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Aspiration-based Reference Dependence

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Dissertação apresentada ao Curso de Mestrado Acadêmico em Economia, Universidade de Brasília, como requisito parcial para a obtenção do título de Mestre em Economia

Universidade de Brasília - UnB

Faculdade de Administração Contabilidade e Economia - FACE

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Programa de Pós-Graduação

Orientador: Prof. Dr. Daniel Oliveira Cajueiro

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Abstract

This paper presents the axiomatization of a choice procedure in the presence of observable but not feasible alternatives, phantom or aspirations, and assimetrically dominated feasible options, decoys or references. The main characteristic of the representation theorem here stated is that aspirations induce references, which attract agent's attention for a specific feasible subset, where she acts like a standard utility maximizer. It turns out that some external factor invisible to the model maker may influence the choice, which means the decision maker may pick an aspiration which attracts dominated alternatives. Moreover, we highlight the existence of a psychological and endogenous metric which evaluates similarity between alternatives and provide further explanation on the eventual choice of strictly dominated alternatives.

Keywords: Choice Correspondences, Decoy, Phantom, Endogenous Metric, Strictly Dominated Alternatives.

Resumo

Esse artigo axiomatiza o procedimento de escolha que contempla alternativas observáveis mas que não são factíveis, chamadas de aspirações ou alternativas fantasma, e opções factíveis assimetricamente dominadas, que são referências ou iscas. A principal característica do teorema de representação obtido é que as aspirações induzem referências, as quais atraem a atenção do agente para um conjunto específico de alternativas factíveis, no qual o agente maximiza sua função de utilidade. Assim sendo, a escolha do agente depende de fatores externos que são invisíveis ao modelador, o que significa que o indivíduo pode escolher uma aspiração que atrai alternativas dominadas. Mais ainda, demonstra-se a existência de uma métrica endógena e individual que avalia a similaridade entre as alternativas e analisa-se o procedimento de escolha no qual a escolha é uma alternativa estritamente dominada à luz do modelo.

Palavras-chave: Correspondências de Escolha, Alternativa Isca, Alternativa Fantasma, Métrica endógena, Alternativa Estritamente Dominada.

Sumário

1 Introduction

Its well known that the regularity hypothesis can fail on choice models . The regularity hypothesis says that the introduction of a new alternative into the feasible set cannot shift preferences towards some alternative previously feasible. However, there are violations violations of this behavior. Generally, the inclusion of some specific alternatives on a choice problem draw attention of the agent to other ones, may it be feasible or not. Decisions often tend not to follow standard predictions of traditional rational models based on the regularity hypothesis. However, often we can model these violations as choice problems where rationality plays a different role when compared with traditional utility maximization.

Under the stochastic theory of choice, for instance, Manzini and Mariotti (2018) shows that a model which originally satisfies the regularity hypothesis gets some nice descriptive power once that property is dropped, while still maintaining some consistency properties. Concerning consumer behavior, their model accommodates the inclusion of choice reversals induced by the introduction of some alternatives and is able to explain some social phenomena in which a specific alternative becomes more attractive not by self modifications, but due to changes on the surrounding environment.

Huber, Payne and Puto (1982) presents one of the first violations of the regularity hypothesis. They show that the regularity hypothesis does not hold when considering the attraction effect. However, Huber, Payne and Puto (1982) developed a model in which an alternative originally not chosen in the presence of a second one can be chosen in a similar setup when we add an asymmetrically dominated option to the situation.

This kind of alternative is said to be a dominated decoy, particularly an asymmetrically dominated decoy. Generally speaking, a decoy r is an alternative which is dominated by some feasible y, in the sense that all the attributes of y are greater than or equal the attributes of r, with at least one strictly greater, but is not dominated by another option. There exists in the literature another kind of decoy, said non-dominated decoy, which does not require a dominating alternative. Both of them are examples of choice reversals, which violates the regularity hypothesis.

The asymmetric dominance or attraction effect is an example of endogenous reference dependence. Ok, Ortoleva and Riella (2015) axiomatize a setup with dominated decoys, in which some alternatives are able to exert the attraction effect, called *reference points*, once they are feasible. Pettibone and Wedell (2000) analyze the effects of non-dominated decoys over the choice of agents, in particular unavailable decoys which dominate all the other alternatives. It turns out that it draws attention to the most similar feasible one. This specific attraction effect is called *aspirational* and axiomatically modeled by Guney, Richter and Tsur (2018).

Another common reference dependence on literature is the status quo bias. Masatlioglu and Ok (2005), Masatlioglu and Ok (2014), Riella and Teper (2014) model these well documented phenomena. In this setup, the current state of an agent becomes a standard for ranking other alternatives. Riella and Teper (2014), for instance, consider an alternative to be a candidate to replace status quo only if the chances of the agent incurring a severe loss when compared to the status quo is not too high.

At the present work, we analyze both kinds of decoys previously described, that is those which are aspirations and those which are reference points. It turns out that aspirations, options she wants but cannot pursue, induce reference points. The wide implications of the inclusion of aspiration and reference points enrich firm models (Hart and Moore (2007), Hart (2008), Hart (2009)), consumer choice under price discrimination (Carbajal and Ely (2016)), financial modeling (Mihm (2016)), game theory (Sawa and Zusai (2014)) and welfare analysis (Genicot and Ray (2017)), for example.

Previous works which do experiments on how references and attraction effect change decision maker preferences are many. Wedell (1991) analyzes the role of assymetrically dominated alternatives when individuals gamble, Sueocurry and Pitts (1995) does the same for voting setups, Trueblood (2012) considers investigative scenarios and Highhouse (1996) for job market.

All these models fit better specific situations and allow the rationality hypothesis to explain apparent violations. Therefore, in some cases where traditional models do not fit it, does not mean agents behave irrationally They just suggest that agents rationality acts in a different manner, such that the mechanism of choice of agents could be more sophisticated.

This paper aims to give a further contribution to the aspiration based model of Guney, Richter and Tsur (2018). We develop a choice model in which observable but not feasible alternatives, aspirations, have influence over the choice of the agents. Under some reasonable conditions, it turns out that first, the agent identify, on the observables subset, their aspirations, which maximize their utility function. Any feasible aspirations are chosen. If that is not the case, the agent proceed to identify, on the feasible subset, the most similar alternatives to each one of the aspirations, the references, according to an endogenous metric. Those references draw the attention of the agent for a subset of the alternatives. Finally the agent acts like a standard utility maximizer when looking to the feasible attracted options.

The novel aspects of our model when comparing to Guney, Richter and Tsur (2018) is the possibility of the existence of more than one aspiration and, therefore, more than one reference. Furthermore, unlike them, here the references are not necessarily chosen. They highlight some other alternatives, where the agent maximizes her utility. The existence of the attraction region, precludes he existence of a choice problem where an strictly dominated alternative is chosen, what we show as some inconsistency of Guney, Richter and Tsur (2018) model.

Haselhuhn (2014) addresses the importance of unpacking a single aspiration into many others. The study shows that market participants obtain higher chances of success in getting closing to their aspirations and, then, achieving better payoffs when they consider not only a single best path, but many. Concerning the possibility of agents having multiple reference points, Koop and Johnson (2010) point out that they can impact behavior even within the same choice context.

This setup allows us to capture different aspects the agent may value when maximizing her utility, that is, *status quo*, alternatives that are high ranked in some aspects by the individual but perform poorly on other and so on. Next we give some more examples to motivate how aspirations and references can be used to explain some situations where rationality seems not to hold.

Suppose a family faces a downgrade on its income, which makes the consumption of the previously optimal bundle unfeasible. They will initially, over their new income constraint, look for a bundle that is similar to the original. It turns out that, after identifying this reference bundle, the family may choose a bundle that is simply much better than the reference, despite being significantly different than the other one.

Consider now someone willing to run for president of a country, but prohibited by some legal requirement. The closest feasible alternative, then, becomes running for the governor's office of one of the country's provinces. However, could be better for her to run for a chair on the lower house. In fact, when raking these two options she considers a chair on the lower house a dominant option, so che chooses it, despite the governor position having the power of being a reference point.

On both examples above, note that the two stage procedure is crucial. When comparing the alternatives with the reference, agents consider different aspects of their preference when, for instance, they compare the same alternatives with their aspiration. On the second example, if the governor is subordinate to the president and the members of the lower house are, in any sense, subordinate to both, when comparing running for governor or the lower house, once both are subordinate to the president, this aspect could no longer be crucial when ranking the last two and it turns out that the lower house chair offers better payoffs.

Same holds for the bundle example. When comparing the two cheaper bundles, one key aspect of the decision is: which bundle makes the family closer to get the original bundle in the future. On such situations, a lot of psychological factors come into play, and they do not mean the rationality is gone.

Another situation that clarifies how important is to consider that not always the most similar option to the aspiration is actually chosen is to consider the scientific modeling itself. A good question when researching natural or human phenomena tries to answer why things we observe actually happen, that is, scientific research tries to uncover laws behind the reality.

Of course, when it comes to modeling the reality, the true process behind it is not feasible but observable. One can make several assumptions that becomes her abstraction of reality closest to the facts. However, more assumptions we make in order to minimize the distance from our model to reality makes the problem less tractable and the real purpose to explain the facts fail. It turns out that an ultra realistic model could lead the researcher to a simpler model, with some strong but reasonable and intuitive axioms, that, although more distant from the reality, explains it better, is more accessible and could ensure a Nobel prize in the future.

Such situations motivates the main goal of this paper, which ensure more flexibility to the choice of the decision maker, enabling her not to necessarily choose the most similar alternative to the aspiration, like Guney, Richter and Tsur (2018). While doing so, we can also derive the endogenous metric that they obtain, which evaluates a psychological distance between alternatives. In addition, we explore further properties of the model here developed.

The metric obtained gives the foundation of the metric with economic meaning described by the fancy work of Loi and Matta (2008), for example, which provides that from any metric, there exists a Riemannian metric on the equilibrium manifold (originally described by Balasko (1975), Balasko (1979)). This study enriches and gives further development to market equilibrium analysis and a fine geometrical comprehension of it with some novel approach.

The equilibrium manifold is said to be simply the usual set of pairs of prices and endowments in which aggregate demand equals aggregate supply. It turns out that this set behave locally like Euclidean space and allows to formally describe the transition of some equilibrium point to another better one through a smooth transition, which is empirically desired from the viewpoint of agents.

However, they simply consider an *a priori* metric with economic meaning and do not investigate why there is such a metric and which are its properties, as stated at Loi and Matta (2008, p.1380). The aspiration-based model with reference-dependence, then, grants the existence of a metric endogenously obtained from choice problem with which agents measure psycological distances between alternatives. From that metric with economic meaning, for example, Loi and Matta (2008) shows that we can obtain a Riemannian metric on the equilibrium manifold and talk about geodesics on such a setup.

This work is structured as follows. On the next chapter we state the axioms and the setup of the model. Chapter 3 provides the representation theorems and Chapter 4 presents further properties of our model. There we present further explanations on the choice of strictly dominated alternatives, discuss that the attraction effect becomes stronger as the distance of the aspiration and the reference becomes greater which was previously suggested by Pettibone and Wedell (2000), Pettibone and Wedell (2007) and Sperlich and Uriarte (2019) and talk about the transitiveness of the attaction effect. Finally, Chapter 5 contains some concluding remarks.

2 The Aspiration-based References Model: Setup and Definitions

We will follow the setup in Guney, Richter and Tsur (2018) with a finite space of alternatives X. The set of all non-empty subsets of X is denoted by $\mathcal{P}(X)$. A choice problem is defined as a pair (S, T), $S, T \in \mathcal{P}(X)$, $S \subseteq T$. The interpretation is that when the agent faces this choice problem, she observes all the alternatives in T but only the alternatives in S are available for choice. The set of all choice problems is denoted by $\mathcal{C}(X)$. A choice correspondence is an application $c : \mathcal{C}(X) \to \mathcal{P}(X)$ such that for each $(S, T) \in \mathcal{C}(X)$, $c(S, T) \subseteq S$.

This framework allows us to consider choice procedures that are affected by unavailable alternatives. In what follows, we will make use of a collection of binary relations we identify from the agent's choices.

Definition 1. *Let $T \in \mathcal{P}(X)$ and $x, y \in T$. We define a binary relation $\succ_T \subseteq X \times X$ by $x \succ_T y$ if, and only if, there exists $S \subseteq T$ with $y \in S$ such that one of the following holds: i) $x \notin c(S \cup \{x\}, T) \neq c(S, T)$ or ii) $x \in c(S \cup \{x\}, T) \neq c(\{x\} \cup c(S, T), \{x\} \cup c(S, T))$.*

This binary relation essentially captures the existence of at least one situation where one can argue that x acts as a reference point in the presence of y. In the (i) case, the choice from (S, T) is different from the choice from $(S \cup \{x\}, T)$ and x is not chosen in $(S \cup \{x\}, T)$. The presence of x is somehow affecting the relative ranking of the other options. That is, x is acting as a reference point. Something similar is happening in (ii) as well. There, although x is a choice from $(S \cup \{x\}, T)$, the choice from $(S \cup \{x\}, T)$ differs from the choice from $(\{x\} \cup c(S, T), \{x\} \cup c(S, T))$. If we understand the choice from $(\{x\} \cup c(S, T), \{x\} \cup c(S, T))$ as being free from aspiration-based effects and, consequently, reference free, we again identify x as a reference point in $(S \cup \{x\}, T)$.

Now we proceed to the axioms that ensure some structure to the decision maker's behavior.

2.1 Axioms

The first two postulates were previously introduced by Guney, Richter and Tsur (2018).

Axiom 1. ***(WARP for Aspirations – A-WARP)*** *For any $S, T \in \mathcal{P}(X), S \subseteq T$, $c(T, T) \cap S \neq \emptyset$ implies that $c(S, S) = c(T, T) \cap S$.*

Axiom 2. ***(Independence of Non-aspirational Alternatives – INA)*** *For any* $S, T_1, T_2 \in \mathcal{P}(X), S \subseteq T_1 \cap T_2$, $c(T_1, T_1) = c(T_2, T_2)$ *implies* $c(S, T_1) = c(S, T_2)$.

Call a choice problem of the form (T, T) an unrestricted choice problem, that is, all observable options are available for choice. Furthermore, aspirations are the choices from the observable subset. That said, Axiom 1 simply says that the standard Weak Axiom of Revealed Preference is satisfied when we look at the restriction of c to unrestricted choice problems. On the other hand, Axiom 2 imposes that only alternatives which are chosen when all observable alternatives are available may influence the decion maker's choices in restricted choice problems.

The next postulate imposes that reference effects in problems with a single aspiration be acyclic.

Axiom 3. ***(Reference Acyclicity)*** *For any* $T \in \mathcal{P}(X)$ *such that* $|c(T, T)| = 1$, $\succ_T$ *is acyclic.*

The three axioms above discipline behavior in choice problems with a single aspiration point. The two postulates below present two alternative ways to deal with problems with multiple aspirations.

Axiom 4. ***(A-Separability)*** *For any* $Y, Y', S \in \mathcal{P}(X)$ *such that* $S \subset Y, Y'$, *if* $c(Y, Y) \cap c(Y \cup Y', Y \cup Y') \neq \emptyset$ *and* $c(Y', Y') \cap c(Y \cup Y', Y \cup Y') \neq \emptyset$ *then* $c(S, Y \cup Y') = c(S, Y) \cup c(S, Y')$.

Axiom 5. ***(A-Optimality)*** *For any* $Y, Y', S \in \mathcal{P}(X)$ *such that* $S \subset Y, Y'$, *if* $c(Y, Y) \cap c(Y \cup Y', Y \cup Y') \neq \emptyset$ *and* $c(Y', Y') \cap c(Y \cup Y', Y \cup Y') \neq \emptyset$ *then* $c(S, Y \cup Y') = c(c(S, Y) \cup c(S, Y'), c(S, Y) \cup c(S, Y'))$.

First, the **A** on the above axioms stands for aspirations. A-Separability means that the aspirations the agent identify on a choice problem influence the choices she make independently. For a choice problem with more than one aspiration, we have that, on a given day, some external factor, not seen by the model maker, leans her to focus at an specific aspiration, ignoring all other ones. We note that, although Axiom 4 is written for two observable sets. A simple inductive argument guarantees it generalizes for any finite number of observable sets.

In contrast, A-Optimality imposes that the decision maker takes into account the choices induced by each aspiration in isolation, but chooses only the optimal ones among them. We are now ready to state our two main results.

A-Separability stands for the mood-driven aspirations, similar to mood-driven choices, described by Mihm and Ozbek (2018). As they said, there are many mood-driven factors – such as addiction, temptation, or inattention – that can cause the agent to make inferior choices leading to a conflict with long-term objectives. The same way they analyze the role of self-regulation (when agents predict the mood could mess up the choice) for choices itself, the agents could regulate themselves when it comes to choose aspiration as well.

3 Representation Theorems

In what follows, we state our two main representation theorems and discuss their main properties.

Theorem 1. *(A – Separability) A choice correspondence satisfies Axioms 1, 2, 3 and 4 if and only if there exists a function $u \in \mathbb{R}^X$, an injective similarity function $v_y \in \mathbb{R}^X$ for each $y \in X$ and a correspondence $Q : X \times X \rightrightarrows X$ such that*

$$c(S,T) = \bigcup_{a \in a(T)} \arg\max u\,(S \cap Q(r(S,a),a)), \text{ for any } (S,T) \in \mathcal{C}(X),$$

where $a(T) = \arg\max_{x \in T} u(x)$ and, for each $a \in a(T)$, $r(S,a) = \arg\max_{x \in S} v_a(x)$.

Theorem 2. *(A – Optimality) A choice correspondence satisfies Axioms 1, 2, 3 and 5 if and only if there exists a function $u \in \mathbb{R}^X$, a injective similarity function $v_y \in \mathbb{R}^X$ for each $y \in X$ and a correspondence $Q : X \times X \rightrightarrows X$ such that*

$$c(S,T) = \arg\max u \left(S \cap \bigcup_{a \in a(T)} Q(r(S,a),a) \right), \text{ for any } (S,T) \in \mathcal{C}(X),$$

where $a(T) = \arg\max_{x \in T} u(x)$ and, for each $a \in a(T)$, $r(S,a) = \arg\max_{x \in S} v_a(x)$.

In the representation above, the DM has a utility function u. Given a choice problem (S,T), she first maximizes u over all the alternatives she can perceive in order to identify her aspiration points. For each aspiration point $a \in T$ the DM associates a unique feasible alternative which is the most similar one to that aspiration. That option is $r(S,a)$ and called a reference, which, together with the aspiration, creates an additional mental constraint represented by $Q(r(S,a),a)$. Then, the DM limits herself to choose only from the feasible alernatives on that region, what is done by maximizing the utility function u on $S \cap Q(r(S,a),a)$.

When the choice problem has more than one aspiration, it turns out that the representation above is agnostic about which aspiration will be active and simply consider choosable the choices induced by each aspiration.

The main difference between Theorem 1 and Theorem 2 stands on the final stage of the choice procedure. If A-Optimality holds, the decision maker will identify all aspirations but the ones which attract the highest utility options are the ones that really matter for choice. However, in case A-Separabiliy holds, her choice may depend on some external factor, which leads her to focus on a single aspiration in a given day.

Under this setup we can provide further characterization on how agent identifies the most similar alternative to another on our model. It turns out that the similarity between alternatives can be evaluated by a psychological metric, the same way previously described by Guney, Richter and Tsur (2018).

Corollary 1. *Let* $(S,T) \in \mathcal{C}(X)$ *such that* $a(T) = \arg\max_{x \in T} u(x)$*. Then, there exists a metric* $d \in \mathbb{R}^{X^2}$ *such that for each* $a \in a(T)$ *there is a single* $r_a = \arg\min_{x \in S} d(x,a)$*.*

We, then, keep the nice structure regarding the metric with economic meaning previously obtained at Guney, Richter and Tsur (2018). On the next chapter, we explore the properties of the choice procedure with these representations.

4 Properties

4.1 On the choice of strictly dominated alternatives

On this section we provide further explanation on the case agents may choose a strictly dominated alternative by another feasible one. To do so, we start with the problem in which a traditional agent whose preference is continuous, strictly convex and strictly monotonic. We know that these hypothesis ensures the existence of a strictly increasing utility function that represents agent's preference. Furthermore, let the set of all alternatives be any finite subset of $\mathbb{R}^n$ and the endogenous metric d to be the Euclidean metric. Under the Guney, Richter and Tsur (2018) model we can state the following proposition.

Proposition 1. *For any aspiration, there always exists a choice problem where the individual chooses a strictly dominated option.*

When considering the agent chooses solely based on the minimization of the distance between aspiration and reference, there always exists a choice problem with the reference and an alternative which strictly dominates it, in which the last one is not chosen. There is no reasonable explanation for such a situation. It turns out that under aspiration-based model with reference dependence can't be shown that such a situation always exist but moreover, whenever it happens, it is explained because the dominant option is not attracted by the reference. Once the agent is a standard utility maximizer on the attracted region, if it is attracted by the reference, it will necessarily be chosen.

To better illustrate how Proposition 1 holds for the solo-aspiration based model,

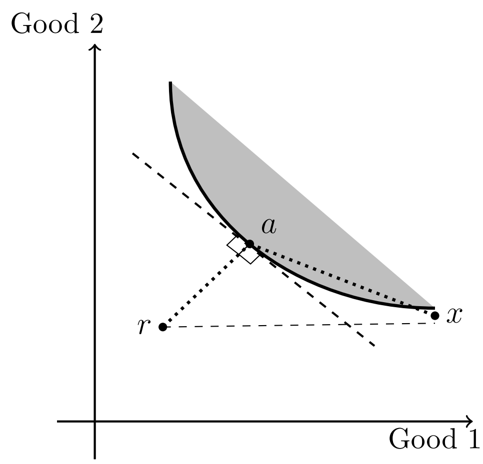

Figura 1 – The existence of a strictly dominant alternative which is not chosen.

consider the set of alternatives be a subset of $\mathbb{R}^2$ (see Figure 1). When a is the aspiration and the model imposes the agent chooses the closest feasible option to it, say r, we can always obtain an alternative x dominated by the aspiration whose coordinates are strictly higher than those of r but x is not chosen because it is far from a.

Why exactly an strictly dominant alternative is not attracted by some reference? Such a question is out of this paper scope. We focused on the existence of the attraction effect, which represents an additional stage on choice procedure.

5 Conclusion

In this paper, we have proposed an axiomatization which leads to two representation theorems for choice procedure with observable but not feasible options, which we call aspiration-based model with reference dependence. Our main results point out that an individual identifies aspirations, considering the whole set of observables, and then each aspiration induce a reference which drives attention to some other feasible options. Furthermore, the representations here stated present some novel aspects when compared to Guney, Richter and Tsur (2018), once they allow agents to identify more than one aspiration. The way the agent looks for aspirations leads to different representation theorems.

Our main contribution is the endogenous attracted region, induced by the references, agents identify, from the metric previously obtained by Guney, Richter and Tsur (2018), and how the decision maker behaves on them. It turns out that each aspiration influence the choice independently, however the DM could be of two types: *i*) the one who restrict herself to optimal attracted options, from the attracted options from each aspiration; or *ii*) the one who pick any of the aspirations due to some external factor invisible to the model and consider only the attracted options by it.

The developed model is specially useful for applications. Once attraction effect is transitive for references and as such follows that the procedure to identify which alternatives are attracted by another one can be described by the evaluation of multi criteria, it leads to many other experiments. Future works can also use the model to solve Pettibone and Wedell (2007) puzzle on how aspiration distance from feasible subset affects the attraction region, while the reference remains the same. It may lead to a better understanding of framing strategy.

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A Proofs

A.1 Proof of Theorem 1

In order to prove Theorem 1, we first state the following lemma, which is useful to prove Theorem 2 as well.

Lemma 1. *Let* $(S, T) \in \mathcal{C}(X)$*. If a choice correspondence c satisfies Axioms 1, 2 and 3 then exists a function* $u \in \mathbb{R}^X$*, a similarity function* $v_y \in \mathbb{R}^X$ *for each* $y \in X$ *and a correspondence* $Q : X \times X \rightrightarrows X$ *such that if* $a(T) = \arg\max_{x \in T} u(x)$*, there is an element* $r(S, a) = \arg\max_{x \in S} v_a(x)$ *for each* $a \in a(T)$*. Moreover,*

$$c(S, S \cup \{a\}) = \arg\max u(S \cap Q(r(S, a))), a \in a(T)$$

Demonstração. Suppose a choice correspondence satisfies Axioms 1, 2 and 3. We first define a binary relation such that $x \succeq y \Leftrightarrow x \in c(\{x, y\}, \{x, y\})$. Follows from WARP that $\succeq$ is a complete preorder defined on a finite set, then there is a $\succeq$-maximal element and $\succeq$ can be represented by an utility function $u \in \mathbb{R}^X$. Next, for $y \in X$, define $X_y = \{y\} \cup \{x \in X \mid u(y) > u(x)\}$ such that the transitive closure of $\succ_{X_y}$, defined on $\Delta_X \cup X^2$, by Axiom 3, have a linear order extension. Moreover, define $L_y := \{(y, x) \mid x \in X_y\}$ such that, the union of L_y with the transitive closure of $\succ_{X_y}$ is still a linear order and, then, can be represented by an injective function $v_y \in \mathbb{R}^X$.

For each pair x, y such that $u(y) > u(x)$, we define

$$Q(x, y) := \{z \in X_y : x \succ_T z \text{ and } z \in c(\{x, z\}, \{x, y, z\}) \text{ or } x \not\succ_T z \text{ and } z \in c(\{x, z\}, \{x, z\})\}$$

Fix T such that $c(T, T) = \{a\}$ and let $r = \arg\max_{x \in S} v_a(x)$. Now we show that $c(S, T) = \arg\max_{x \in S \cap Q(r,a)} u(x)$. To do so, fix $x \in c(S, T)$ and $y \in \arg\max_{x \in S \cap Q(r,a)} u(x)$.

Let $S \setminus \{x, r\} = \{z_1, ..., z_m\}$. Once r is unique, by acyclicity follows that $c(S \setminus \{z_1\}, T) = c(S, T) \setminus \{z_1\}$. In fact, it follows from the both two cases on definition of salience relation. Repeating this reasoning, we get that $c(\{x, r\}, T) = c(S \setminus \{z_1, ..., z_m\}, T) = c(S, T) \setminus \{z_1, ..., z_m\}$ and, then, $x \in c(\{x, r\}, T)$. If $r \succ_T x$, follows that $x \in Q(r, a)$. On the other hand, if $r \not\succ_T x$, we must have $x \in c(\{x, r\}, \{x, r\})$, otherwise, $x \succ_T r$, which contradicts the definition of r. Similarly, acyclicity implies $x \in c(\{x, y, r\}, T) = c(\{x\} \cup c(\{y, r\}, T), \{x\} \cup c(\{y, r\}, T))$. Since $y \in c(\{y, r\}, T)$, we have that $u(x) \geq u(y)$, due to Axiom 1.

Conversely, note that or $y \in c(\{x\} \cup c(\{y, r\}, T), \{x\} \cup c(\{y, r\}, T))$, when the same reasoning we used before gives us that $c(\{x, y, r\}, T) = c(S, T) \setminus (S \setminus \{x, y, r\})$ and then

$y \in c(S,T)$, or $r \not\succ_T y$ and $y \in c(\{r,y\},\{r,y\})$, case in which we verify $y \in c(\{y,r\},T)$ and then $y \in c(S,T)$, otherwise, $r \succ_T y$, what is a contradiction. □

Now we proceed to show the Theorem 1. It is routine to show only that the axioms imply the representation, so we assume a choice correspondence c that satisfies Axioms 1, 2, 3 and 4. From Lemma 1, we get $u \in \mathbb{R}^X$, a similarity function $v_y \in \mathbb{R}^X$ for each $y \in X$ and a correspondence $Q : X \times X \rightrightarrows X$. Next, we omit the index $a \in a(T)$ of the union to keep it clean.

We show then that $c(S,T) = \bigcup \arg\max u\,(S \cap Q(r(S,a),a))$. Fix $x \in c(S,T)$ and $y \in \bigcup \arg\max u(S \cap Q(r(S,a),a))$.Note Axiom 4 can be easily extended to a finite union with the Axiom 2, that is $c\,(S,T) = \bigcup\limits_{a\in c(T,T)} c(S, S \cup \{a\})$. Then $x \in \bigcup c(S, S \cup \{a\})$, which implies $x \in c(S, S \cup \{a\})$ for some a and then, the result follows.

Conversely, note $y \in \bigcup c(S, S, \cup\{a\})$ and then $y \in c(S,T)$.

A.2 Proof of Theorem 2

Again, we show only that axioms imply the representation. Then, we just need to show that $c(S,T) = \arg\max u\,(S \cap (\bigcup Q(r(S,a),a)))$. Fix $x \in c(S,T)$ and $y \in \arg\max u(S \cap (\bigcup Q(r(S,a),a)))$. By a simple inductive argument, using Axiom 1 and 2, we can extend Axiom 5 for a finite number of subsets. Follows then that $c(S,T) = c\left(\bigcup\limits_{a\in c(T,T)} c(S, S \cup \{a\}), \bigcup\limits_{a\in c(T,T)} c(S, S \cup \{a\})\right)$. From this fact follows that $x \in c\,(\bigcup c(S, S \cup \{a\}), \bigcup c(S, S \cup \{a\}))$ and then $x \in \arg\max u(\bigcup c(S, S \cup \{a\}))$, which implies $u(x) \geq u(y)$.

Conversely, note $y \in c(S, S \cup \{a\})$ for some a, which implies $y \in \bigcup c(S, S, \cup\{a\})$. We had already proven that $u(x) \geq u(y)$. Then $u(x) = u(y)$ and, from the extension of Axiom 4, then, $y \in c(S,T)$.

A.3 Proof of Corollary 1

Now, define $\hat{d} : X \times X \to \mathbb{R}$ as follows:

$$\hat{d}(x,y) = \begin{cases} v_x(x) - v_x(y), \text{if } u(x) \geq u(y); \\ v_y(y) - v_y(x), \text{if } u(x) < u(y) \end{cases}$$

Note that $\hat{d}$ is non-negative, symmetric and reflexive and, then, a semimetric. As a semimetric we know $\hat{d}$ has an extension d that satisfies the triangle inequality (see Guney, Richter and Tsur (2018)).

Now we just need to show that $r(S,a) = \arg\min_{x\in S} d(x,a)$. Once d is an extension of $\hat{d}$, we have that $\arg\min_{x\in S} d(x,a) = \arg\min_{x\in S} \hat{d}(x,a)$. Note that $a = \arg\max_{x\in T} u(x)$, then, $\hat{d}(x,a) = v_a(a) - v_a(x)$. Recall $r(S,a) = \arg\max_{x\in S} v_a(x)$, which implies that $r(S,a) = \arg\min_{x\in S} d(x,a)$.

A.4 Proof of Proposition 1

Fix any $a \in \mathbb{R}^n_{++}$ and define the upper contour set as $U_{\succeq}(a) := \{x \in \mathbb{R}^n : x \succeq a\}$. By assumption $U_{\succeq}(a)$ is closed and convex. Since a is a boundary point of $U_{\succeq}(a)$ there is a supporting hyperplane H of $U_{\succeq}(a)$ that touches a with orthogonal direction h. Let $r = a - \alpha h$ for some α and λ be such that $r + \lambda e^i \in H$ for every canonical vector e^i. Because $\succeq$ is strictly convex, there is i^* such that $a \succ r + \lambda e^{i^*}$. By continuity, $a \succ r + (\lambda+\delta)e^{i^*}$, for δ small. Note that $d(a, r + (\lambda+\delta)e^{i^*}) > d(a,r)$. The continuity of both $\succeq$ and d implies that for ε small enough, $a \succ r + (\lambda+\delta)e^{i^*} + \varepsilon(1,\cdots,1)$ and $d(a, r + (\lambda+\delta)e^{i^*} + \varepsilon(1,\cdots,1)) > d(a,r)$. Define $x := r + (\lambda+\delta)e^{i^*} + \varepsilon(1,\cdots,1)$. Then, the problem $(\{x,r\},\{x,r,a\})$ does the job.