we perform counterfactual exercises by changing key parameters. These exercises do not change any of the patterns attained in the model if the appropriate re-calibration is executed when necessary. Among the tested parameters, only those associated with the intensive margin and the payroll tax have any effects over the degree of misallocation in the formal sector, which fluctuates more when the intensive margin is larger. Besides, a more productive informal sector leads to a smaller formal sector, smaller government revenues, but larger total GDP.

The work is structured as follows. In section 2, we discuss the related literature; and in section 3, we present the baseline version of the model, with the intensive margin only. Next, we calibrate and present the main results attained from this model in section 4. Then, we extend the model by adding the extensive margin, re-calibrate and present the new results in section 5. In section 6, we do some other counterfactual and robustness exercises, and, finally, we conclude in section 7. The appendix contains a more detailed description of the model's solution, it's basic computational algorithm and the tables that support the counterfactual exercises in section 6.

2 Literature Overview

This work is more closely related to the literature that studies the aggregate consequences of informality. Ulyssea (2018), for instance, makes the distinction between the extensive and the intensive margins - the study of the latter being a major contribution of his work. In his paper, he focuses on studying how formalization policies affect output. He also builds on a distinction found on the literature between three types of informal businesses: 1) those that do not formalize due to high entry costs to the formal sector; 2) those that remain informal to benefit from cost advantages; and 3) those that are too unproductive to become formal. He estimates that they correspond to roughly 9.3%, 41.9%, and 48.8% of informal firms respectively. Both classifications (intensive vs. extensive margin and the types of informal businesses) are relevant for our work.

Erosa; Fuster; Martínez (2020) follow a similar path, but add financial frictions and an occupational choice. They find that, although the elimination of both margins of informality leads to output and TFP gains, the elimination of the intensive margin alone leads to a decrease in both. This is because hiring informal workers can aid formal firms in circumventing financial restrictions, since it is much cheaper for the firm to hire them. In their model, the effect of eliminating both margins is different from the sum of the individual effects, which justifies the study of both margins in conjunction. We follow their lead by modelling informality in a setup where entrepreneurs are credit constrained.

De Paula; Scheinkman (2010) build an equilibrium model of tax avoidance to evaluate if the credit method used to collect value-added tax generates informality chains - that is, a correlation between clients and suppliers in their formalization status - and test the implications of the model empirically using data from the Brazilian economy. One of the main predictions of the model is that an increase in tax rate directly leads to an increase in informality, and they conjecture, supported by numerical simulations, a net (general equilibrium) effect on the same direction. They find that, indeed, VAT collection through the credit method leads to the transmission of informality through vertical relationships between firms. Besides, their work indicates that the introduction of the SIMPLES program[1] generated some incentives to formalization. Likewise, we find that a lower VAT tax induces a movement towards the formal sector.

Finally, La Porta; Shleifer (2014, 2008) make the case for the "dual view of informality", that is, the view that informal and formal economies are separate. They also delve deep into the causes of the negative correlation between the prevalence of informality

[1] SIMPLES is a tax program, instituted in 1996 in Brazil by the federal government, with the aim of simplifying social security contributions and reducing VAT rates for small firms (that is, with less that $R\$\,1,000,000$ in annual revenues).

(as self-employment) in an economy and GDP per capita. They point as likely explanations the supply-side effect of education gains for entrepreneurs and a demand-side effect. The latter arises from the fact that in poor economies, fixed costs are potentially not covered by the demand for modern products, given that this demand is lower than in rich economies, especially if inequality is high. We follow their work by using a dual markets model for studying the extensive margin.

The present work is also related to the literature that is directly concerned with the behavior and the effects of informality along business cycle fluctuations. Lambert; Pescatori; Toscani (2020) build an open-economy DSGE model with a formal and an informal sector to assess how labor informality affects the economy's response to shocks in Latin America. In their model, they incorporate the aforementioned La Porta; Shleifer's demand-side effect. They find that informality plays the role of a shock absorber; in particular, GDP, consumption and unemployment are more volatile in its absence. We find similar results with a different kind of model.

Fiess; Fugazza; Maloney (2010) try to account for the observed episodes of procyclicality of self-employment in some countries. They show that whether formal and informal labor markets are integrated or dualistic depends on the presence or absence of binding wage rigidities, and, depending on the nature of the shock, this can lead to either procyclical or countercyclical movements of informality. In the case of severe recessions, though, informality works more dualistically, as rigidities become more binding. This result strengthens the relevance of studying recessions through the use of a dual markets model.

Castillo; Montoro (2012) build a New Keynesian Model with labor market frictions to study how informality affects inflation. They find that informality causes a "buffer" effect that reduces the fluctuations on inflation that arise from demand shocks. Besides, economies with a large prevalence of informality respond more to changes in interest rate, in the sense that real output can be stimulated with a lesser impact on inflation. With productivity shocks, nevertheless, informality amplifies the effects of these shocks on inflation and output, though this effect is marginal. We find results on the opposite direction for a TFP shock.

The present work is also related to the misallocation literature. In their classical paper, Hsieh; Klenow (2009) build a model of monopolistic competition with heterogeneous firms to study the effects of misallocation of resources over aggregate total factor productivity. They model labor and capital distortions as implicit taxes. They find bigger dispersion of marginal product in China and India than in the United States and a great potential for growth in those countries by reducing the degree of misallocation. Vasconcelos (2017) applies their model to the Brazilian manufacturing sector and finds that this sector operates at around 50% of its efficient level of output and that small firms are more strongly affected by changes in misallocation during periods of crisis.

On the other hand, Midrigan; Xu (2014) concentrate on capital misallocation. They model a dualistic economy with a traditional sector - that uses labor as its sole input - and a modern sector. Following the literature, they model financial frictions as a collateral constraint. They conclude that the accumulation of inner funds over time allows bigger firms to outgrow credit constraints. Therefore, financial frictions cannot generate large misallocation losses within the modern sector. These frictions, however, make the entry into the modern sector more difficult for firms in the traditional sector. We model the informal sector in a similar way. Besides, as in their work, misallocation has less negative consequences than it is usual in the literature.

Finally, the model here presented is also related to the literature that studies business cycles in economies with financial frictions. Kiyotaki (1998) presents two theoretical models in which financial frictions have the effect of amplifying shocks and making them more persistent. Building upon the foundation laid out by Kiyotaki, Reis (2013) builds a model of an open economy with credit frictions to study how financial integration with foreign economies, in the absence of financial deepening, can lead to an increase in the production of non-tradables relative to tradable goods, leading to a fall in productivity. We follow Reis (2013) closely in the way we model financial frictions.

3 Basic Model

The basic model features only the intensive margin and it studies how this margin reacts to a TFP shock as well as its effects on the economy - characterized by capital misallocation - as a whole. It is a theoretical model; therefore, we use it to indicate the direction of informality's behavior and effects along the cycle. It features a representative, hand-to-mouth, household, a continuum of entrepreneurs and a (budget balance) government. Entrepreneurs differ according to their productivity, which they draw every period. More productive entrepreneurs enter the market at $t+1$ by borrowing money (up to a constraint) at time t from less productive ones, who remain inactive at $t+1$. The misallocation of capital between entrepreneurs is modelled closely following Reis (2013), and indirectly Kiyotaki (1998), through the use of a collateral constraint, as it is usual in the literature. We also follow Reis by assuming perfect foresight. Therefore, there is no uncertainty regarding the value of the collateral for a loan.

We model informality, as in Erosa; Fuster; Martínez (2020), as an expected cost (a fine) of the company being caught with informal workers. Therefore, we do not focus on hiring frictions, as it is usual in the business cycle literature that features informality, for two main reasons: 1) it allows us to more easily treat informality as a size dependent policy; and 2) it adds to the literature by bringing a different perspective from what has been brought so far. There is a labor market where formal and informal labor are perfect substitutes and the decision, by the entrepreneurs, of hiring either of the two is based on their relative cost. Moreover, we suppose that the nominal wage is fully rigid at the steady state value, and that prices fluctuate in order to close the labor market[1].

3.1 Household

The representative household consumes a homogeneous good produced by a continuum of entrepreneurs. The household solves the following problem:

$$\max_{\hat{C}_t,\hat{N}_t} \quad u(\hat{C}_t,\hat{N}_t,G_t) = ln\left(\hat{C}_t - \chi^{\varphi}\frac{\hat{N}_t^{1+\varphi}}{1+\varphi}\right) + ln(G_t), \tag{3.1}$$

$$\text{s.t} \quad p_t\hat{C}_t = w_t\hat{N}_t, \tag{3.2}$$

$$\hat{C}_t \geq 0, \tag{3.3}$$

[1] Since real wages vary to close the labor market, the assumption of wage rigidity does not affect the results obtained. Since we do not have enough degrees of freedom to determine both variables, we choose to assume more flexibility in prices than in nominal wages.

in which φ is the inverse of the Frisch elasticity of labor supply, χ is a preference parameter and G_t is government spending. This household is a hand-to-mouth worker, as their problem is static. The household does not distinguish between formal and informal work.

3.2 Entrepreneurs

We suppose that companies differ according to their productivity. Every period t, they draw an idiosyncratic productivity factor z_{t+1} from a static distribution F(z) over the interval $[\underline{z}, \bar{z}]$. The production function for the entrepreneur who draws z_t is given by:

$$y_t(z_t, \Phi_t, k_t, n_t) = z_t \Phi_t k_t^{\alpha} n_t^{1-\alpha}, \tag{3.4}$$

in which $\alpha \in (0, 1)$ and Φ_t is the total factor productivity (TFP). For simplicity of notation, we will henceforth suppress in most of the equations that follow the choice variable's dependence on z_t, however these quantities are company-specific and vary according to their productivity.

The TFP follows an AR(1) process of the form:

$$\Phi_t = \bar{\Phi} + \rho(\Phi_{t-1} - \bar{\Phi}) + \epsilon_t, \tag{3.5}$$

in which $\bar{\Phi}$ is the average TFP and ϵ_t is an exogenous productivity shock. Moreover, we suppose, as in Reis (2013), that these draws are *i.i.d.* and that the capital stock fully depreciates from one period to the next. We make these assumptions in order to facilitate aggregation and keep the problem tractable.

As in Erosa; Fuster; Martínez (2020), formal and informal workers are perfect substitutes and there is no difference in productivity between them. However, either type has unique costs associated to it. Thus, each entrepreneur faces a cost of hiring informally:

$$\eta(n_{i,t}, n_{f,t}, z_t, \Phi_t) = \eta^i z_t \Phi_t n_{i,t} \left(\frac{n_{i,t}}{n_{i,t} + n_{f,t}} \right)^{\omega+1}, \tag{3.6}$$

in which η^i is a cost scaling parameter and $\omega > 0$. This function is interpreted as an expected cost of having to pay a fine for having workers hired *"off the books"* when an inspection takes place. It also constitutes, therefore, a probabilistic revenue for the government. Moreover, the entrepreneur pays a payroll tax, τ_p, for its formal workers in addition to the value-added tax (VAT), τ_y, that it pays over its output.

The informality cost function implies that less productive companies are more intensive in informal labor than more productive entrepreneurs. This definition incorporates the empirical fact that bigger companies hire proportionally fewer informal workers than smaller businesses given their higher probability of being inspected (Ulyssea, 2018). Productivity - which encompasses both the idiosyncratic and the aggregate terms - is used

here as a proxy for size in order to facilitate aggregation (see Appendix A for clarifications on this point). Finally, as $\omega > 0$, the costs of hiring informal workers increase as more informal workers are hired and decrease as the entrepreneur hires more formal workers.

This way, the entrepreneur chooses $\{c_t, n_{f,t}, n_{i,t}, k_{t+1}, b_{t+1}\}_{t=0}^{\infty}$ to solve the problem:

$$\max \mathbb{E}_t \left[\sum_{t=0}^{\infty} \beta^t ln(c_t) \right] \quad \text{subject to} \tag{3.7}$$

$$p_t c_t + k_{t+1} + b_t = R_t^k + \frac{b_{t+1}}{R_t}, \tag{3.8}$$

$$R_t^k = (1-\tau_y) p_t z_t \Phi_t k_t^{\alpha} n_t^{1-\alpha} - w_t n_t - \tau_p w_t n_{f,t} - w_t \eta(n_{i,t}, n_{f,t}, z_t, \Phi_t), \tag{3.9}$$

$$b_t \leq \theta R_t^k, \tag{3.10}$$

$$n_t = n_{f,t} + n_{i,t}, \tag{3.11}$$

$$c_t, k_{t+1} \geq 0, \tag{3.12}$$

in which $\theta \in (0,1)$ indicates how tight the collateral constraint is, p_t is the price level, b_t is the payment of debt incurred on the previous period, when positive, and the money received for lending in $t-1$, when negative. The equations suggest that some entrepreneurs will lend money at a rate of R_t (and produce no units of the consumption good in $t+1$), while others will produce and, for such an end, will incur in debt, which will have to be paid on the next period. The decision will ultimately depend on how high $R_t^k(z_{t-1})$ is. Inflation is defined as usual, as $\pi_t = \frac{p_t - p_{t-1}}{p_{t-1}}$, and is equal to 0 at the steady state.

If there was no credit constraint, only the most productive entrepreneur would produce, and all the others would lend her money. However, we suppose, as in Reis (2013), that the constraint will bind even for the most productive entrepreneur. This way, there will be entrepreneurs with $z_t < \bar{z}$ in business, which generates the heterogeneity needed to properly model informality. As a consequence, the return, for each entrepreneur $z_t \in [0,1]$, is given by:

$$R_{t,t+1}(z_{t+1}) = \max \left\{ R_t, \frac{R_{t+1}^k(z_{t+1}) - b_{t+1}(z_t)}{k_{t+1}(z_t) - \frac{b_{t+1}(z_{t+1})}{R_t}} \right\}, \tag{3.13}$$

in which the second term in the curly brackets is the return from investing in capital with the entrepreneur's own resources. It follows that z_t^*, the cutoff value of z that satisfies $R_{t,t+1}(z_t) = R_t$, is such that only entrepreneurs with productivity $z_t \in (z_t^*, \bar{z}]$ will choose to produce.

We assume that the marginal worker is formal. Though this may not be true for smaller businesses, since the majority of workers in the formal sector are formal, it is a reasonable assumption to make for tractability's sake. The first order condition for the amount of labor demanded yields:

$$n_t = \left[\frac{(1-\tau_y)(1-\alpha) p_t z_t \Phi_t}{(1+\tau_p) w_t} \right]^{\frac{1}{\alpha}} k_t. \tag{3.14}$$

Moreover, the optimality condition for the entrepreneurs implies that the marginal cost of hiring formal workers must be the same as the marginal cost of hiring informal workers. Therefore, the following expression must hold:

$$w_t \frac{\partial \eta(\cdot)}{\partial n_i} = w_t \frac{\partial \eta(\cdot)}{\partial n_f} + \tau_p. \tag{3.15}$$

Thus, entrepreneurs will choose the ratio of informal to formal workers that will minimize labor costs.

We can, finally, obtain an expression for entrepreneur's profits:

$$R_t^k = w_t \left[\frac{(1-\tau_y)(1-\alpha)p_t z_t \Phi_t}{(1+\tau_p)w_t}\right]^{\frac{1}{\alpha}} \left[\frac{\alpha(1+\tau_p)}{1-\alpha} + \frac{(1+\omega)}{(\eta^i z_t \Phi_t)^{\frac{1}{1+\omega}}} \left(\frac{\tau_p}{2+\omega}\right)^{\frac{2+\omega}{1+\omega}}\right] k_t$$

$$R_t^k = x_t k_t. \tag{3.16}$$

This means that profits are a linear function of the capital stock, where x_t is the average profit rate per unit of capital. This is possible due to the fact that we use entrepreneur's productivity as a proxy for size in the cost function (3.6). Besides, we can obtain the following expression for capital accumulation:

$$k_{t+1} = \frac{\beta R_t q_t}{R_t - \theta x_{t+1}} \tag{3.17}$$

where $q_t := x_t k_t - b_t$ corresponds the the entrepreneur's net worth at t (see Appendix A). This means that capital accumulation increases with the profit rate, x_{t+1}, and decreases with interest rate, R_t. In addition to this, since both labor demand, given by (3.14), and informal labor demand (see Appendix A) are linear in the capital stock, the cost of hiring informal workers is linear on capital as well, according to (3.6). As a result, when the interest rate falls, both capital accumulation and the cost associated to the intensive margin increase.

Note that (3.14) and (3.6) imply that informality, in our model, does not affect the size of the businesses, but it affects their profits. Therefore, informality's effects over aggregate variables operate through the misallocation channel only in the model. This results from using productivity as a proxy for size in the cost function (3.6), which means that the variation in the ratio n_i/n between businesses is not due to their endogenous hiring decision, but instead to their idiosyncratic productivity. However, note that informality co-moves with size over time due to the fact that aggregate productivity, Φ, affects its cost, which gives us a cross-sectional and a time series dimension to informal hiring. Therefore, our main contribution consists in the isolation of the misallocation channel of informality through the business cycle.

3.3 Government

The government is conceived very simply. It transfers resources from companies to households in the form of government spending. Therefore, government spending follows:

$$G_t = \int_{z^*}^{\bar{z}} [\tau_y y_t(z) + \tau_p n_{f,t}(z) + \eta(n_{i,t}, n_{f,t}, z_t, \Phi_t)] dF(z), \tag{3.18}$$

We keep the government simple so that we focus on modelling informality itself. It is fair to note that, although the money obtained from fines on businesses that hire informal workers is part of government's revenue, we will focus on the resources obtained from taxation when we talk about government revenues later on, in the results sections. The reason for this emphasis is that these are more standard sources of government revenue in the literature, and, here, the fines are the mechanism to model informality, not being of interest in themselves. Besides, the fines are not quantitatively relevant. Therefore, government revenue is given by:

$$GRev_t = \tau_y Y_t + \tau_p (N_t - N_{i,t}), \tag{3.19}$$

in which Y_t, N_t and $N_{i,t}$ are aggregate output, total and informal labor respectively.

3.4 Market clearing

The market clearing conditions are:

$$\hat{C}_t + \int_{\underline{z}}^{\bar{z}} [c_t + k_{t+1}] \, dF(z) + G_t = \int_{z^*}^{\bar{z}} [z_t \Phi_t k_t^{\alpha} n_t^{1-\alpha} - \eta(n_{i,t}, n_{f,t}, z_t, \Phi_t)] dF(z), \tag{3.20}$$

$$\int_{z^*}^{\bar{z}} b_{t+1} dF(z) = - \int_{\underline{z}}^{z^*} b_{t+1} dF(z), \tag{3.21}$$

$$\hat{N}_t = \int_{z^*}^{\bar{z}} [n_{f,t} + n_{i,t}] dF(z). \tag{3.22}$$

When these equations are satisfied, the goods market, the labor market and the loanable funds market are in equilibrium. We impose that the labor and loans market clear, and suppose, by Walras' Law, that the goods market clears as well.

4 Main Results

In this section, we present the databases used for the calibration of the model, the calibration output as well as the main results. We focus on the results that come from shutting down informality and leave other counterfactual exercises for a later section.

4.1 Productivity Distribution

We suppose a bounded Pareto distribution for the idiosyncratic productivity factor because it is a good approximation to the size distribution of entrepreneurs. The density function has the form:

$$\frac{\mu \underline{z}^{\mu} z^{-\mu-1}}{1-\left(\frac{\underline{z}}{\bar{z}}\right)^{\mu}},$$

in which $z \in [\underline{z}, \bar{z}]$ and $\underline{z}$ and $\bar{z}$ are the distribution's lower and upper bounds. Moreover, the cumulative distribution function is given by:

$$\frac{1-\underline{z}^{\mu} z^{-\mu}}{1-\left(\frac{\underline{z}}{\bar{z}}\right)^{\mu}}.$$

Note that μ is the parameter that governs how fat the tail of the distribution is: the lower μ is, the fatter is the tail, that is, the proportion of larger, more productive entrepreneurs in this economy gets larger as μ is decreased.

4.2 Calibration

Since our emphasis is of a theoretical nature, we stick to reasonable values based on the literature for most of the parameters. The other ones are set in order to approximately match some moments to the Brazilian data. We use mostly data from the Brazilian Geography and Statistics Institute (IBGE, *Instituto Brasileiro de Geografia e Estatística*, in Portuguese), and, in specific, from PNAD (*Pesquisa Nacional por Amostra de Domiclios*) to calibrate most labor market parameters. We exclude data from 2020 to eliminate the impact of the COVID-19 pandemic in the targeted moments, and try to use recent data (from 2012 onward) whenever it is available.

We set $\alpha = 0.3$ at its standard value. Besides, in the model β represents the fraction of assets that is invested by entrepreneurs. We set it at $\beta = 0.86$ to match the average savings rate as an approximation for $1-\beta$, which, according to IBGE's national accounts, was of approximately 0.149 between 2012-2019. We set total factor productivity's persistence at $\rho = 0.589$, the value estimated by Ellery Jr.; Gomes; Sachsida (2002).

Table 1 – Baseline Model - Calibration Summary

Parameter	Value	Target
α	0.3	Standard K/Y ratio
β	0.86	Savings rate
θ	0.25	Share of bank financing of non-financial firms
$\bar{\Phi}$	1	Assumed without loss of generality
ρ	0.589	Ellery Jr.; Gomes; Sachsida (2002)
φ	1	Reis (2013)
χ	0.339	Fraction of formal sector workers in labor force
η^i	1	Fraction of informal workers among formal sector workers
ω	0.8454	Erosa; Fuster; Martínez (2020)
τ_y	0.2925	Erosa; Fuster; Martínez (2020)
τ_p	0.29	Erosa; Fuster; Martínez (2020)
$\underline{z}$	0.3	Assumed as the lowest feasible lower bound
$\bar{z}$	1	Assumed without loss of generality
μ	0.6309	Distribution of the total number of workers hired

Moreover, we suppose, as in Reis (2013) and in much of the literature, that the inverse of the Frisch elasticity of labor supply is $\varphi = 1$. Then, we set $\chi = 0.339$ to match the fraction of workers in the formal sector with respect to the population in the labor force, according to PNAD. We subtract the public sector workers from the labor force, because we do not model this sector, and obtain an average for this ratio of $0,50365$ for the period between 2012 and 2019.

Next, we follow Erosa; Fuster; Martínez (2020) by setting the parameter that governs how the demand for informal workers by formal entrepreneurs varies with business size $\omega = 0.8454$ and calibrating $\eta^i = 1$ in order to reach the proportion of the workers in the formal sector that are informal. According to PNAD, this fraction was of around 0.19 for 2012-2019. We also follow the authors by setting the taxes at the values of $\tau_y = 0.2925$ and $\tau_p = 0.29$.

There are three parameters related to the companies' productivity distribution. We assume $\bar{z} = 1$ for simplicity, and set $\underline{z} = 0.3$ because it is approximately the smallest value for which the model behaves properly. We try to mimic the fact that, according to IBGE's 2018 Enterprises Central Register's Statistics[1], companies at the 98^{th} percentile hire approximately $5,556$ times more workers than companies at the 87^{th} percentile. The quantity of labor demanded by each entrepreneur (3.14) can be re-written as:

$$n_t = \left[\frac{(1-\tau_y)(1-\alpha)p_t z_t \Phi_t}{(1+\tau_p)w_t}\right]^{\frac{1}{\alpha}} \frac{\beta q_{t-1}}{\left(1-\frac{\theta x_t}{R_{t-1}}\right)}.$$

[1] *Estatísticas do Cadastro Central de Empresas*, in Portuguese

Although we do not calculate each entrepreneur's own demand for labor, since we assume the productivity draws are *i.i.d.*[2], we can estimate relative labor demands. This value can only be exactly achieved on section 5's extension to the model, but not in the baseline version. Therefore, we set $\mu = 0.6309$ to replicate the $x_{SS}(z_{SS})$ distribution for companies in business in section 5's extended model.

Finally, with respect to the financial friction, we set the value at $\theta = 0.25$ as in Reis (2013). He uses the share of bank financing of non-financial corporations as a proxy for θ, and Rodrigues Júnior; Melo (1999) indicate that this is a reasonable value for Brazil in the 1990s. In the absence of more recent data on this, we keep the value used by Reis (2013). We summarize the calibration outcomes in Table 1.

4.3 Results

We now present the main results obtained from the baseline model. First, we impose a negative TFP shock $\epsilon_t = -0.05$ at $t = 1$, which will affect the collateral constraint at $t = 0$. Then, we simulate the TFP series up to $t = 10$. Figure 1 shows how the cutoff value z^* and the fraction of workers who are informal, N_i/N, evolve over time.

Notice that the marginal entrepreneur is less productive when TFP is at a lower level, and the cutoff value z^* returns to its former, steady state, level as TFP recovers. Likewise, as expected, both total labor N and total informal labor, N_i, drop during the recession, as can be seen in panels (c) and (d). However, the fraction of workers that are informal, N_i/N, increases on the aftermath of the productivity shock before it returns to its previous value. This means that the number of formal workers drops by more than that of informal workers.

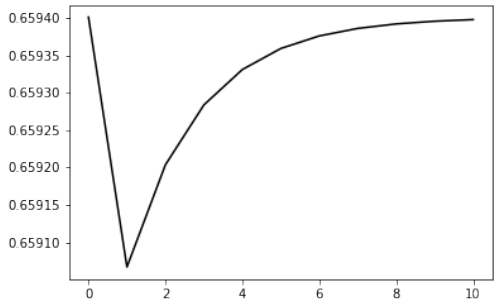
We also do a counterfactual exercise, in which we increase the cost of hiring informal workers to an arbitrarily high value ($\eta_t^i = 10,000$) so as to nearly shut down the intensive margin[3]. We wish to evaluate how the possibility of hiring workers informally affects the paths of output, inflation, real interest rates, government revenue and of the cutoff value z^*. Figure 2 shows how the fraction of workers who are informal behaves. Notice that, in both cases, this fraction goes up, though, in the alternative model, it is nearly zero, which is what was intended by increasing the costs of informality.

In Figure 3, we compare the impulse-response functions generated by the baseline model (in full lines) and the alternative model with close to no informality (in dashed lines). We examine the behavior of the idiosyncratic productivity factor level for the marginal producer z^*. The steady-state value for z^* in the baseline economy is approximately

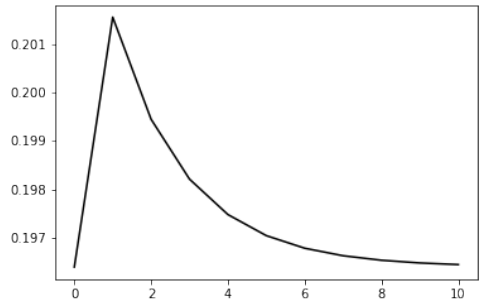
[2] This implies that the net worth at $t-1$, q_{t-1}, is, on average, the same for every company.

[3] As $\eta^i \to \infty$, informality, N_i, tends to zero. Thus, by "nearly" we mean that we approach full elimination of the intensive margin asymptotically.

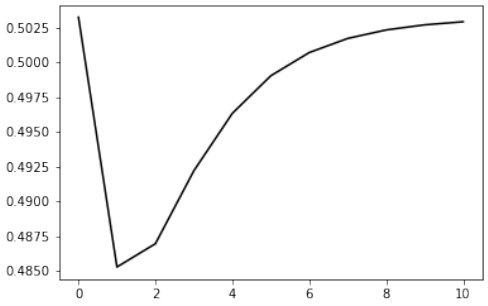
Figure 1 – Baseline Model - Cutoff Value and Hiring After Productivity Shock



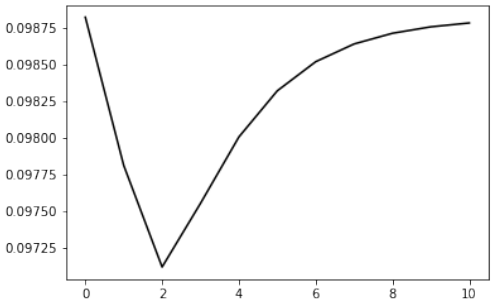
(a) Cutoff value, z^*



(b) Fraction of informal workers in total, N_i/N



(c) Total labor, N



(d) Total informal labor, N_i

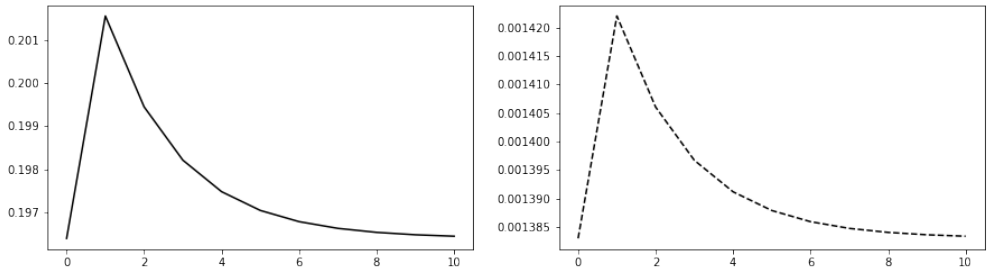
Notes: The figure displays how, following a negative productivity shock, the cutoff value, z^*, decreases and then recovers. The same happens with total labor and informal labor. Nevertheless, panel (b) indicates that entrepreneurs keep proportionally more informal workers in a recession than in periods of normality.

$z^*_{SS} = 0.6594$, whereas in the alternative, "no intensive margin", model, the value is $z^*_{SS} = 0.6718$. This suggests that, when hiring informal workers is feasible, some low productivity producers remain active, whereas they would be out of business if it was unfeasible.

Moreover, panel (a) in Figure 3 indicates that, when this economy receives an exogenous negative supply shock, the cutoff value drops by significantly more in the baseline economy than in the counterfactual one. We plot the first differences in Figure 4 to allow for better comparison. Given how z^* varies in the baseline economy, its variation in the alternative, high η^i, economy is unnoticeable. This indicates that, in a recession, companies that are more intensive in informal labor - that is, smaller and less productive companies - gain some competitive advantage with respect to bigger, less informal labor-intensive businesses. This is the main result obtained through this exercise.

Figure 3 indicates that employment goes down in both cases, but, more importantly, that the steady state level employment, N_{SS}, at the beginning of the series, is larger in the economy in which informal workers can be hired (0.5032 versus 0.4956). This indicates that the impossibility of reducing costs by hiring informal workers makes entrepreneurs

Figure 2 – Baseline Model - Fraction of Informal Workers in Total, N_i/N



Notes: The figure plots the path of the fraction of informal workers in total, N_i/N, on the baseline model (on the left) and on the alternative, high η^i, version (on the right). Notice that there is a difference in scale.

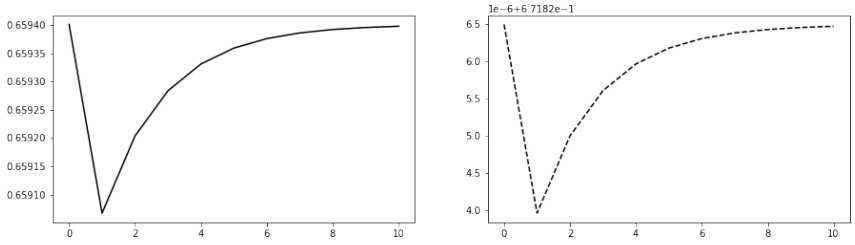
hire less workers overall. Also, this higher hiring cost can be the reason for fewer companies being in business in the counterfactual economy. As a consequence of this, real output is smaller at the steady state and at the lowest point of the series in the case in which there is less informal hiring. This is no surprise, since the literature suggests that shutting down the intensive margin reduces the aggregate product. (Erosa; Fuster; Martínez, 2020; Ulyssea, 2018).

Furthermore, inflation increases initially, as expected given that this economy receives a negative supply shock. It is interesting to note, though, that there is a period with a mild deflation in both cases after the initial increase. Additionally, note that, unlike what is found by Castillo; Montoro (2012), here, the baseline economy has slightly less volatile inflation than the economy with higher costs to the intensive margin. It is important to note that, in their paper, inflation becomes less volatile in the presence of informality when the economy receives a demand (rather than supply) shock. Here, since labor costs are diminished through informal hiring, the "buffer" effect of informality found in their work also operates for a TFP shock.

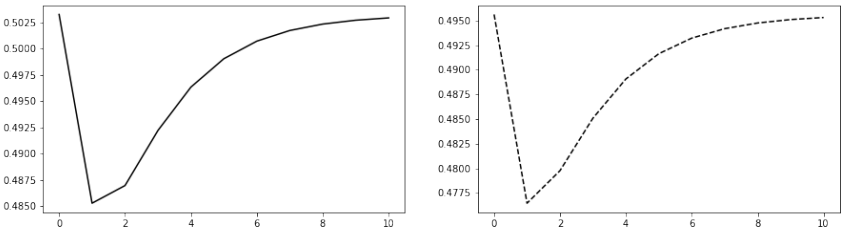
Also, panel (e) in Figure 3 indicates that government revenue is larger in the economy without informality at the steady state in spite of the fact that both output and employment are lower. This is due to the fact that almost all of the employed workers in the economy with high costs of informal hiring are formal. This leads to a bigger base from which to collect payroll taxes in comparison to the baseline economy. In section 6, we will check if this result still holds under alternative parametrization.

To gain intuition on the dynamics of the model, we show, in Figure 5, how the interest rate, R, responds to the TFP shock. R initially goes down as the TFP shock decreases the entrepreneur's profits. Also, the fact that entrepreneurial returns are increasing on z over the interval $(z_t^*, \bar{z}]$ implies that the fall in the cutoff value decreases the interest rate. Besides, interest rate recovers faster than aggregate productivity and cutoff value. This is due to the fact that rising prices increase revenues for the entrepreneurs and

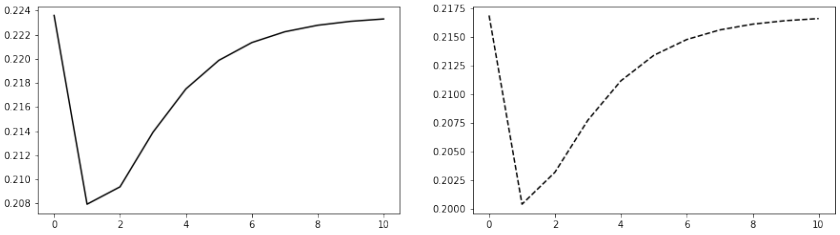
Figure 3 – Baseline Model - Effects of "Shutting Down" Informality on the Economy's Response to the Productivity Shock



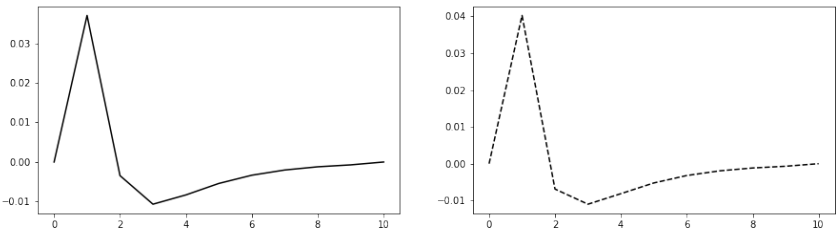
(a) Cutoff value, z^*



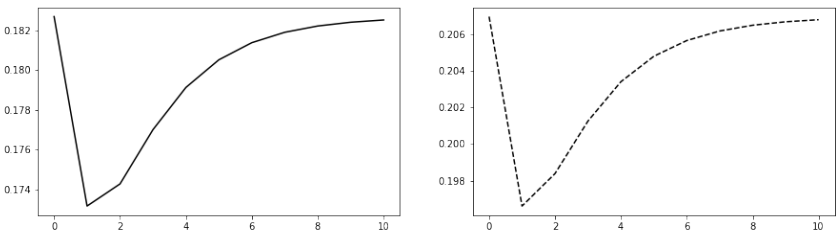
(b) Employment, N



(c) Real output, Y



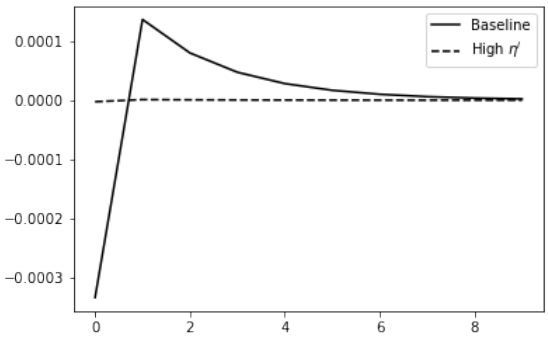
(d) Inflation, π



(e) Government Revenue

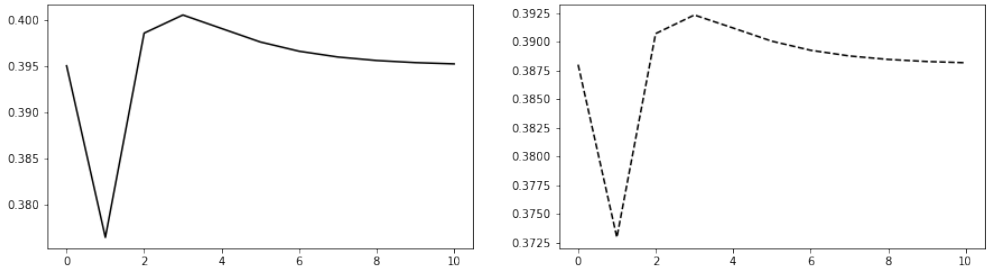
Notes: The figure plots the paths of each variable for the baseline model (on the left) and the alternative, high η^i, version of the model (on the right). Notice that there is a significant difference in scale between the two versions for the cutoff value, z^*.

Figure 4 – Baseline Model - First Differences With Respect to z^*



Notes: Here, we present, on the same graph, the first differences with respect to the cutoff value, z^*, for the baseline version of the model (full line) and the alternative version (dashed line). The latter remains nearly constant, whereas the former fluctuates more significantly along the business cycle.

Figure 5 – Baseline Model - Interest Rate's Responses to the Productivity Shock



Notes: Here, we present, on the same graph, the first path of the interest rate R_t, for the baseline version of the model (full line) and the alternative version (dashed line), with no intensive margin.

decreases real wages, which implies a faster recovery for profits and for R. This helps to explain why the interest rate rises a little above its initial level before returning to it.

Note that, although in the economy with the intensive margin the cutoff value, z^*, is lower than in the economy without it, the steady state interest rate is higher in the former, low η^e, economy (approximately 0.3951) than in the latter (approximately 0.388). This reflects the fact that the possibility of hiring informal workers implies a higher profit rate for the entrepreneurs. Therefore, output is larger in the economy with informality due to the fact that, in the model, 1) informal hiring increases entrepreneurial profit rates and 2) a lower cutoff value, *ceteris paribus*, implies a lower interest rate. Both phenomena produce more capital accumulation. Finally, the graph indicates higher interest rate volatility in the presence of the intensive margin.

Thus, the fall in TFP decreases entrepreneurial profits and the interest rate. Businesses shrink in face of the lower profits, which decreases the cost of the intensive

margin. This, in turn, creates a competitive advantage for less productive entrepreneurs, leading to a fall in the cutoff value, z^*. This fall, in turn, pushes the interest downwards even further, which is the reason for the higher interest rate volatility in the case in which hiring informal workers is possible. Note that this also means that a fall in the cutoff value, z^*, has, to some degree, a stimulating effect over the economy through the reduction of the interest rate. On the other hand, the fall in the interest rate stimulates some capital accumulation, which increases the cost of hiring informal workers and, therefore, pushes the cutoff value upwards to a certain degree.

Next, we examine, in Table 2, the standard deviations for GDP, employment, inflation, interest rate, government revenue and informality ratio. The latter is, as already noted, larger in the economy without the intensive margin. The same happens to the interest rate, due to the general equilibrium effects discussed above. With these exceptions, all the other variables are less volatile in the baseline model. Thus, this suggests that informality, in the intensive margin, has stabilizing effects over the economy.

Table 2 – Baseline Model - Standard Deviations

Variable	Baseline model	No intensive margin
Output	5.745×10^{-3}	5.781×10^{-3}
Employment	6.577×10^{-3}	6.726×10^{-3}
Inflation	1.269×10^{-2}	1.377×10^{-2}
Interest Rate	6.169×10^{-3}	4.938×10^{-3}
Government Revenue	3.382×10^{-3}	3.580×10^{-3}
Informality ratio, N_i/N	1.635×10^{-3}	1.229×10^{-5}

In summary, in the model, the intensive margin of informality is associated with a higher degree of misallocation, given that, at any point, there are more unproductive companies in an economy with the intensive margin than without it. Additionally, during recessions, the possibility of hiring informal workers gives smaller, less productive companies a competitive advantage. Moreover, informal hiring leads to higher profit rates for the entrepreneurs and more capital accumulation. Finally, this margin of informality leads to higher output and employment, lower inflation (resulting from a negative supply shock) and less volatility overall, though it is detrimental to government tax collection and produces more interest rate volatility due to general equilibrium effects.

5 Adding the Extensive Margin

In this section, we add the extensive margin of informality to this economy. Furthermore, we also explain how we calibrate the new parameters and present the main results that come from this extension of the baseline model.

5.1 A Model with the Extensive Margin

The way we add the extensive margin to the model is still inspired by Reis (2013), by making households produce and consume informal goods, and to choose how much labor to allocate to the informal and formal sectors. We model the informal sector as Midrigan; Xu (2014) model the traditional sector, that is, by supposing that it produces only with labor input and that it has a lower productivity level than the formal sector.

The household's problem then becomes:

$$\max_{\hat{C}_t, \hat{L}_t} \quad u(\hat{C}_t, \hat{L}_t, G_t) = ln\left(\hat{C}_t - \chi^{\varphi}\frac{\hat{L}_t^{1+\varphi}}{1+\varphi}\right) + ln(G_t), \tag{5.1}$$

$$\text{s.t} \quad p_t\hat{C}_t = (1-\eta^e)p_t^i\hat{z}\hat{M}_t^{\alpha_i} - w_t\hat{M}_t + w_t\hat{L}_t, \tag{5.2}$$

$$\hat{L}_t = \hat{M}_t + \hat{N}_t, \tag{5.3}$$

$$\hat{C}_t \geq 0,$$

in which, as before, $\hat{N}_t$ is the amount of labor supplied to the formal sector (both as formal and informal labor); $\hat{L}_t$ is the total amount of labor supplied by the household at t, $\hat{M}_t$ is the labor supplied to the informal sector, p_t^i is the price of the informal good and $\eta^e \in [0,1]$ is the cost of informality in the extensive margin. We suppose that $\hat{z} < z_t^*$ for every period t and that $0 < \alpha_i < 1$, which implies that the informal sector operates with decreasing returns to scale: a natural assumption since it is observed that informal companies usually remain small (La Porta; Shleifer, 2014). Moreover, naturally it follows that:

$$MP_{\hat{M}}^i = w_t. \tag{5.4}$$

That is, the marginal productivity of labor in the informal sector must be equal the nominal wage.

We include the demand channel of informality present in La Porta; Shleifer (2014) by considering, as in Lambert; Pescatori; Toscani (2020), that the good consumed by the household, $\hat{C}$, is a composite of the formal good, $\hat{C}^f$, and the informal good, $\hat{C}^i$, in the following quasilinear form:

$$\hat{C}_t = \hat{C}_t^f + \lambda(\hat{C}_t^i)^{\delta}. \tag{5.5}$$

The cost of this consumption good is given by:

$$p_t \hat{C}_t = p_t^f \hat{C}_t^f + p_t^i \hat{C}_t^i. \tag{5.6}$$

The optimality condition implies that:

$$\delta \lambda (\hat{C}_t^i)^{\delta - 1} = \frac{p_t^i}{p_t}, \tag{5.7}$$

$$p_t^f = p_t. \tag{5.8}$$

Therefore, although we suppose for simplicity that entrepreneurs consume only formal goods, there is no need to alter the definition of inflation and of price level, since the price for the composite good is the same as the price of the formal good. The market clearing condition for the informal sector is:

$$\hat{C}_t^i = \hat{z} \hat{M}_t^{\alpha_i}. \tag{5.9}$$

Finally, the extensive margin of informality, in the way defined here, accounts only for those companies that are too unproductive to enter the formal sector, which, as already stated before, accounts for approximately $48,8\%$ of the informal firms (Ulyssea, 2018). This is the case because the model is not complex enough to account for entry and operational costs. However, as indicated by Fiess; Fugazza; Maloney (2010), labor markets become more dualistic when wages are rigid and a severe recession ensues, which means this is a reasonable assumption to make here. For simplicity, we suppose that $\eta^e = 0$ in the baseline case, since assuming that it is an increasing function of the informal firm's size (as it is usual in the literature) would constitute an unnecessary complication. We will use this parameter to more easily shut down the extensive margin later.

5.2 Calibration

In this extended version of the model, we target the same moments we did in the previous section, with some additional moments stemming from the extensive margin. To begin, we fix $\alpha_i = 0.5$ without loss of generality[1]. We also set $\delta = 0.6$, following the estimation made by Lambert; Pescatori; Toscani (2020). Although they estimate this parameter for Colombia, in the absence of an estimation of such parameter for Brazil, we use their value as an approximation and calibrate the other parameters to match Brazilian data.

Erosa; Fuster; Martínez (2020) indicate that around 30% of all informal workers in Brazil belong to the informal sector. In addition, La Porta; Shleifer (2008, 2014), using

[1] The qualitative results obtained by changing this parameter are the same (see section 6). Besides, it is not possible to separate α_i from $\hat{z}$ in the calibration.

data from the World Bank Enterprise Survey, indicate that the ratio of value added by informal firms to value added by small formal firms[2] in Brazil is 0.5. Again, similarly to what was done in the calibration of the baseline model, by estimating how much larger the demand for workers by the average and marginal entrepreneurs is, we can estimate the marginal entrepreneur's added value. These moments are achieved by setting $\lambda = 0.235$ and $\hat{z} = 0.108$, which implies that, as argued by La Porta; Shleifer, informal companies are much less productive than formal firms. Finally, we set $\eta^e = 0$ in the baseline case, and raise it later as a counterfactual exercise. Table 3 summarizes the changes in parameters with respect to the previous section.

Table 3 – Extended Model - Calibration Summary

Parameter	Value	Target
χ	0.3096	Fraction of formal sector workers in labor force
μ	0.6046	Distribution of the total number of workers hired
δ	0.6	Lambert; Pescatori; Toscani (2020)
α_i	0.5	Assumed without loss of generality
λ	0.235	Fraction of informal workers on the extensive margin
$\hat{z}$	0.108	Ratio of formal and informal value added
η^e	0	Assumed for the baseline case

5.3 Results

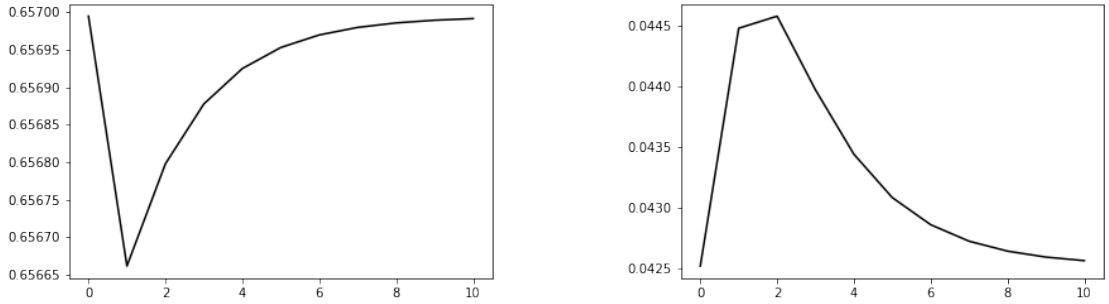
As before, the economy is hit by a negative TFP shock $\epsilon_t = -0.05$ at $t = 1$. Figure 5 shows how the cutoff value z^* and the employment of informal workers in the extensive margin $\hat{M}$ evolve over time. The graph for the cutoff value z^* is as before. As expected, labor in the extensive margin of informality increases as GDP decreases, given the demand effect (La Porta; Shleifer, 2014; Lambert; Pescatori; Toscani, 2020). This is also in line with the stylized fact that the share of formal employment is procyclical (Bosch; Esteban-Pretel, 2012). Movements in formal employment N and in the fraction of informal workers in the formal sector N_i/N also behave as they did in the previous model.

We once again do the exercise of nearly shutting down the intensive margin by increasing the cost of hiring informal workers to $\eta_t^i = 10,000$. The results of this exercise are represented in Figure 7. We omit the dynamics of z^*, inflation, interest rate and government revenue because they react just as they did when we previously eliminated the intensive margin. Again, the baseline model is represented by the full lines, and the alternative model with close to no informality by the dashed lines.

Shutting down the intensive margin has the effect of increasing informal labor on the extensive margin both at the steady state and (even more so) at the lowest point of

[2] Following the World Bank classification.

Figure 6 – Extended Model - Cutoff Value and the Extensive Margin After Productivity Shock



(a) Cutoff value, z^* (b) Extensive margin, $\hat{M}$

Notes: The figure plots how the cutoff value, z^*, and the informal labor on the extensive margin, $\hat{M}$, react to a negative productivity shock on the formal sector. The former reacts just as it did on the original version of the model, while the extensive margin becomes larger during the recession.

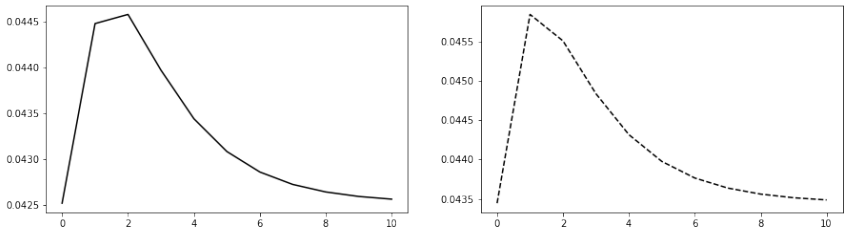
the recession. This means that part of the informal workers are replaced from the formal to the informal sector. Furthermore, panel (b) indicates that total employment in the formal sector decreases as expected. The increase on the extensive margin employment is not enough to prevent total employment (on the formal and informal sectors) to decrease, as can be seen in panel (c).

Finally, formal sector output decreases if we eliminate the intensive margin, and, although informal sector output increases as a result of the increase in hiring, total real GDP still goes down. This reflects the large size of the informal sector in Brazil[3]. Table 4 shows the standard deviations in both versions of the model. The standard deviations are larger for the economy without the intensive margin for almost all variables: formal and total output, formal sector, informal sector and total employment, inflation and government revenues. As in the original version of the model, the informality ratio and the interest rate are less volatile in the absence of the intensive margin. Therefore, the patterns observed on the previous section are maintained here.

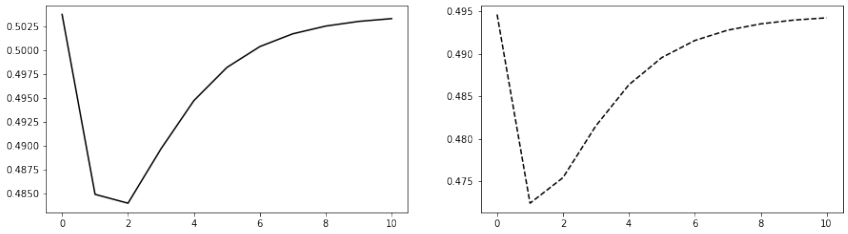
We assess the consequences of nearly shutting down the extensive margin. This is accomplished by increasing its cost parameter to $\eta^e = 0.999$. Figure 8 presents the results of this exercise over the labor market variables, whereas Figure 9 presents its effects over the outstanding variables, except for the interest rate, represented in Figure 10. This increase in cost decreases employment along the extensive margin, as can be seen in Figure 8, panel (a). However, the ratio of informal workers in the formal sector, N_i/N, does not change, since shutting down the extensive margin does not affect the costs of hiring informal workers on the intensive margin.

[3] As we will see in the next section.

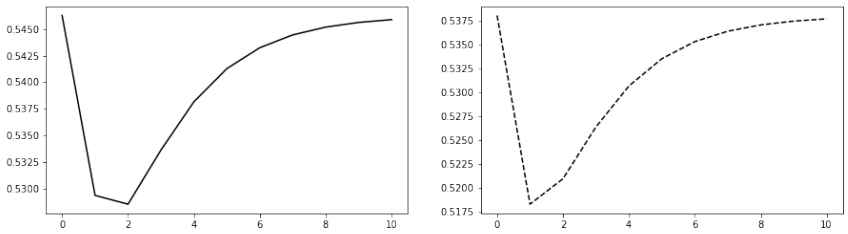
Figure 7 – Extended Model - Effects of "Shutting Down" the Intensive Margin on the Economy's Response to the Productivity Shock



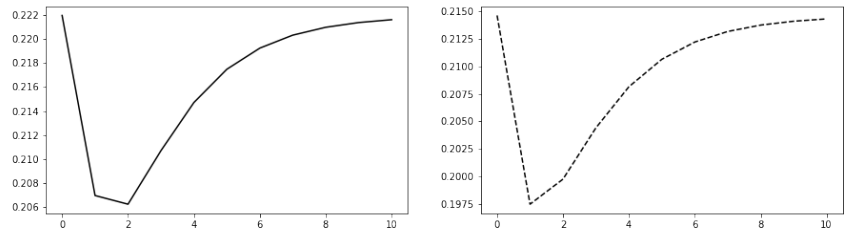
(a) Extensive margin, M



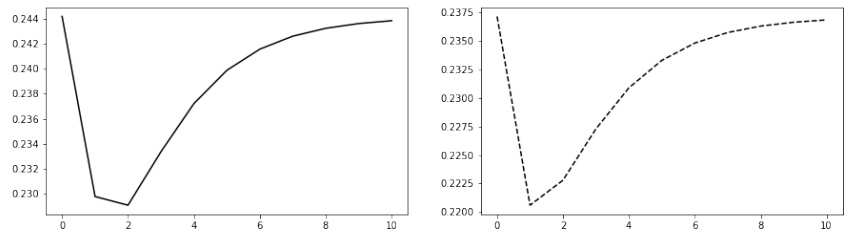
(b) Employment in the formal sector, N



(c) Total employment



(d) Formal sector real output



(e) Total real GDP

Notes: The figure displays the paths of each variable for the baseline model (on the left) and the version of the model with nearly no intensive margin (on the right).

Total employment does not change, which means that all these previously informal workers will be absorbed by the formal sector, being divided proportionally as formal and informal workers. This is due to the fact that, unlike the case in which the intensive margin is shut down, real wages do not change when we eliminate the extensive margin. This result is possible since, in the baseline case, aggregate net worth is approximately $Q_{SS} = 0.04752$ at the steady state, whereas in the case with no extensive margin, $Q_{SS} = 0.05153$, which implies that formal companies become less constrained with the increased labor force and can, now, accumulate more capital.

Table 4 – Extended Model - Standard Deviations: Baseline vs. The Two Margins

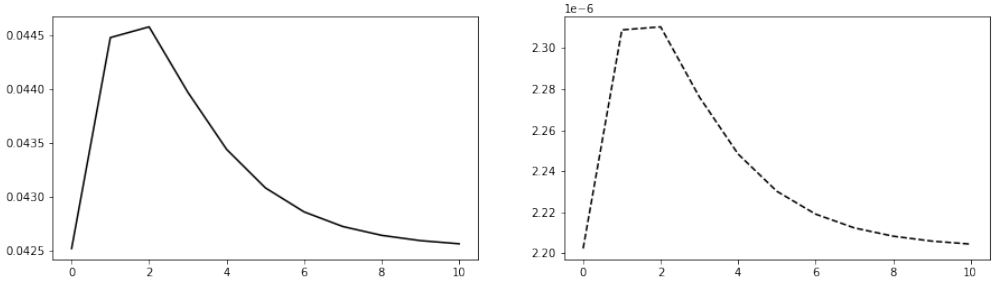
Variable	Baseline model	No intensive margin	No extensive margin
Formal Sector Output	5.951×10^{-3}	6.156×10^{-3}	5.997×10^{-3}
Total GDP	5.749×10^{-3}	5.938×10^{-3}	5.995×10^{-3}
Formal Sector Employment	7.526×10^{-3}	7.944×10^{-3}	6.919×10^{-3}
Extensive Margin	7.8×10^{-4}	8.538×10^{-4}	4.149×10^{-8}
Total Employment	6.746×10^{-3}	7.091×10^{-3}	6.919×10^{-3}
Inflation	1.116×10^{-2}	1.306×10^{-2}	1.168×10^{-2}
Interest Rate	8.202×10^{-3}	5.366×10^{-3}	7.576×10^{-3}
Government Revenue	3.652×10^{-3}	4.042×10^{-3}	3.540×10^{-3}
Informality ratio N_i/N	1.502×10^{-3}	1.237×10^{-5}	1.502×10^{-3}

Notes: The table displays the standard deviations of each of our key variables for the baseline version of the model and for both of the alternative, counterfactual, versions. There is an unequivocal decrease in volatility when the intensive margin is eliminated, whereas the results for the elimination of the extensive margin are more varied.

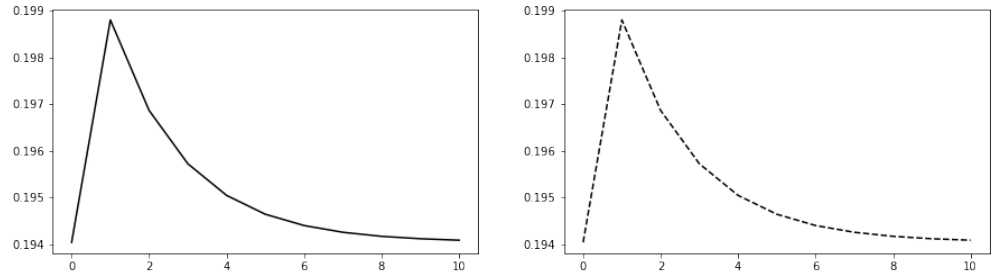
Formal sector output, as can be seen on panel (a), Figure 9, increases along the entire path when the extensive margin is eliminated, which is expected, given that more workers go to the formal sector in this economy. Total real GDP, however, decreases in the absence of the extensive margin. This result is in line with the literature on dual markets, since increasing the cost of informality here implies essentially driving the representative informal business out of the market, since the informal business cannot become formal due to the fact that its productivity is too low (Ulyssea, 2020). Figure 10 shows the response of the rate of interest, R, to the productivity shock in both scenarios. The figure implies that the interest rate's path is almost unchanged by the elimination of the extensive margin.

Finally, we analyze the standard deviations. The results are more varied than in the case of the intensive margin. The elimination of the extensive margin decreases the standard deviations of formal sector employment and government revenue with respect to the baseline version of the model. This is due to the fact that, since workers from the formal sector will not go to the informal sector, not only formal employment fluctuates less, but so do government revenues from the value added tax. Given that the informality ratio is not significantly affected by the elimination of the extensive margin, then the volatility of government revenue out of the payroll tax is not affected either. The elimination of the extensive margin also lowers interest rate volatility. This is likely due to the fact that

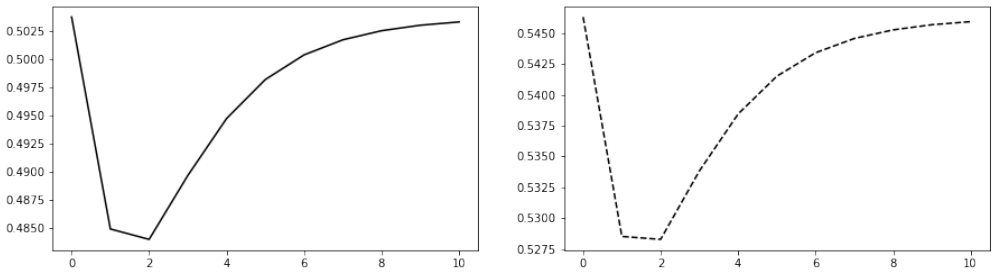
Figure 8 – Extended Model - Effects of "Shutting Down" the Extensive Margin on Employment's Response to the Productivity Shock



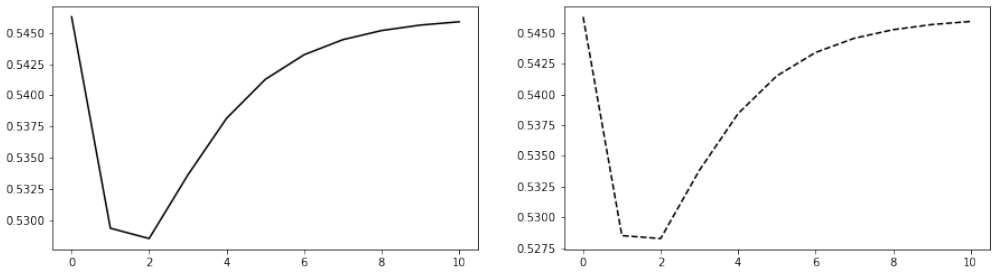
(a) Extensive margin employment, M



(b) Formal sector informality ratio, N_i/N



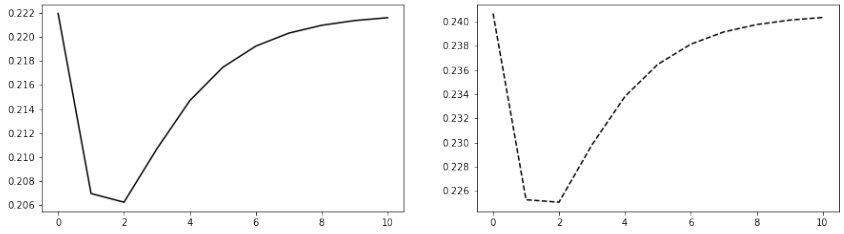
(c) Formal sector employment, N



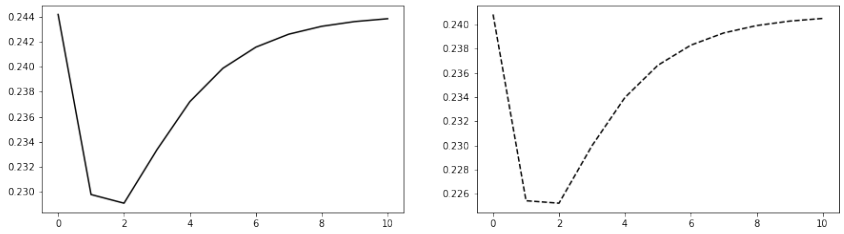
(d) Total employment

Notes: The figure shows how each of the employment variables evolve after a negative productivity shock on the baseline model (on the left) and on the version of the model with nearly no extensive margin (on the right). Notice that there is a significant difference in scale in panel (a). The only other employment variable starkly affected by the near elimination of the extensive margin is N, the formal sector employment.

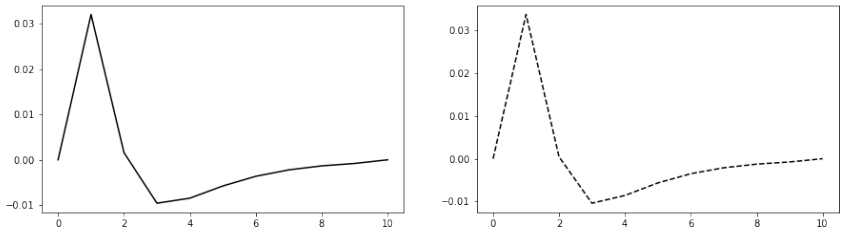
Figure 9 – Extended Model - Effects of "Shutting Down" the Extensive Margin on the Other Variables' Response to the Productivity Shock



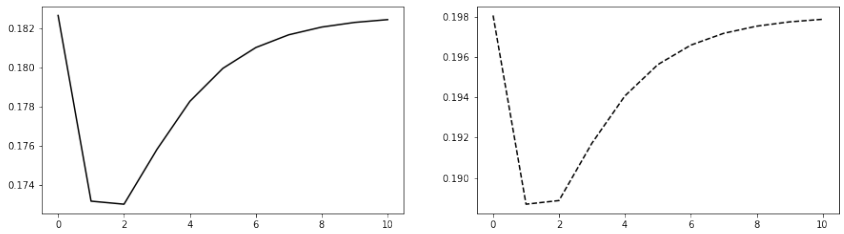
(a) Formal sector real output



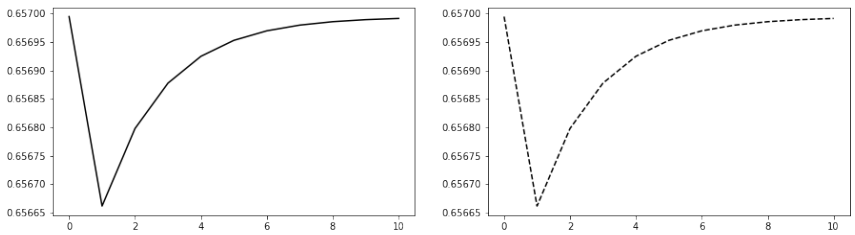
(b) Total real GDP



(c) Inflation



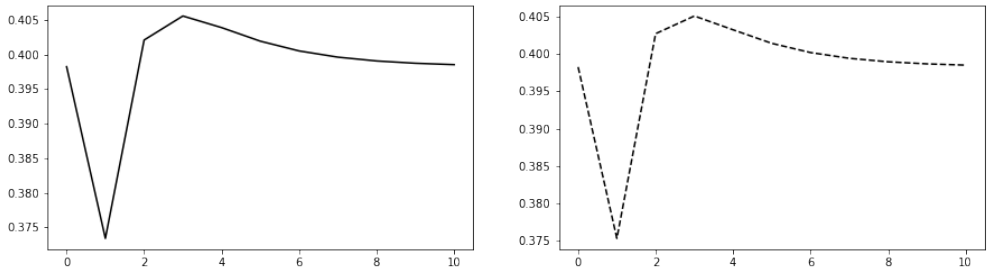
(d) Government revenue



(e) Cutoff value, z^*

Notes: The figure displays the paths of each variable for the baseline model (on the left) and the alternative, high η^e, version of the model (on the right). Formal sector output and government revenue increase with the near elimination of the extensive margin, whereas total GDP decreases.

Figure 10 – Extended Model - Interest Rate's Responses to the Productivity Shock



Notes: Here, we present, on the same graph, the first path of the interest rate R_t, for the baseline version of the model (full line) and the alternative version (dashed line), with no extensive margin.

prices respond more intensely to the TFP shock in the economy with no extensive margin, which alleviates the fall in the profit rate, x, by affecting both entrepreneurial revenues and real wages.

Eliminating the extensive margin, on the other hand, yields more volatility than the baseline case for total GDP, formal output, total employment and inflation, and even more so than the elimination of the intensive margin in the case of the first of these. These increases in volatility are, however, mostly mild. This large increase in total GDP volatility is due to the fact that, once the extensive margin is eliminated, employment cannot be smoothed by optimally allocating part of the labor force to the informal sector. The increase in volatility in formal sector output is due to the increased inflation volatility.

In a summary, this section indicates that shutting down the intensive margin leads to layoffs in the formal sector and that these workers are absorbed by the informal sector. Given the latter's low productivity, however, this leads to a decrease in total GDP. Moreover, the elimination of the intensive margin increases this economy's volatility, except for the interest rate and informality ratio volatility, which decreases upon the elimination of this margin. On the other hand, shutting down the extensive margin does not affect the fraction of workers that are informal in the formal sector, but it does increase formal sector hiring and product. However, the closure of informal businesses leads to a lower total GDP. Finally, the effects of the absence of the extensive margin over this economy's volatility are ambiguous.

6 Further Exercises

Although the model accomplishes its purpose of assessing how informality affects the degree of misallocation in the economy and the co-movements of the variables are largely in line with the literature, the way we model financial frictions understates the negative impact of such misallocation over GDP. This is due to the fact that the model overestimates the increase in product that comes from less productive companies entering the market, since there is no exit dynamics. As a result, some caution is necessary in assessing the quantitative implications of some of the exercises done in the present section. Besides, this feature of the model and the assumption of full depreciation generate a paradoxical response in GDP when we change the parameter θ. Therefore, we focus on doing counterfactual exercises on the remaining parameters.

In this section, we change many of the parameters in the model in order to better understand its functioning, its implications and to which extent its results hold for different values of these parameters. We analyze how the steady state level and volatility of each variable is altered when we change these parameters, and how the effects of the elimination of either the intensive or extensive margin are affected by this change. Since there is a narrower range of feasible values for the formal sector parameters, the changes in these parameters are more moderate than the ones we do for the informal sector. The tables containing the results of these counterfactual exercises can all be found on Appendix B.

Change in φ: We change the value for the inverse Frisch elasticity of labor supply from $\varphi = 1$ to $\varphi^{'} = 0.9$. This change leads to a decrease in the formal sector output and in total employment at the steady state. The ratio of informal workers on the formal sector N_i/N, the cutoff value z^*, the interest rate R and the extensive margin are unaffected. The elimination of both the intensive and the extensive margin maintains the same patterns observed on the baseline model with respect to the steady state level of the variables as well as to volatility.

The economy becomes, overall, less volatile with the lower φ, though the volatility of the ratio N_i/N is not affected and the volatility of the interest rate R increases. This result might appear enigmatic at first since the reduction in φ means an increase on the Frisch elasticity of labor supply, and indeed, it is difficult to interpret. However, note that inflation becomes less volatile, which, in turn, given that nominal wage is fixed at the steady state level, leads to lower volatility in real wages. Less inflation volatility also helps to account for the higher interest rate volatility since, in this case, the profit rate also becomes more volatile. Since the decrease in inflation volatility is small, it is not enough

to account for the (more significant) decrease in employment volatility and increase in interest rate volatility.

Change in ω: We lower the value of ω from $\omega = 0.8454$ to $\omega^{'} = 0.5$. This reduction decreases the extent to which the costs of hiring informal workers rises as entrepreneurs hire more of them. Such a change leads to a mild decrease in GDP, more concentrated on the formal sector; a mild increase on the extensive margin; a decrease in N_i/N; a mild decrease in the interest rate R; and an increase in z^*. Again, this decrease in GDP is at least partially due to the increase in z^* and is, therefore, probably overstated. Besides, the rise in z^* indicates that a flatter informality cost distribution (through the decrease in ω) implies in less competitive advantage for smaller businesses in recessions. Moreover, the decrease in N_i/N can be interpreted as following from a decrease in the degree to which the smaller companies (that are more numerous) hire informal workers. Besides, all variables become more volatile as ω decreases, except for the interest rate.

The patterns of how the economy responds to the elimination of each of the margins of informality remain, overall, the same both for the steady state levels of the variables and for their volatility. Nevertheless, the increase in volatility due to the elimination of the intensive margin is notably smaller than with the baseline calibration. There is one change though: a lower ω decreases the volatility of the cutoff value z^* in the economy with no intensive margin - the opposite of what happens in the baseline economy. Furthermore, since cutoff volatility increases mildly in the baseline model, the fall in interest rate's standard deviation is a consequence of a smaller variation in profits when ω falls, especially on the lower end of the productivity distribution. This strengthens the hypothesis that smaller businesses become more competitive in recessions due to informality.

Change in τ_y: We raise the value added tax from $\tau_y = 0.2925$ to $\tau_y^{'} = 0.33$. This brings about a decrease in GDP; an increase on the extensive margin - in line with De Paula; Scheinkman (2010); - and no change whatever on the intensive margin nor on the interest rate. This result means that this change in taxation produces no allocative distortion within the formal section - but it leads to a distortion in allocation between formal and informal sectors. The levels and volatility of the model's variables keep the same patters of response to the elimination of the extensive and the intensive margin as in the baseline calibration. Nevertheless, although the formal economy becomes less volatile after the increase in taxes, the informal sector and the interest rate become more volatile. In the case of the latter, it is likely due to the lower inflation volatility.

Change in τ_p: We raise the payroll tax from $\tau_p = 0.29$ to $\tau_p^{'} = 0.33$. The results of this exercise indicate that GDP decreases; the extensive margin grows; the rate of informality in the formal sector also increases as expected; and the cutoff value z^* decreases. This latter result indicates that this change in payroll taxation level increases the degree of misallocation on this economy, favoring smaller, less productive businesses; it also produces

misallocation between both sectors, by increasing the size of the less productive informal sector. The interest rate increases slightly as τ_p goes up, which indicates that the profit rate of the marginal producer is larger. This probably means that a higher payroll tax creates a competitive advantage for businesses that evade it the most through informal hiring. No changes are observed in the level responses of the model's variables to the increase in the costs of either margin of informality.

The economy as a whole becomes more volatile, though the volatility of the informality ratio decreases. Besides, volatility patterns are maintained following the elimination of the extensive margin, except for the interest rate. Interest rate volatility decreases in the baseline economy when the extensive margin is eliminated, but it increases slightly in the economy with a higher τ_p. In fact, interest rate becomes significantly more volatile when the payroll tax is increased. This is due to the fact that this increase makes formal sector output volatility go up significantly - even more so in an economy with no informal sector. Furthermore, the patterns are inverted when the intensive margin is eliminated, that is, the elimination of such margin leads to lower volatility. This probably means that uncertainty about the degree of misallocation due to the presence of the intensive margin leads to this result. Since informal hiring is more relevant in an economy with higher τ_p, misallocation within the formal sector fluctuates more. When the intensive margin is eliminated, however, the effect of increasing taxes is to decrease volatility.

Here, it is important to make one observation. In the model, the decrease in GDP is less significant when we increase the payroll tax than in the case where we increase τ_y to the same value. This is not in line with the literature, since it is commonly considered that payroll taxes are more distortionary. This result is due to the fact that the model underestimates the negative consequences of higher misallocation of capital as already stated in the beginning of the current section. In fact, the increase on the value added tax produces, as posited previously, no distortion in allocation within the formal sector. Therefore, though each of these results, considered separately, make intuitive sense, a quantitative comparison between the two might produce misleading conclusions.

Change in δ: We change the value of the δ from $\delta = 0.6$ to $\delta' = 0.8$. This brings about an increase in formal sector output, and also in total GDP (though smaller). The informal sector, however, becomes smaller. In fact, this parameter changes leads to a reduction on the demand for the informal good, which is reflected in a smaller price for these goods. Since the equilibrium steady state wage is not changed, labor supply remains the same, and workers are relocated towards the formal sector. The intensive margin and the interest rate are, nevertheless, unchanged.

No changes in pattern are produced, neither in volatility nor in steady state level when we eliminate the intensive and the extensive margins. Nevertheless, the effects of eliminating the extensive margin are smaller than on the baseline case. This is due to the

fact that, at steady state, informality is smaller when δ is larger. Besides, formal sector's standard deviation increases with the parameter change, but informal sector, interest rate and government revenue's volatility decreases. These changes in volatility are likely related to the level change that results from raising δ.

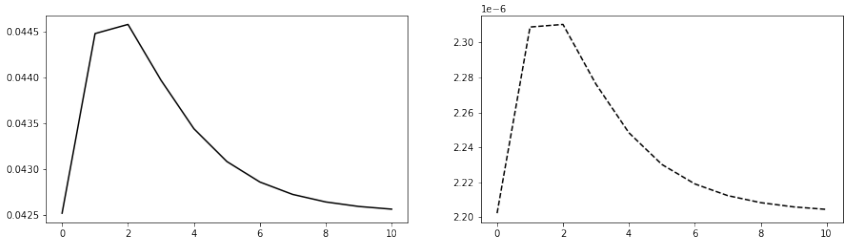
Change in λ: An increase in λ from $\lambda = 0.235$ to $\lambda^{'} = 0.3$ leads to a shrink in the formal sector and an increase in the informal sector. What happens here is precisely the opposite of what happens when we raise δ: the demand for informal goods increases. Again, both total employment, the interest rate and the intensive margin are unaffected. Moreover, level and volatility patterns are maintained following the increase in informality costs along the extensive and intensive margin. However, the elimination of the extensive margin has now larger effects than on the baseline case due to its bigger size.

The same happens with volatility. Formal sector labor volatility increases, and GDP volatility decreases mildly. We can also observe increased extensive margin volatility, which is the likely cause for the higher volatility in formal sector labor in spite of total employment being less volatile (since real wages vary less). Besides, formal sector output becomes less volatile in spite of the higher volatility on its labor due to decreased inflation volatility. This lower inflation volatility is also probably behind the higher interest rate volatility.

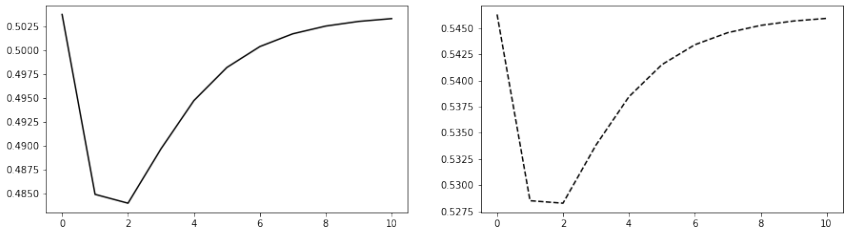
Change in $\hat{z}$: We increase the informal sector productivity from $\hat{z} = 0.108$ to $\hat{z}^{'} = 0.2$. This leads to a shrink of the formal sector, though total GDP increases. Government revenue, as expected, decreases with the formal sector. Intensive margin and the interest rate are left unchanged. Patterns are unchanged following the elimination of both margins. However, with a higher $\hat{z}$, GDP drops by more when the extensive margin is eliminated, and formal sector hiring increases by more. Likewise, the variation in volatility is stronger in the scenario where the extensive margin is eliminated. Volatility decreases for the formal sector and for the economy as whole, but increases for the extensive margin because of its larger size. Volatility also increases for the interest rate, due to lower inflation volatility.

Change in α_i: An increase in the technological parameter α_i from $\alpha_i = 0.5$ to $\alpha_i = 0.7$ leads to a mild increase in formal sector's output and labor. On the other hand, total GDP decreases. The extensive margin also shrinks due to the increase in relative informal good's price. The intensive margin and the interest rate are not affected. Patterns are maintained for the elimination of the intensive margin both with respect to the steady state level of the variables and their volatility. However, total GDP increases if the extensive margin is eliminated. This result is probably due to the fact that the decrease in output on the baseline case is mild when α_i is changed. Therefore, the increase in formal sector output trumps the effect of the elimination of the informal sector, which is smaller with a higher α_i.

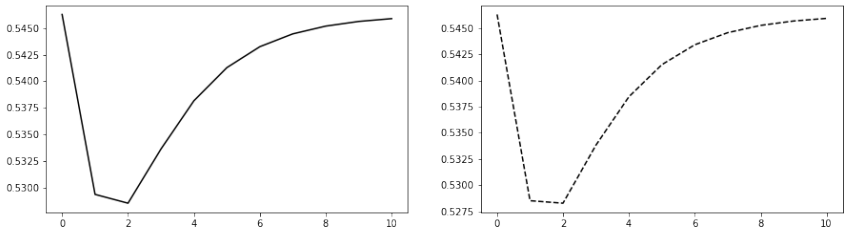
Figure 11 – Change in α_i - Effects of "Shutting Down" the Extensive Margin on the Economy's Response to the Productivity Shock



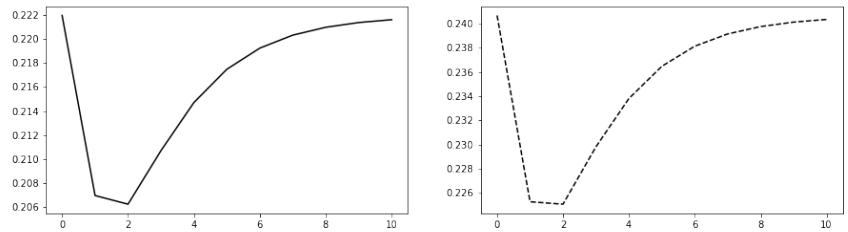
(a) Extensive margin employment, M



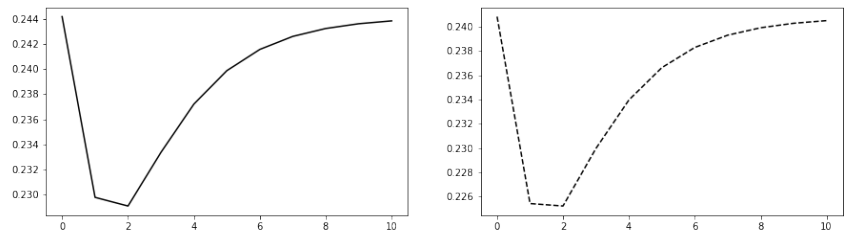
(b) Formal sector employment, N



(c) Total employment



(d) Formal sector output



(e) Total GDP

Notes: The figure shows that the patterns observed on the baseline calibration of the model are re-established when it is re-calibrated with a higher value for α_i. On the left, we show the paths of the variables on the baseline model and, on the right, the paths of the variables on the alternative, high η^e, version. Note that, as before, total real GDP shrinks when the extensive margin is eliminated.

We recalibrate the model to match the moments presented on the previous section with the new value for α_i, and proceed with the elimination of the extensive margin. The results are presented in Figure 11. As can be seen very clearly, the previous patterns are re-established, that is, the informal sector is large enough to lead to a decrease on total GDP upon its elimination. Therefore, the model remains robust to a variation on α_i, and, indeed, the qualitative results obtained on the previous section are not sensitive to the assumption that $\alpha_i = 0.5$. Finally, the economy becomes more volatile overall, but the intensive margin is not affected. Total employment, however, becomes slightly less volatile. Though formal output volatility changes sign in how it responds to the elimination of the extensive margin, this change in standard deviation is slight, and therefore not significant.

All in all, the main conclusions obtained on the previous sections still hold when parameters are changed, since deviations from those patterns are either too small or are eliminated when the model is recalibrated. Besides, all the changes in parameters that affect all formal entrepreneurs equally have no effect on the relative size of the intensive margin with respect to total formal sector labor nor on the degree of misallocation within the formal sector. Moreover, a bigger intensive margin leads to more fluctuations on the degree of misallocation between formal companies. It is also important to note that the net effect over total GDP of the elimination of the extensive margin depends on how small it is: a smaller informal sector might lead to an increase in total output after its elimination. In addition, the behavior of the interest rate in the exercises indicates that 1) its steady state level is not affected by changes in the informal sector, but its volatility is; and 2) the hypothesis that smaller businesses become more competitive in recessions does properly account for the behaviors of the intensive margin and cutoff value in the model.

With respect to taxation, the model here presented also leads to the conclusion that a value added tax is not distortionary, but that a payroll tax is. Nevertheless, it must be added that this result is only valid within the formal sector: both kinds of tax increase the size of the informal sector when augmented. No reliable quantitative comparisons between the two types of tax can be drawn from this model though. Finally, a more productive informal sector, perhaps due to a higher level of human capital for informal entrepreneurs, leads to higher GDP, but tax collection is lower due to the resulting shrink of the formal sector. This result echoes La Porta; Shleifer (2014) and Lambert; Pescatori; Toscani (2020), that argue that the stagnation of productivity on the informal sector leads to its decrease in size over time, as the formal sector becomes more productive.

7 Concluding Remarks

The present work presented a theoretical model that explores the effects and behavior of informality along the business cycle in an economy characterized by capital misallocation. It differs from much of the business cycle literature on informality in that it models the intensive margin of informality through a size-dependent cost function as in Erosa; Fuster; Martínez (2020) and Ulyssea (2018), rather than focusing on entry and exit dynamics. Besides, we extended the model to include the extensive margin following a dualistic approach to informality as the one defended by La Porta; Shleifer (2014). Later, we conducted counterfactual exercises to assess the model's robustness and to further understand its mechanisms.

The results obtained can be summarized by the following points: 1) the presence of the intensive margin gives less productive entrepreneurs a competitive advantage during recessions; 2) the existence of the intensive margin leads to higher profit rates for the entrepreneurs and more capital accumulation; 3) the elimination of either margin decreases total GDP but increases tax collection; 4) the intensive margin smooths fluctuations overall; 5) the extensive margin smooths fluctuation for some variables, does not affect the intensive margin, but it increases formal sector employment and government revenue volatility; 6) only the intensive margin affects the degree of misallocation within the formal sector; 7) either type of tax increase leads to more intersectorial misallocation; and 8) the payroll tax alone affects the degree of misallocation within the formal sector. Moreover, the model is overall robust to changes in parameters provided that the due recalibration is performed when necessary.

It is important to note, however, that the strong assumptions made regarding capital depreciation and entrepreneurs' productivity transition dynamics produce a shortcoming to the model that make it unsuitable for very rigorous quantitative analysis. Therefore, one natural step forwards is to relax these assumptions. Also, the assumption of perfect foresight could also be relaxed in order to model inflation dynamics more realistically by assuming a central bank following Taylor's rule. Finally, another direction in which the present work could be improved is by modelling the extensive margin in a way that also encompasses companies that are productive enough to formalize, but optimally chose not to in order to benefit from reduced costs.

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APPENDIX A – More Details to the Model

A.1 Cost of hiring informal workers

Given that the cost of hiring informal workers is defined by (3.6), the condition (3.15) yields:

$$n_{i,t} = \left[\frac{\tau_p}{\eta^i(2+\omega)z_t\Phi_t}\right]^{\frac{1}{1+\omega}} n_t, \tag{A.1}$$

which is the amount of informal labor demanded by the entrepreneur with idiosyncratic productivity factor z_t. Note that $n_{i,t}$ increases with n_t. However, the more productive the entrepreneur is (and therefore, the larger she is) the smaller is the *fraction* of the workers hired by this entrepreneur that are informal.

A.2 Entrepreneurs

The entrepreneur's hiring decision is static. Since we have supposed that the marginal worker in the formal sector is formal, it follows that:

$$(1-\tau_y)(1-\alpha)p_t z_t \Phi_t \left(\frac{k_t}{n_t}\right)^{\alpha} = (1+\tau_p)w_t,$$

which implies that the amount of labor demanded is given by (3.14).

However, from (3.9), (A.1) and (3.14), we obtain the expression (3.16) for entrepreneur's profits as a linear function of the capital stock. This, coupled with the collateral constraint (3.10), implies that entrepreneur's returns $R_{t,t+1}(z_{t+1})$ can be rewritten as:

$$R_{t,t+1}(z_{t+1}) = \max\left\{R_t, \frac{(1-\theta)x_{t+1}(z_{t+1})}{1-\frac{\theta x_{t+1}(z_{t+1})}{R_t}}\right\}. \tag{A.2}$$

Also, for next period's marginal producer, with productivity term z^*_{t+1}, we must have:

$$R_t = x_{t+1}(z^*_{t+1}). \tag{A.3}$$

Now, we define entrepreneur's net worth, q_t, as $q_t := x_t k_t - b_t$. The results presented in this section allow us to re-write, as in Reis (2013), the entrepreneur's problem as follows:

$$\begin{aligned} &\max \mathbb{E}_t\left[\sum_{t=0}^{\infty}\beta^t ln(c_t)\right] \quad \text{subject to} \\ &q_{t+1} = R_{t,t+1}(z_t)[q_t - p_t c_t], \\ &c_t, q_t \geq 0. \end{aligned}$$

which is a standard problem, that yields the first order conditions:

$$p_t c_t = (1-\beta) q_t, \tag{A.4}$$

$$q_{t+1} = R_{t,t+1}(z_t)\beta q_t. \tag{A.5}$$

Moreover, from the original version of the problem, we obtain the following Euler equation for entrepreneurs that will be active in $t+1$:

$$\frac{\beta R_t}{p_{t+1}c_{t+1}}\left[(1-\theta)(1-\tau_y)MP_k(z_{t+1})\right] = \frac{1}{p_t c_t}\left[R_t - \theta(1-\tau_y)MP_k(z_{t+1})\right], \tag{A.6}$$

in which $MP_k(z_{t+1}) = \alpha p_{t+1} z_{t+1} \Phi_{t+1} \left(\frac{n_{t+1}}{k_{t+1}}\right)^{1-\alpha}$ is the marginal productivity of capital for the entrepreneur with idiosyncratic productivity factor z_{t+1} at $t+1$.

By using (A.6), (A.4) and (A.5), one can obtain the following expression:

$$w_{t+1}^{1-\frac{1}{\alpha}}\left[\frac{(1-\tau_y)(1-\alpha)^{1-\alpha}p_{t+1}z_{t+1}\Phi_t}{(1+\tau_p)^{1-\alpha}}\right]^{\frac{1}{\alpha}} = \frac{R_t}{\alpha\left[\frac{(1-\theta)R_t}{R_{t,t+1}(z_{t+1})}+\theta\right]}. \tag{A.7}$$

This expression gives the value for the post-tax average productivity of capital $(1-\tau_y)\frac{y_{t+1}(z_{t+1})}{k_{t+1}} = w_{t+1}^{1-\frac{1}{\alpha}}\left[\frac{(1-\tau_y)(1-\alpha)^{1-\alpha}p_{t+t}z_{t+1}\Phi_{t+1}}{(1+\tau_p)^{1-\alpha}}\right]^{\frac{1}{\alpha}}$. Finally, note that the entrepreneur's investment in capital stock for the next period is given by:

$$k_{t+1} = \frac{\beta R_{t,t+1}(z_{t+1})q_t}{(1-\theta)x_{t+1}(z_{t+1})}. \tag{A.8}$$

This expression, alongside with (A.2) yields (3.17).

A.3 Loans market

To begin, note that the collateral constraint (3.10) and the definition of q_t imply that the amount of loans b_{t+1} chosen by entrepreneurs who will be, respectively, active and inactive at $t+1$ are:

$$b_{t+1}(z_{t+1}) = \frac{\theta q_{t+1}(z_{t+1})}{1-\theta}, \quad z_{t+1} > z^*_{t+1}, \tag{A.9}$$

$$b_{t+1}(z_{t+1}) = -q_{t+1}(z_{t+1}), \quad z_{t+1} \leq z^*_{t+1}. \tag{A.10}$$

Secondly, the aggregate amount of funds borrowed by entrepreneurs who will be productive in $t+1$ is given by:

$$\int_{z^*_{t+1}}^{\bar{z}} b_{t+1}dF(z) = \int_{z^*_{t+1}}^{\bar{z}} \frac{\theta q_{t+1}}{1-\theta}dF(z) = \int_{z^*_{t+1}}^{\bar{z}} \frac{\theta\beta R_{t,t+1}(z_{t+1})q_t}{1-\theta}dF(z)$$

$$\int_{z^*_{t+1}}^{\bar{z}} b_{t+1} = \frac{\theta\beta Q_t}{1-\theta}\int_{z^*_{t+1}}^{\bar{z}} R_{t,t+1}(z_{t+1})dF(z). \tag{A.11}$$

Note that the aggregation of q_t in the last equality is possible due to the assumption that the draws of the idiosyncratic productivity factor, z_t, are *i.i.d.* Thirdly, the aggregate amount of funds lent by entrepreneurs who will be inactive in $t+1$ is given by:

$$-\int_{\underline{z}}^{z^*_{t+1}} b_{t+1}dF(z) = \int_{\underline{z}}^{z^*_{t+1}} q_{t+1}dF(z) = \int_{\underline{z}}^{z^*_{t+1}} \beta R_t q_t dF(z)$$
$$-\int_{\underline{z}}^{z^*_{t+1}} b_{t+1}dF(z) = \beta R_t Q_t F(z^*_{t+1}), \tag{A.12}$$

in which $F(z^*_{t+1})$ is the cumulative distribution function of z up to z^*_{t+1}. Fourthly, note that the loans market market clearing condition (3.21), along with (A.11) and (A.12), implies that:

$$\frac{(1-\theta)R_t}{\theta} = \frac{\int_{z^*_{t+1}}^{\bar{z}} R_{t,t+1}(z_{t+1})dF(z)}{F(z^*_{t+1})} \tag{A.13}$$

Finally, this market clearing condition leads to the following law of motion for Q:

$$Q_{t+1} = \int_{\underline{z}}^{\bar{z}} q_{t+1}dF(z) = \beta\int_{\underline{z}}^{\bar{z}} R_{t,t+1}(z_{t+1})q_t dF(z)$$
$$= \beta Q_t\left[R_t F(z^*_{t+1}) + \int_{z^*_{t+1}}^{\bar{z}} R_{t,t+1}(z_{t+1})dF(z)\right]$$
$$Q_{t+1} = \frac{\beta R_t F(z^*_{t+1})}{\theta}Q_t. \tag{A.14}$$

A.4 Aggregation

First of all, we aggregate the expression for the post-tax average productivity of capital (A.7) at t to obtain:

$$w_t^{1-\frac{1}{\alpha}} p_t^{\frac{1}{\alpha}}\left[\frac{(1-\tau_y)(1-\alpha)^{1-\alpha}\Phi_t}{(1+\tau_p)^{1-\alpha}}\right]^{\frac{1}{\alpha}}\int_{z^*_t}^{\bar{z}} z_t^{\frac{1}{\alpha}}dF(z) = \frac{R_t F(z^*_t)}{\alpha\theta\left[1+F(z^*_t)\right]}.$$

If one uses the expression for the returns obtained by the marginal producer (A.3), one arrives at the following equation:

$$\frac{[1+F(z^*_t)]}{F(z^*_t)}\int_{z^*_t}^{\bar{z}} z_t^{\frac{1}{\alpha}}dF(z) = \frac{z_t^{*\frac{1}{\alpha}}}{\theta}\left[1+\frac{(1+\omega)(1-\alpha)}{\alpha(1+\tau_p)(\eta^i z^*_t\Phi_t)^{\frac{1}{1+\omega}}}\left(\frac{\tau_p}{2+\omega}\right)^{\frac{2+\omega}{1+\omega}}\right]. \tag{A.15}$$

This equation implies that the cutoff value for z_t depends only on Φ_t. Therefore, knowing the latter for every period ensures we can find the whole series for z^*. Furthermore, since we have supposed a Pareto distribution, we must have:

$$\frac{[1+F(z^*_t)]}{F(z^*_t)}\int_{z^*_t}^{\bar{z}} z_t^{\frac{1}{\alpha}}dF(z) = \frac{\mu\underline{z}^\mu\left(\underline{z}^{\frac{1}{\alpha}-\mu} - z_t^{*\frac{1}{\alpha}-\mu}\right)\left[2-\left(\frac{\underline{z}}{\bar{z}}\right)^\mu - \left(\frac{\underline{z}}{z^*_t}\right)^\mu\right]}{\left(\frac{1}{\alpha}-\mu\right)\left[1-\left(\frac{\underline{z}}{z^*_t}\right)^\mu\right]^2}.$$

Now, using the equation for capital (A.8) into the equation for labor demand (3.14), and aggregating it yields:

$$N_t = \frac{\beta Q_{t-1}}{(1-\theta)w_t} \int_{z_t^*}^{\bar{z}} \left[\frac{R_{t-1,t}(z_t)}{\frac{\alpha(1+\tau_p)}{1-\alpha} + \frac{(1+\omega)}{(\eta^i z_t \Phi_t)^{\frac{1}{1+\omega}}} \left(\frac{\tau_p}{2+\omega}\right)^{\frac{2+\omega}{1+\omega}}} \right] dF(z), \tag{A.16}$$

in which N_t is the total amount of labor demanded at t. Doing the same for the aggregate amount of informal labor demanded, $N_{i,t}$ at t gives:

$$N_{i,t} = \left[\frac{\tau_p}{\eta^i(2+\omega)\Phi_t}\right]^{\frac{1}{1+\omega}} \frac{\beta Q_{t-1}}{(1-\theta)w_t} \int_{z_t^*}^{\bar{z}} \left[\frac{R_{t-1,t}(z_t)}{\frac{\alpha(1+\tau_p) z_t^{\frac{1}{1+\omega}}}{1-\alpha} + \frac{(1+\omega)}{(\eta^i \Phi_t)^{\frac{1}{1+\omega}}} \left(\frac{\tau_p}{2+\omega}\right)^{\frac{2+\omega}{1+\omega}}} \right] dF(z). \tag{A.17}$$

Lastly, by substituting labor demand (3.14) into the production function (3.4) and aggregating it yields the expression for output:

$$Y_t = \frac{(1+\tau_p)w_t}{(1-\tau_y)(1-\alpha)p_t} N_t. \tag{A.18}$$

A.5 Household

In both formulations of the household problem, we must have that labor supply is given by the following expression:

$$\hat{L}_t = \frac{1}{\chi}\left(\frac{w_t}{p_t}\right)^{\frac{1}{\varphi}}. \tag{A.19}$$

However, in the baseline formulation - without the extensive margin -, $\hat{L}_t = \hat{N}_t$. The optimality condition for the labor supply to the informal sector (5.4) implies that:

$$\hat{M}_t = \left(\frac{(1-\eta^e)\alpha_i p_t^i \hat{z}}{w_t}\right)^{\frac{1}{1-\alpha_i}}. \tag{A.20}$$

Now, notice that the market clearing condition for the informal sector (5.9) and the optimality condition for the consumption of the informal good (5.7) imply that the price of the informal good satisfies:

$$(p_t^i)^{\frac{1}{1-\delta}+\frac{\alpha_i}{1-\alpha_i}} = (\delta\lambda p_t)^{\frac{1}{1-\delta}} \left(\frac{w_t}{\alpha_i \hat{z}^{\frac{1}{\alpha_i}}}\right)^{\frac{\alpha_i}{1-\alpha_i}}. \tag{A.21}$$

Finally, we can combine (A.19), (A.20) and (5.3) to obtain the labor supply to the formal sector:

$$\hat{N}_t = \frac{1}{\chi}\left(\frac{w_t}{p_t}\right)^{\frac{1}{\varphi}} - \left(\frac{(1-\eta^e)\alpha_i p_t^i \hat{z}}{w_t}\right)^{\frac{1}{1-\alpha_i}}. \tag{A.22}$$

A.6 Computation Algorithm

To begin, we calculate the steady state. We suppose, without loss of generality, that $\Phi_{SS} = 1$, and use (A.15) to find z^*_{SS}. we can, then, use (A.14) to find R_{SS} since $Q_{t+1} = Q_t$ at the steady state. Now, by normalizing the steady state price level at $p_{SS} = 1$, we can find the wages at the steady state by using (A.3). With the wage level at hand, we can find total labor supply by using (A.19) and (formal sector) output by using (A.18).

In the case of the economy with the extensive margin, we can now also find the amount of labor in the informal ($\hat{M}_{SS}$) and in the formal sector ($\hat{N}_{SS}$) and the informal good price level (p^i_{SS}) by using, respectively, (A.20), (A.22) and (A.21). Next, we use the market clearing condition for the formal sector's labor market, $\hat{N}_{SS} = N_{SS}$, along with (A.16), to find the aggregate steady state amount of aggregate net worth Q_{SS}. Finally, we use (A.17) to find the amount of intensive margin labor hired at the steady state $N_{i,SS}$.

Next, the economy starts, at $t = 0$, at the steady state. Then, we give this economy a one-standard deviation exogenous productivity shock ϵ_t at $t = 1$, and simulate the series up to $t = 10$ by using (3.5). Notice that we already have $Q_{-1} = Q_{SS}$. Besides, as already stated in Section 3, we suppose that wages are fixed at steady state level for all the periods.

Again, we use (A.15) to find z^*_t for every t. Then, for each period, we follow the following steps in order:

1. we calculate the real wage - by finding p_t - that will clear the labor market by using equations (A.16) and (A.19) - and (A.3) to get rid of the R_{t-1} in the entrepreneur's return, $R_{t-1,t}(z_t)$, expression;

2. we use (A.3) to find the equilibrium interest rate R_{t-1} between $t-1$ and t;

3. we find labor supply to the informal ($\hat{M}_t$) and formal sector ($\hat{N}_t$) and the informal good's price level (p^i_t) by using (A.20), (A.22) and (A.21)

4. we calculate the amount of informal labor in formal businesses $N_{i,t}$, by using (A.17), real formal GDP, by using (A.18), real informal GDP by substituting the calculated value for $\hat{M}_t$ into the informal production function, $\hat{z}\hat{M}^{\alpha_i}_T$, governement revenue $GRev_t$ through the use of (3.19) and inflation as usual;

5. Finally, we calculate time t's aggregate net worth through the use of (A.14)

APPENDIX B – Other Tables

Table 5 – Exercises - Level: Baseline vs. Intensive Margin

Variable	Baseline	Baseline $+\Delta$	No intensive margin	No intensive margin $+\Delta$
$\varphi = 1$ to $\varphi^{'} = 0.9$				
Formal Sector Output	2.219×10^{-1}	1.788×10^{-1}	2.147×10^{-1}	1.725×10^{-1}
Total GDP	2.442×10^{-1}	2.011×10^{-1}	2.372×10^{-1}	1.950×10^{-1}
Formal Sector Labor	5.038×10^{-1}	4.059×10^{-1}	4.947×10^{-1}	3.975×10^{-1}
Extensive Margin	4.252×10^{-2}	4.252×10^{-2}	4.345×10^{-2}	4.345×10^{-2}
Informality Ratio	1.940×10^{-1}	1.940×10^{-1}	1.392×10^{-3}	1.392×10^{-3}
Total Employment	5.463×10^{-1}	4.484×10^{-1}	5.381×10^{-1}	4.409×10^{-1}
Interest Rate	3.982×10^{-1}	3.982×10^{-1}	3.910×10^{-1}	3.910×10^{-1}
Gov. Revenue	1.827×10^{-1}	1.472×10^{-1}	2.060×10^{-1}	1.656×10^{-1}
Cutoff value	6.570×10^{-1}	6.570×10^{-1}	6.694×10^{-1}	6.694×10^{-1}
$\omega = 0.8454$ to $\omega^{'} = 0.5$				
Formal Sector Output	2.219×10^{-1}	2.205×10^{-1}	2.147×10^{-1}	2.146×10^{-1}
Total GDP	2.442×10^{-1}	2.428×10^{-1}	2.372×10^{-1}	2.371×10^{-1}
Formal Sector Labor	5.038×10^{-1}	5.020×10^{-1}	4.947×10^{-1}	4.946×10^{-1}
Extensive Margin	4.252×10^{-2}	4.270×10^{-2}	4.345×10^{-2}	4.345×10^{-2}
Informality Ratio	1.940×10^{-1}	1.653×10^{-1}	1.392×10^{-3}	3.669×10^{-4}
Total Employment	5.463×10^{-1}	5.447×10^{-1}	5.381×10^{-1}	5.381×10^{-1}
Interest Rate	3.982×10^{-1}	3.968×10^{-1}	3.910×10^{-1}	3.910×10^{-1}
Gov. Revenue	1.827×10^{-1}	1.860×10^{-1}	2.060×10^{-1}	2.062×10^{-1}
Cutoff value	6.570×10^{-1}	6.594×10^{-1}	6.694×10^{-1}	6.695×10^{-1}
$\tau_y = 0.0.2925$ to $\tau_y^{'} = 0.33$				
Formal Sector Output	2.219×10^{-1}	1.971×10^{-1}	2.147×10^{-1}	1.905×10^{-1}
Total GDP	2.442×10^{-1}	2.206×10^{-1}	2.372×10^{-1}	2.143×10^{-1}
Formal Sector Labor	5.038×10^{-1}	4.579×10^{-1}	4.947×10^{-1}	4.493×10^{-1}
Extensive Margin	4.252×10^{-2}	4.752×10^{-2}	4.345×10^{-2}	4.856×10^{-2}
Informality Ratio	1.940×10^{-1}	1.940×10^{-1}	1.392×10^{-3}	1.392×10^{-3}
Total Employment	5.463×10^{-1}	5.054×10^{-1}	5.381×10^{-1}	4.978×10^{-1}
Interest Rate	3.982×10^{-1}	3.982×10^{-1}	3.910×10^{-1}	3.910×10^{-1}
Gov. Revenue	1.827×10^{-1}	1.721×10^{-1}	2.060×10^{-1}	1.930×10^{-1}
Cutoff value	6.570×10^{-1}	6.570×10^{-1}	6.694×10^{-1}	6.694×10^{-1}
$\tau_p = 0.29$ to $\tau_p^{'} = 0.33$				
Formal Sector Output	2.219×10^{-1}	2.151×10^{-1}	2.147×10^{-1}	2.068×10^{-1}

Total GDP	2.442×10^{-1}	2.378×10^{-1}	2.372×10^{-1}	2.298×10^{-1}
Formal Sector Labor	5.038×10^{-1}	4.870×10^{-1}	4.947×10^{-1}	4.765×10^{-1}
Extensive Margin	4.252×10^{-2}	4.425×10^{-2}	4.345×10^{-2}	4.538×10^{-2}
Informality Ratio	1.940×10^{-1}	2.005×10^{-1}	1.392×10^{-3}	1.492×10^{-3}
Total Employment	5.463×10^{-1}	5.312×10^{-1}	5.381×10^{-1}	5.219×10^{-1}
Interest Rate	3.982×10^{-1}	3.995×10^{-1}	3.910×10^{-1}	3.910×10^{-1}
Gov. Revenue	1.827×10^{-1}	1.914×10^{-1}	2.060×10^{-1}	2.175×10^{-1}
Cutoff value	6.570×10^{-1}	6.548×10^{-1}	6.694×10^{-1}	6.694×10^{-1}
$\delta = 0.6$ to $\delta^{'} = 0.8$				
Formal Sector Output	2.219×10^{-1}	2.322×10^{-1}	2.147×10^{-1}	2.249×10^{-1}
Total GDP	2.442×10^{-1}	2.472×10^{-1}	2.372×10^{-1}	2.401×10^{-1}
Formal Sector Labor	5.038×10^{-1}	5.270×10^{-1}	4.947×10^{-1}	5.183×10^{-1}
Extensive Margin	4.252×10^{-2}	1.932×10^{-2}	4.345×10^{-2}	1.982×10^{-2}
Informality Ratio	1.940×10^{-1}	1.940×10^{-1}	1.392×10^{-3}	1.392×10^{-3}
Total Employment	5.463×10^{-1}	5.463×10^{-1}	5.381×10^{-1}	5.381×10^{-1}
Interest Rate	3.982×10^{-1}	3.982×10^{-1}	3.910×10^{-1}	3.910×10^{-1}
Gov. Revenue	1.827×10^{-1}	1.911×10^{-1}	2.060×10^{-1}	2.159×10^{-1}
Cutoff value	6.570×10^{-1}	6.570×10^{-1}	6.694×10^{-1}	6.694×10^{-1}
$\lambda = 0.235$ to $\lambda^{'} = 0.3$				
Formal Sector Output	2.219×10^{-1}	2.141×10^{-1}	2.147×10^{-1}	2.068×10^{-1}
Total GDP	2.442×10^{-1}	2.406×10^{-1}	2.372×10^{-1}	2.336×10^{-1}
Formal Sector Labor	5.038×10^{-1}	4.860×10^{-1}	4.947×10^{-1}	4.765×10^{-1}
Extensive Margin	4.252×10^{-2}	6.027×10^{-2}	4.345×10^{-2}	6.159×10^{-2}
Informality Ratio	1.940×10^{-1}	1.940×10^{-1}	1.392×10^{-3}	1.392×10^{-3}
Total Employment	5.463×10^{-1}	5.463×10^{-1}	5.381×10^{-1}	5.381×10^{-1}
Interest Rate	3.982×10^{-1}	3.982×10^{-1}	3.910×10^{-1}	3.910×10^{-1}
Gov. Revenue	1.827×10^{-1}	1.762×10^{-1}	2.060×10^{-1}	1.985×10^{-1}
Cutoff value	6.570×10^{-1}	6.570×10^{-1}	6.694×10^{-1}	6.694×10^{-1}
$\hat{z} = 0.108$ to $\hat{z}^{'} = 0.2$				
Formal Sector Output	2.219×10^{-1}	2.089×10^{-1}	2.147×10^{-1}	2.015×10^{-1}
Total GDP	2.442×10^{-1}	2.626×10^{-1}	2.372×10^{-1}	2.558×10^{-1}
Formal Sector Labor	5.038×10^{-1}	4.742×10^{-1}	4.947×10^{-1}	4.644×10^{-1}
Extensive Margin	4.252×10^{-2}	7.211×10^{-2}	4.345×10^{-2}	7.368×10^{-2}
Informality Ratio	1.940×10^{-1}	1.940×10^{-1}	1.392×10^{-3}	1.392×10^{-3}
Total Employment	5.463×10^{-1}	5.463×10^{-1}	5.381×10^{-1}	5.381×10^{-1}
Interest Rate	3.982×10^{-1}	3.982×10^{-1}	3.910×10^{-1}	3.910×10^{-1}
Gov. Revenue	1.827×10^{-1}	1.719×10^{-1}	2.060×10^{-1}	1.934×10^{-1}
Cutoff value	6.570×10^{-1}	6.570×10^{-1}	6.694×10^{-1}	6.694×10^{-1}
$\alpha_i = 0.5$ to $\alpha_i^{'} = 0.7$				

Formal Sector Output	2.219×10^{-1}	2.233×10^{-1}	2.147×10^{-1}	2.159×10^{-1}
Total GDP	2.442×10^{-1}	2.345×10^{-1}	2.372×10^{-1}	2.274×10^{-1}
Formal Sector Labor	5.038×10^{-1}	5.068×10^{-1}	4.947×10^{-1}	4.975×10^{-1}
Extensive Margin	4.252×10^{-2}	3.952×10^{-2}	4.345×10^{-2}	4.056×10^{-2}
Informality Ratio	1.940×10^{-1}	1.940×10^{-1}	1.392×10^{-3}	1.392×10^{-3}
Total Employment	5.463×10^{-1}	5.463×10^{-1}	5.381×10^{-1}	5.381×10^{-1}
Interest Rate	3.982×10^{-1}	3.982×10^{-1}	3.910×10^{-1}	3.910×10^{-1}
Gov. Revenue	1.827×10^{-1}	1.838×10^{-1}	2.060×10^{-1}	2.072×10^{-1}
Cutoff value	6.570×10^{-1}	6.570×10^{-1}	6.694×10^{-1}	6.694×10^{-1}

Table 6 – Exercises - Level: Baseline vs. Extensive Margin

Variable	Baseline	Baseline $+\Delta$	No intensive margin	No intensive margin $+\Delta$
$\varphi = 1$ to $\varphi^{'} = 0.9$				
Formal Sector Output	2.219×10^{-1}	1.788×10^{-1}	2.407×10^{-1}	1.975×10^{-1}
Total GDP	2.442×10^{-1}	2.011×10^{-1}	2.408×10^{-1}	1.977×10^{-1}
Formal Sector Labor	5.038×10^{-1}	4.059×10^{-1}	5.463×10^{-1}	4.484×10^{-1}
Extensive Margin	4.252×10^{-2}	4.252×10^{-2}	2.202×10^{-6}	2.202×10^{-6}
Informality Ratio	1.940×10^{-1}	1.940×10^{-1}	1.940×10^{-1}	1.940×10^{-1}
Total Employment	5.463×10^{-1}	4.484×10^{-1}	5.463×10^{-1}	4.484×10^{-1}
Interest Rate	3.982×10^{-1}	3.982×10^{-1}	3.982×10^{-1}	3.982×10^{-1}
Gov. Revenue	1.827×10^{-1}	1.472×10^{-1}	1.981×10^{-1}	1.626×10^{-1}
Cutoff value	6.570×10^{-1}	6.570×10^{-1}	6.570×10^{-1}	6.570×10^{-1}
$\omega = 0.8454$ to $\omega^{'} = 0.5$				
Formal Sector Output	2.219×10^{-1}	2.205×10^{-1}	2.407×10^{-1}	2.393×10^{-1}
Total GDP	2.442×10^{-1}	2.428×10^{-1}	2.408×10^{-1}	2.394×10^{-1}
Formal Sector Labor	5.038×10^{-1}	5.020×10^{-1}	5.463×10^{-1}	5.447×10^{-1}
Extensive Margin	4.252×10^{-2}	4.270×10^{-2}	2.202×10^{-6}	2.211×10^{-6}
Informality Ratio	1.940×10^{-1}	1.653×10^{-1}	1.940×10^{-1}	1.653×10^{-1}
Total Employment	5.463×10^{-1}	5.447×10^{-1}	5.463×10^{-1}	5.447×10^{-1}
Interest Rate	3.982×10^{-1}	3.968×10^{-1}	3.982×10^{-1}	3.968×10^{-1}
Gov. Revenue	1.827×10^{-1}	1.860×10^{-1}	1.981×10^{-1}	2.018×10^{-1}
Cutoff value	6.570×10^{-1}	6.594×10^{-1}	6.570×10^{-1}	6.594×10^{-1}
$\tau_y = 0.0.2925$ to $\tau_y^{'} = 0.33$				

Formal Sector Output	2.219×10^{-1}	1.971×10^{-1}	2.407×10^{-1}	2.175×10^{-1}
Total GDP	2.442×10^{-1}	2.206×10^{-1}	2.408×10^{-1}	2.177×10^{-1}
Formal Sector Labor	5.038×10^{-1}	4.579×10^{-1}	5.463×10^{-1}	5.054×10^{-1}
Extensive Margin	4.252×10^{-2}	4.752×10^{-2}	2.202×10^{-6}	2.461×10^{-6}
Informality Ratio	1.940×10^{-1}	1.940×10^{-1}	1.940×10^{-1}	1.940×10^{-1}
Total Employment	5.463×10^{-1}	5.054×10^{-1}	5.463×10^{-1}	5.054×10^{-1}
Interest Rate	3.982×10^{-1}	3.982×10^{-1}	3.982×10^{-1}	3.982×10^{-1}
Gov. Revenue	1.827×10^{-1}	1.721×10^{-1}	1.981×10^{-1}	1.899×10^{-1}
Cutoff value	6.570×10^{-1}	6.570×10^{-1}	6.570×10^{-1}	6.570×10^{-1}
$\tau_p = 0.29$ to $\tau_p^{'} = 0.33$				
Formal Sector Output	2.219×10^{-1}	2.151×10^{-1}	2.407×10^{-1}	2.346×10^{-1}
Total GDP	2.442×10^{-1}	2.378×10^{-1}	2.408×10^{-1}	2.348×10^{-1}
Formal Sector Labor	5.038×10^{-1}	4.870×10^{-1}	5.463×10^{-1}	5.312×10^{-1}
Extensive Margin	4.252×10^{-2}	4.425×10^{-2}	2.202×10^{-6}	2.292×10^{-6}
Informality Ratio	1.940×10^{-1}	2.005×10^{-1}	1.940×10^{-1}	2.005×10^{-1}
Total Employment	5.463×10^{-1}	5.312×10^{-1}	5.463×10^{-1}	5.312×10^{-1}
Interest Rate	3.982×10^{-1}	3.995×10^{-1}	3.982×10^{-1}	3.995×10^{-1}
Gov. Revenue	1.827×10^{-1}	1.914×10^{-1}	1.981×10^{-1}	2.088×10^{-1}
Cutoff value	6.570×10^{-1}	6.548×10^{-1}	6.570×10^{-1}	6.548×10^{-1}
$\delta = 0.6$ to $\delta^{'} = 0.8$				
Formal Sector Output	2.219×10^{-1}	2.322×10^{-1}	2.407×10^{-1}	2.407×10^{-1}
Total GDP	2.442×10^{-1}	2.472×10^{-1}	2.408×10^{-1}	2.407×10^{-1}
Formal Sector Labor	5.038×10^{-1}	5.270×10^{-1}	5.463×10^{-1}	5.463×10^{-1}
Extensive Margin	4.252×10^{-2}	1.932×10^{-2}	2.202×10^{-6}	1.932×10^{-7}
Informality Ratio	1.940×10^{-1}	1.940×10^{-1}	1.940×10^{-1}	1.940×10^{-1}
Total Employment	5.463×10^{-1}	5.463×10^{-1}	5.463×10^{-1}	5.463×10^{-1}
Interest Rate	3.982×10^{-1}	3.982×10^{-1}	3.982×10^{-1}	3.982×10^{-1}
Gov. Revenue	1.827×10^{-1}	1.911×10^{-1}	1.981×10^{-1}	1.981×10^{-1}
Cutoff value	6.570×10^{-1}	6.570×10^{-1}	6.570×10^{-1}	6.570×10^{-1}
$\lambda = 0.235$ to $\lambda^{'} = 0.3$				
Formal Sector Output	2.219×10^{-1}	2.141×10^{-1}	2.407×10^{-1}	2.407×10^{-1}
Total GDP	2.442×10^{-1}	2.406×10^{-1}	2.408×10^{-1}	2.409×10^{-1}
Formal Sector Labor	5.038×10^{-1}	4.860×10^{-1}	5.463×10^{-1}	5.463×10^{-1}
Extensive Margin	4.252×10^{-2}	6.027×10^{-2}	2.202×10^{-6}	3.122×10^{-6}
Informality Ratio	1.940×10^{-1}	1.940×10^{-1}	1.940×10^{-1}	1.940×10^{-1}
Total Employment	5.463×10^{-1}	5.463×10^{-1}	5.463×10^{-1}	5.463×10^{-1}
Interest Rate	3.982×10^{-1}	3.982×10^{-1}	3.982×10^{-1}	3.982×10^{-1}
Gov. Revenue	1.827×10^{-1}	1.762×10^{-1}	1.981×10^{-1}	1.981×10^{-1}

Cutoff value	6.570×10^{-1}	6.570×10^{-1}	6.570×10^{-1}	6.570×10^{-1}
$\hat{z} = 0.108$ to $\hat{z}' = 0.2$				
Formal Sector Output	2.219×10^{-1}	2.089×10^{-1}	2.407×10^{-1}	2.407×10^{-1}
Total GDP	2.442×10^{-1}	2.626×10^{-1}	2.408×10^{-1}	2.411×10^{-1}
Formal Sector Labor	5.038×10^{-1}	4.742×10^{-1}	5.463×10^{-1}	5.463×10^{-1}
Extensive Margin	4.252×10^{-2}	7.211×10^{-2}	2.202×10^{-6}	3.735×10^{-6}
Informality Ratio	1.940×10^{-1}	1.940×10^{-1}	1.940×10^{-1}	1.940×10^{-1}
Total Employment	5.463×10^{-1}	5.463×10^{-1}	5.463×10^{-1}	5.463×10^{-1}
Interest Rate	3.982×10^{-1}	3.982×10^{-1}	3.982×10^{-1}	3.982×10^{-1}
Gov. Revenue	1.827×10^{-1}	1.719×10^{-1}	1.981×10^{-1}	1.981×10^{-1}
Cutoff value	6.570×10^{-1}	6.570×10^{-1}	6.570×10^{-1}	6.570×10^{-1}
$\alpha_i = 0.5$ to $\alpha_i^{'} = 0.7$				
Formal Sector Output	2.219×10^{-1}	2.233×10^{-1}	2.407×10^{-1}	2.407×10^{-1}
Total GDP	2.442×10^{-1}	2.345×10^{-1}	2.408×10^{-1}	2.407×10^{-1}
Formal Sector Labor	5.038×10^{-1}	5.068×10^{-1}	5.463×10^{-1}	5.463×10^{-1}
Extensive Margin	4.252×10^{-2}	3.952×10^{-2}	2.202×10^{-6}	2.657×10^{-6}
Informality Ratio	1.940×10^{-1}	1.940×10^{-1}	1.940×10^{-1}	1.940×10^{-1}
Total Employment	5.463×10^{-1}	5.463×10^{-1}	5.463×10^{-1}	5.463×10^{-1}
Interest Rate	3.982×10^{-1}	3.982×10^{-1}	3.982×10^{-1}	3.982×10^{-1}
Gov. Revenue	1.827×10^{-1}	1.838×10^{-1}	1.981×10^{-1}	1.981×10^{-1}
Cutoff value	6.570×10^{-1}	6.570×10^{-1}	6.570×10^{-1}	6.570×10^{-1}

Table 7 – Exercises - Volatility: Baseline vs. Intensive Margin

Variable	Baseline	Baseline $+\,\Delta$	No intensive margin	No intensive margin $+\,\Delta$
$\varphi = 1$ to $\varphi^{'} = 0.9$				
Formal Sector Output	5.951×10^{-3}	5.058×10^{-3}	6.156×10^{-3}	5.211×10^{-3}
Total GDP	5.749×10^{-3}	4.860×10^{-3}	5.938×10^{-3}	4.998×10^{-3}
Formal Sector Labor	7.526×10^{-3}	6.780×10^{-3}	7.944×10^{-3}	7.131×10^{-3}
Extensive Margin	7.800×10^{-4}	7.636×10^{-4}	8.538×10^{-4}	8.334×10^{-4}
Informality Ratio	1.502×10^{-3}	1.502×10^{-3}	1.237×10^{-5}	1.237×10^{-5}
Total Employment	6.746×10^{-3}	6.017×10^{-3}	7.091×10^{-3}	6.298×10^{-3}
Inflation	1.116×10^{-2}	1.077×10^{-2}	1.306×10^{-2}	1.254×10^{-2}
Interest Rate	8.202×10^{-3}	8.673×10^{-3}	5.767×10^{-3}	6.342×10^{-3}
Gov. Revenue	3.652×10^{-3}	3.187×10^{-3}	4.042×10^{-3}	3.543×10^{-3}

Cutoff value	1.051×10^{-4}	1.051×10^{-4}	7.965×10^{-7}	7.965×10^{-7}
$\omega = 0.8454$ to $\omega^{'} = 0.5$				
Formal Sector Output	5.951×10^{-3}	6.109×10^{-3}	6.156×10^{-3}	6.156×10^{-3}
Total GDP	5.749×10^{-3}	5.899×10^{-3}	5.938×10^{-3}	5.937×10^{-3}
Formal Sector Labor	7.526×10^{-3}	7.759×10^{-3}	7.944×10^{-3}	7.945×10^{-3}
Extensive Margin	7.800×10^{-4}	8.107×10^{-4}	8.538×10^{-4}	8.540×10^{-4}
Informality Ratio	1.502×10^{-3}	1.727×10^{-3}	1.237×10^{-5}	4.024×10^{-6}
Total Employment	6.746×10^{-3}	6.949×10^{-3}	7.091×10^{-3}	7.091×10^{-3}
Inflation	1.116×10^{-2}	1.217×10^{-2}	1.306×10^{-2}	1.306×10^{-2}
Interest Rate	8.202×10^{-3}	6.787×10^{-3}	5.767×10^{-3}	5.764×10^{-3}
Gov. Revenue	3.652×10^{-3}	3.841×10^{-3}	4.042×10^{-3}	4.044×10^{-3}
Cutoff value	1.051×10^{-4}	1.060×10^{-4}	7.965×10^{-7}	2.485×10^{-7}
$\tau_y = 0.0.2925$ to $\tau_y^{'} = 0.33$				
Formal Sector Output	5.951×10^{-3}	5.367×10^{-3}	6.156×10^{-3}	5.548×10^{-3}
Total GDP	5.749×10^{-3}	5.155×10^{-3}	5.938×10^{-3}	5.319×10^{-3}
Formal Sector Labor	7.526×10^{-3}	7.071×10^{-3}	7.944×10^{-3}	7.464×10^{-3}
Extensive Margin	7.800×10^{-4}	8.665×10^{-4}	8.538×10^{-4}	9.475×10^{-4}
Informality Ratio	1.502×10^{-3}	1.502×10^{-3}	1.237×10^{-5}	1.237×10^{-5}
Total Employment	6.746×10^{-3}	6.205×10^{-3}	7.091×10^{-3}	6.516×10^{-3}
Inflation	1.116×10^{-2}	1.105×10^{-2}	1.306×10^{-2}	1.290×10^{-2}
Interest Rate	8.202×10^{-3}	8.338×10^{-3}	5.767×10^{-3}	5.938×10^{-3}
Gov. Revenue	3.652×10^{-3}	3.561×10^{-3}	4.042×10^{-3}	3.937×10^{-3}
Cutoff value	1.051×10^{-4}	1.051×10^{-4}	7.965×10^{-7}	7.965×10^{-7}
$\tau_p = 0.29$ to $\tau_p^{'} = 0.33$				
Formal Sector Output	5.951×10^{-3}	6.872×10^{-3}	6.156×10^{-3}	5.965×10^{-3}
Total GDP	5.749×10^{-3}	6.626×10^{-3}	5.938×10^{-3}	5.742×10^{-3}
Formal Sector Labor	7.526×10^{-3}	8.752×10^{-3}	7.944×10^{-3}	7.750×10^{-3}
Extensive Margin	7.800×10^{-4}	9.702×10^{-4}	8.538×10^{-4}	8.895×10^{-4}
Informality Ratio	1.502×10^{-3}	5.472×10^{-4}	1.237×10^{-5}	1.327×10^{-5}
Total Employment	6.746×10^{-3}	7.782×10^{-3}	7.091×10^{-3}	6.861×10^{-3}
Inflation	1.116×10^{-2}	1.584×10^{-2}	1.306×10^{-2}	1.300×10^{-2}
Interest Rate	8.202×10^{-3}	2.199×10^{-2}	5.767×10^{-3}	5.831×10^{-3}
Gov. Revenue	3.652×10^{-3}	4.272×10^{-3}	4.042×10^{-3}	4.237×10^{-3}
Cutoff value	1.051×10^{-4}	1.221×10^{-4}	7.965×10^{-7}	9.427×10^{-7}
$\delta = 0.6$ to $\delta^{'} = 0.8$				
Formal Sector Output	5.951×10^{-3}	5.998×10^{-3}	6.156×10^{-3}	6.216×10^{-3}
Total GDP	5.749×10^{-3}	5.837×10^{-3}	5.938×10^{-3}	6.041×10^{-3}
Formal Sector Labor	7.526×10^{-3}	7.256×10^{-3}	7.944×10^{-3}	7.663×10^{-3}
Extensive Margin	7.800×10^{-4}	4.209×10^{-4}	8.538×10^{-4}	4.637×10^{-4}

Informality Ratio	1.502×10^{-3}	1.502×10^{-3}	1.237×10^{-5}	1.237×10^{-5}
Total Employment	6.746×10^{-3}	6.835×10^{-3}	7.091×10^{-3}	7.199×10^{-3}
Inflation	1.116×10^{-2}	1.142×10^{-2}	1.306×10^{-2}	1.342×10^{-2}
Interest Rate	8.202×10^{-3}	7.883×10^{-3}	5.767×10^{-3}	5.366×10^{-3}
Gov. Revenue	3.652×10^{-3}	3.611×10^{-3}	4.042×10^{-3}	3.976×10^{-3}
Cutoff value	1.051×10^{-4}	1.051×10^{-4}	7.965×10^{-7}	7.965×10^{-7}
$\lambda = 0.235$ to $\lambda^{'} = 0.3$				
Formal Sector Output	5.951×10^{-3}	5.924×10^{-3}	6.156×10^{-3}	6.120×10^{-3}
Total GDP	5.749×10^{-3}	5.687×10^{-3}	5.938×10^{-3}	5.864×10^{-3}
Formal Sector Labor	7.526×10^{-3}	7.759×10^{-3}	7.944×10^{-3}	8.188×10^{-3}
Extensive Margin	7.800×10^{-4}	1.092×10^{-3}	8.538×10^{-4}	1.193×10^{-3}
Informality Ratio	1.502×10^{-3}	1.502×10^{-3}	1.237×10^{-5}	1.237×10^{-5}
Total Employment	6.746×10^{-3}	6.667×10^{-3}	7.091×10^{-3}	6.995×10^{-3}
Inflation	1.116×10^{-2}	1.093×10^{-2}	1.306×10^{-2}	1.275×10^{-2}
Interest Rate	8.202×10^{-3}	8.477×10^{-3}	5.767×10^{-3}	6.113×10^{-3}
Gov. Revenue	3.652×10^{-3}	3.693×10^{-3}	4.042×10^{-3}	4.105×10^{-3}
Cutoff value	1.051×10^{-4}	1.051×10^{-4}	7.965×10^{-7}	7.965×10^{-7}
$\hat{z} = 0.108$ to $\hat{z}^{'} = 0.2$				
Formal Sector Output	5.951×10^{-3}	5.904×10^{-3}	6.156×10^{-3}	6.093×10^{-3}
Total GDP	5.749×10^{-3}	5.427×10^{-3}	5.938×10^{-3}	5.579×10^{-3}
Formal Sector Labor	7.526×10^{-3}	7.907×10^{-3}	7.944×10^{-3}	8.342×10^{-3}
Extensive Margin	7.800×10^{-4}	1.296×10^{-3}	8.538×10^{-4}	1.413×10^{-3}
Informality Ratio	1.502×10^{-3}	1.502×10^{-3}	1.237×10^{-5}	1.237×10^{-5}
Total Employment	6.746×10^{-3}	6.612×10^{-3}	7.091×10^{-3}	6.929×10^{-3}
Inflation	1.116×10^{-2}	1.078×10^{-2}	1.306×10^{-2}	1.254×10^{-2}
Interest Rate	8.202×10^{-3}	8.665×10^{-3}	5.767×10^{-3}	6.350×10^{-3}
Gov. Revenue	3.652×10^{-3}	3.718×10^{-3}	4.042×10^{-3}	4.143×10^{-3}
Cutoff value	1.051×10^{-4}	1.051×10^{-4}	7.965×10^{-7}	7.965×10^{-7}
$\alpha_i = 0.5$ to $\alpha_i^{'} = 0.7$				
Formal Sector Output	5.951×10^{-3}	6.003×10^{-3}	6.156×10^{-3}	6.212×10^{-3}
Total GDP	5.749×10^{-3}	5.830×10^{-3}	5.938×10^{-3}	6.023×10^{-3}
Formal Sector Labor	7.526×10^{-3}	7.617×10^{-3}	7.944×10^{-3}	8.047×10^{-3}
Extensive Margin	7.800×10^{-4}	8.783×10^{-4}	8.538×10^{-4}	9.657×10^{-4}
Informality Ratio	1.502×10^{-3}	1.502×10^{-3}	1.237×10^{-5}	1.237×10^{-5}
Total Employment	6.746×10^{-3}	6.739×10^{-3}	7.091×10^{-3}	7.081×10^{-3}
Inflation	1.116×10^{-2}	1.114×10^{-2}	1.306×10^{-2}	1.303×10^{-2}
Interest Rate	8.202×10^{-3}	8.227×10^{-3}	5.767×10^{-3}	5.802×10^{-3}
Gov. Revenue	3.652×10^{-3}	3.690×10^{-3}	4.042×10^{-3}	4.088×10^{-3}
Cutoff value	1.051×10^{-4}	1.051×10^{-4}	7.965×10^{-7}	7.965×10^{-7}

Table 8 – Exercises - Volatility: Baseline vs. Extensive Margin

Variable	Baseline	Baseline $+\Delta$	No intensive margin	No intensive margin $+\Delta$
		$\varphi = 1$ to $\varphi^{'} = 0.9$		
Formal Sector Output	5.951×10^{-3}	5.058×10^{-3}	5.997×10^{-3}	5.119×10^{-3}
Total GDP	5.749×10^{-3}	4.860×10^{-3}	5.995×10^{-3}	5.117×10^{-3}
Formal Sector Labor	7.526×10^{-3}	6.780×10^{-3}	6.919×10^{-3}	6.215×10^{-3}
Extensive Margin	7.800×10^{-4}	7.636×10^{-4}	4.149×10^{-8}	4.091×10^{-8}
Informality Ratio	1.502×10^{-3}	1.502×10^{-3}	1.502×10^{-3}	1.502×10^{-3}
Total Employment	6.746×10^{-3}	6.017×10^{-3}	6.919×10^{-3}	6.215×10^{-3}
Inflation	1.116×10^{-2}	1.077×10^{-2}	1.168×10^{-2}	1.140×10^{-2}
Interest Rate	8.202×10^{-3}	8.673×10^{-3}	7.576×10^{-3}	7.908×10^{-3}
Gov. Revenue	3.652×10^{-3}	3.187×10^{-3}	3.540×10^{-3}	3.087×10^{-3}
Cutoff value	1.051×10^{-4}	1.051×10^{-4}	1.051×10^{-4}	1.051×10^{-4}
		$\omega = 0.8454$ to $\omega^{'} = 0.5$		
Formal Sector Output	5.951×10^{-3}	6.109×10^{-3}	5.997×10^{-3}	6.166×10^{-3}
Total GDP	5.749×10^{-3}	5.899×10^{-3}	5.995×10^{-3}	6.164×10^{-3}
Formal Sector Labor	7.526×10^{-3}	7.759×10^{-3}	6.919×10^{-3}	7.141×10^{-3}
Extensive Margin	7.800×10^{-4}	8.107×10^{-4}	4.149×10^{-8}	4.321×10^{-8}
Informality Ratio	1.502×10^{-3}	1.727×10^{-3}	1.502×10^{-3}	1.727×10^{-3}
Total Employment	6.746×10^{-3}	6.949×10^{-3}	6.919×10^{-3}	7.141×10^{-3}
Inflation	1.116×10^{-2}	1.217×10^{-2}	1.168×10^{-2}	1.278×10^{-2}
Interest Rate	8.202×10^{-3}	6.787×10^{-3}	7.576×10^{-3}	6.076×10^{-3}
Gov. Revenue	3.652×10^{-3}	3.841×10^{-3}	3.540×10^{-3}	3.727×10^{-3}
Cutoff value	1.051×10^{-4}	1.060×10^{-4}	1.051×10^{-4}	1.060×10^{-4}
		$\tau_y = 0.0.2925$ to $\tau_y^{'} = 0.33$		
Formal Sector Output	5.951×10^{-3}	5.367×10^{-3}	5.997×10^{-3}	5.420×10^{-3}
Total GDP	5.749×10^{-3}	5.155×10^{-3}	5.995×10^{-3}	5.418×10^{-3}
Formal Sector Labor	7.526×10^{-3}	7.071×10^{-3}	6.919×10^{-3}	6.401×10^{-3}
Extensive Margin	7.800×10^{-4}	8.665×10^{-4}	4.149×10^{-8}	4.636×10^{-8}
Informality Ratio	1.502×10^{-3}	1.502×10^{-3}	1.502×10^{-3}	1.502×10^{-3}
Total Employment	6.746×10^{-3}	6.205×10^{-3}	6.919×10^{-3}	6.401×10^{-3}
Inflation	1.116×10^{-2}	1.105×10^{-2}	1.168×10^{-2}	1.168×10^{-2}
Interest Rate	8.202×10^{-3}	8.338×10^{-3}	7.576×10^{-3}	7.576×10^{-3}

Gov. Revenue	3.652×10^{-3}	3.561×10^{-3}	3.540×10^{-3}	3.439×10^{-3}
Cutoff value	1.051×10^{-4}	1.051×10^{-4}	1.051×10^{-4}	1.051×10^{-4}
$\tau_p = 0.29$ to $\tau_p^{'} = 0.33$				
Formal Sector Output	5.951×10^{-3}	6.872×10^{-3}	5.997×10^{-3}	6.971×10^{-3}
Total GDP	5.749×10^{-3}	6.626×10^{-3}	5.995×10^{-3}	6.969×10^{-3}
Formal Sector Labor	7.526×10^{-3}	8.752×10^{-3}	6.919×10^{-3}	8.049×10^{-3}
Extensive Margin	7.800×10^{-4}	9.702×10^{-4}	4.149×10^{-8}	5.210×10^{-8}
Informality Ratio	1.502×10^{-3}	5.472×10^{-4}	1.502×10^{-3}	5.472×10^{-4}
Total Employment	6.746×10^{-3}	7.782×10^{-3}	6.919×10^{-3}	8.049×10^{-3}
Inflation	1.116×10^{-2}	1.584×10^{-2}	1.168×10^{-2}	1.681×10^{-2}
Interest Rate	8.202×10^{-3}	2.199×10^{-2}	7.576×10^{-3}	2.220×10^{-2}
Gov. Revenue	3.652×10^{-3}	4.272×10^{-3}	3.540×10^{-3}	4.118×10^{-3}
Cutoff value	1.051×10^{-4}	1.221×10^{-4}	1.051×10^{-4}	1.221×10^{-4}
$\delta = 0.6$ to $\delta^{'} = 0.8$				
Formal Sector Output	5.951×10^{-3}	5.998×10^{-3}	5.997×10^{-3}	5.997×10^{-3}
Total GDP	5.749×10^{-3}	5.837×10^{-3}	5.995×10^{-3}	5.996×10^{-3}
Formal Sector Labor	7.526×10^{-3}	7.256×10^{-3}	6.919×10^{-3}	6.919×10^{-3}
Extensive Margin	7.800×10^{-4}	4.209×10^{-4}	4.149×10^{-8}	4.264×10^{-9}
Informality Ratio	1.502×10^{-3}	1.502×10^{-3}	1.502×10^{-3}	1.502×10^{-3}
Total Employment	6.746×10^{-3}	6.835×10^{-3}	6.919×10^{-3}	6.919×10^{-3}
Inflation	1.116×10^{-2}	1.142×10^{-2}	1.168×10^{-2}	1.168×10^{-2}
Interest Rate	8.202×10^{-3}	7.883×10^{-3}	7.576×10^{-3}	7.576×10^{-3}
Gov. Revenue	3.652×10^{-3}	3.611×10^{-3}	3.540×10^{-3}	3.540×10^{-3}
Cutoff value	1.051×10^{-4}	1.051×10^{-4}	1.051×10^{-4}	1.051×10^{-4}
$\lambda = 0.235$ to $\lambda^{'} = 0.3$				
Formal Sector Output	5.951×10^{-3}	5.924×10^{-3}	5.997×10^{-3}	5.997×10^{-3}
Total GDP	5.749×10^{-3}	5.687×10^{-3}	5.995×10^{-3}	5.995×10^{-3}
Formal Sector Labor	7.526×10^{-3}	7.759×10^{-3}	6.919×10^{-3}	6.919×10^{-3}
Extensive Margin	7.800×10^{-4}	1.092×10^{-3}	4.149×10^{-8}	5.881×10^{-8}
Informality Ratio	1.502×10^{-3}	1.502×10^{-3}	1.502×10^{-3}	1.502×10^{-3}
Total Employment	6.746×10^{-3}	6.667×10^{-3}	6.919×10^{-3}	6.919×10^{-3}
Inflation	1.116×10^{-2}	1.093×10^{-2}	1.168×10^{-2}	1.168×10^{-2}
Interest Rate	8.202×10^{-3}	8.477×10^{-3}	7.576×10^{-3}	7.576×10^{-3}
Gov. Revenue	3.652×10^{-3}	3.693×10^{-3}	3.540×10^{-3}	3.540×10^{-3}
Cutoff value	1.051×10^{-4}	1.051×10^{-4}	1.051×10^{-4}	1.051×10^{-4}
$\hat{z} = 0.108$ to $\hat{z}^{'} = 0.2$				
Formal Sector Output	5.951×10^{-3}	5.904×10^{-3}	5.997×10^{-3}	5.997×10^{-3}
Total GDP	5.749×10^{-3}	5.427×10^{-3}	5.995×10^{-3}	5.993×10^{-3}
Formal Sector Labor	7.526×10^{-3}	7.907×10^{-3}	6.919×10^{-3}	6.919×10^{-3}

Extensive Margin	7.800×10^{-4}	1.296×10^{-3}	4.149×10^{-8}	7.036×10^{-8}
Informality Ratio	1.502×10^{-3}	1.502×10^{-3}	1.502×10^{-3}	1.502×10^{-3}
Total Employment	6.746×10^{-3}	6.612×10^{-3}	6.919×10^{-3}	6.919×10^{-3}
Inflation	1.116×10^{-2}	1.078×10^{-2}	1.168×10^{-2}	1.168×10^{-2}
Interest Rate	8.202×10^{-3}	8.665×10^{-3}	7.576×10^{-3}	7.576×10^{-3}
Gov. Revenue	3.652×10^{-3}	3.718×10^{-3}	3.540×10^{-3}	3.540×10^{-3}
Cutoff value	1.051×10^{-4}	1.051×10^{-4}	1.051×10^{-4}	1.051×10^{-4}
$\alpha_i = 0.5$ to $\alpha_i^{'} = 0.7$				
Formal Sector Output	5.951×10^{-3}	6.003×10^{-3}	5.997×10^{-3}	5.997×10^{-3}
Total GDP	5.749×10^{-3}	5.830×10^{-3}	5.995×10^{-3}	5.997×10^{-3}
Formal Sector Labor	7.526×10^{-3}	7.617×10^{-3}	6.919×10^{-3}	6.919×10^{-3}
Extensive Margin	7.800×10^{-4}	8.783×10^{-4}	4.149×10^{-8}	6.071×10^{-9}
Informality Ratio	1.502×10^{-3}	1.502×10^{-3}	1.502×10^{-3}	1.502×10^{-3}
Total Employment	6.746×10^{-3}	6.739×10^{-3}	6.919×10^{-3}	6.919×10^{-3}
Inflation	1.116×10^{-2}	1.114×10^{-2}	1.168×10^{-2}	1.168×10^{-2}
Interest Rate	8.202×10^{-3}	8.227×10^{-3}	7.576×10^{-3}	7.576×10^{-3}
Gov. Revenue	3.652×10^{-3}	3.690×10^{-3}	3.540×10^{-3}	3.540×10^{-3}
Cutoff value	1.051×10^{-4}	1.051×10^{-4}	1.051×10^{-4}	1.051×10^{-4}