ISSN 0101-4161

Decisions on Investment Allocation in the Post-Keynesian Growth Model ♦

Ricardo Azevedo Araujo
Professor - Departamento de Economia, Universidade de Brasília (UnB)
Campus Darcy Ribeiro - Asa Norte - CEP: 70910-900, Brasíla - DF
E-mail: rsaaraujo@unb.br

Joanílio Rodolpho Teixeira
Professor - Departamento de Economia, Faculdade de Estudos Sociais Aplicados
Universidade de Brasília (UnB) - Campus Darcy Ribeiro - Asa Norte - CEP: 70910-900, Brasíla - DF
E-mail: joanilioteixeira@hotmail.com

Recebido em 05 de setembro de 2011. Aceito em 09 de abril de 2012.

Abstract
In this article the growth models of Feldman (1928) and Mahalanobis (1953) are extended to consider the analysis of decisions of investment allocation in the context of the Post-Keynesian Growth Model. By adopting this approach it is possible to introduce distributive features in the Feldman-Mahalanobis model that allows us to determine the rate of investment allocation according to the equilibrium decisions of investment and savings. Finally, an additional condition is added to the Post-Keynesian Growth Model in order to fully characterise the equilibrium path in an extended version of this framework, where capital goods are also needed to produce capital goods.

Keywords
Post-Keynesian growth model, structural change, multi-sector models.

JEL Classification
E21, O11

Resumo
Neste artigo os modelos de crescimento e alocação de investimento a la Feldman--Mahalanobis são estendidos para considerar a análise de decisões de alocação de investimento no contexto do modelo de crescimento Pós-Keynesiano. Ao adotar essa abordagem é possível introduzir características distributivas no modelo de Feldman-Mahalanobis que nos permitem determinar a taxa de alocação de investimentos de acordo com as decisões de equilíbrio entre investimento e poupança. Finalmente, uma condição adicional é acrescida ao modelo de crescimento Pós-Keynesiano, a fim de

♦ A preliminary version of this paper was presented in the 39th Brazilian Meeting of Economics in Foz do Iguaçu, Brazil, December 6 to 9, 2011. Financial support from CNPq is acknowledged.

caracterizar plenamente o caminho de equilíbrio em uma versão estendida deste, em que bens de capital também são necessários para produzir bens de capital.

Palavras-Chave
Modelo Pós-Keynesiano de crescimento, mudança estrutural, modelos multisetoriais

1. Introduction

The Post-Keynesian growth model – PKGM hereafter – designates the growth models that were initially developed by Kaldor (1956) and Robinson (1956, 1962) and extended by Dutt (1984), Rowthorn (1982) as well as by Bhaduri and Marglin (1990). Some characteristics of these models are worth to remember: (i) the functional distribution of income plays an important role in the determination of macroeconomic variables and growth rates. Besides (ii) there is an inversion of causality direction between savings and investment, as assumed by the Neoclassical economics: investment is shown to determine savings and not the reverse.

The PKGM[1] passes through three principal phases that are labelled as 'generations'. Although Kaldor (1956) has built his seminal model on the notion of full capacity utilization, Dutt (1984) and Rowthorn (1982), working independently, have built what is known as the second generation of the PKGM by endogenizing the rate of capacity utilization in the lines of Steindl (1952). One of the main contributions of this generation is the possibility of disequilibrium and the presence of a stagnationist[2] regime in which an increase in the profit share implies a reduction in capacity utilization. The key assumption behind this result is that the growth rate of investment is a function not only of the profit rate, as in Kaldor-Robinson but also of the rate of capacity utilization.

Bhaduri and Marglin (1990) have challenged this view by considering that the growth rate of investment is a straight function not of the profit rate but of the profit share. According to them the profit rate has already been implicitly considered in the equation of the growth

1 See Stockhammer (1999) for a survey of the PKGM.
2 In a stagnationist regime a redistribution of income towards profits may result in a smaller rate of capacity utilization and economic growth [Blecker (1989)]. Taylor (1985, p. 383) considers that in a 'stagnationist view' both the growth rate and the level of capacity utilization can be different under different conditions of income distribution and/or macroeconomic policy.

rate of investment through its relation with the rate of capacity utilization. Hence by substituting the profit rate by the profit share in the expression of the growth rate of investment avoids to consider twice the effects of the former. One of the properties of the third generation model, as it became known, is the possibility of a 'non-stagnationist' regime[3] in which eventual falls in consumption due to a lower real wage are overcompensated by an increase in investment led by a profit share expansion. Although these characteristics are shared by other models in the Post-Keynesian tradition there is a remarkable lack of theoretical cohesion between them and the PKGM – This was an argument highlighted by Pasinetti (2005, p. 839-40) to explain why the Keynesian School has somewhat failed as a successful alternative paradigm to mainstream economics. Of course some effort was made in order to establish connections among these approaches or even to build a general PKGM. Intending to build reconciliation between the Kaleckian effective demand and Sraffian normal prices Lavoie (2003), for instance, has built a bridge between the PKGM and the Sraffian model. While the former focuses mainly on a determination of economic growth from the interaction between technical progress and evolution of demand patterns the latter focuses on this issue from a class struggle point of view that allows it to consider the existence of different regimes in which an increase in the participation of wages or profits lead the expansion.

In the present paper we show that the cross-fertilization between the PKGM and models in the structural economic dynamic tradition may render new results to central issues of economic growth such as investment allocation and structural change. However, a key methodological difference between the two approaches is that the PKGM consider national economies in the aggregate.[4] At this stage, it is important to note that one of the major criticisms Post-Keynesians leveled against the Neoclassical model is that it aggregates the whole economy into one sector, rendering the model incapable of performing an analysis of structural change.

[3] Accordingly in a non-stagnationist regime a redistribution of income towards profits may result in higher rates of capacity utilization and economic growth.

[4] In fact in his analysis Kalecki (1968) considers an economy with three compartments that can be viewed as a first approximation to a multi-sectoral analysis. Besides, his digression on mark-ups relies implicitly on reasoning that accrues from a multi-sectoral viewpoint since he considers crucial the comparison between sectoral and average mark-ups.

According to Pasinetti (1981, 1993), structural change refers to variations in the structure of an economy, and should be understood as related to the existence of particular rates of technological progress and also demand levels for final consumption goods. One sector models cannot take into account structural change because in such approaches any increase in per capita income is transformed into a higher level of consumption of the same final good. Furthermore, implicit in the Neoclassical representation is a well-known and strict definition of balanced growth, assuming that growth is non-inflationary and full-capacity utilization.

By ignoring structural change, the PKGM overlooks some crucial dimensions of economic growth, calling this approach into question. In order to overcome this limitation of the PKGM here its analysis is performed in a two-sector framework in the lines suggested by Feldman-Mahalanobis, hereafter F-M model. Feldman (1928) and Mahalanobis (1953) models, are generally used as benchmarks to study the effects of the investment allocation on economic growth.[5] In order to introduce a normative criterion to these approaches, Bose (1968) and Weitzman (1971) established an optimum rate of investment allocation in a context of dynamic optimisation of consumption. However, these analyses did not take into account the composition of consumption demand. In order to mitigate the limitations of the F-M model in relation to the passive role of per capita consumption demand, Araujo and Teixeira (2002) have shown that the F-M model may be treated as a particular case of Pasinetti's model of structural change. In this case it was possible to establish the rate of investment allocation which guarantees that the economy is in its stable growth path.

Araujo and Teixeira (2002) have also introduced a normative criterion to define the rate of investment allocation but it is important to note that their result is only normative and it remains the question of what is the actual the rate of investment allocation. Here we answer this question by showing that the PKGM may be treated as an aggregated version of the F-M model. This fact is not a novelty since both models are vertically integrated.[6] By following this ap-

[5] Dutt (1990, p. 120) considers that no discussion related to models with investment and consumption good sectors is complete without considering the F-M contribution. This view is shared by Halevi (1996) but this author points to the needy of introducing the demand side in the F-M analysis.

[6] Roughly speaking a sector is vertically integrated if it produces only one final good by using

proach it is possible to determine the rate of investment allocation compatible with the equilibrium in the credit market given by the PKGM growth model. Then it is possible to compare this rate with the normative one obtained from the F-M model as found by Araujo and Teixeira (2002). These results point to the importance of the credit market in determining the existing conditions of capital accumulation. If the decisions on investment allocation were distorted as a consequence of wrong expectations of savers and investors then less capital may be accumulated than what is necessary to endow the economy with the required capital goods to keep the economy in equilibrium.

This paper is structured as follows: in the next section we present a brief overview of the PKGM. In section 3 we show that the PKGM may be disaggregated into a two sector model in the lines of the F-M model by using the device of vertical integration. Furthermore the rate of investment allocation is also derived and it is compared with the one warranted rate of investment allocation obtained from the Pasinetti's model. Section 4 searches to extend these results to a more disaggregated economy. Section 5 summarizes the results.

2. A Two-Sector Version of the PKGM

A possible departing point to establish a bridge between the two approaches is to consider the relationship $r = \pi u$ in a two sector framework. This is an important point since although vertically integrated 'industries' are merely weighted combinations of real industries Steedman (1992, p. 149) it is possible to particularize to each sector a profit share, a rate of capacity utilization and a rate of profit, and to establish a relation among these variables in a two sector economy. According to Bhaduri and Marglin (1990, p.377) in the PKGM "we can think of the representative firm as vertically integrated using directly and indirectly a constant amount of labour per unit of final output." Araujo and Teixeira (2002) and Halevi (1996)

only labour and capital goods as inputs. No intermediate goods are allowed in this representation. As pointed out by Lavoie (1997, p. 453), "the concept of vertical integration, although extensively but implicitly used in macroeconomic analysis, has always been difficult to seize intuitively". What is behind this affirmation is that models that are aggregated in one or two sectors are based on the device of vertical integration. This range of vision is confirmed by Scazzieri (1990, p.26) for whom "[a]ny given economic system may generally be partitioned into a number of distinct subsystems, which may be identified according to a variety of criteria."

also have shown that the F-M model may be seen as a particular case of Pasinetti (1981) by using the device of vertical integration. Then by focusing on the degree of aggregation it is possible to say that the main difference between the PKGM and the F-M model is that while the former is aggregated in one sector the latter is aggregated in two sectors. But the device adopted to build these models is the same, namely vertical integration.

This view is also supported by other authors such as Lavoie (1997) and Scazzieri (1990) for whom the concept of vertical integration has been extensively but implicitly used in macroeconomic analysis.[7] From this standpoint let us consider a two-sector version of the PKGM. In what follows we consider that the capitalist economies are characterized by the tendency of levelling between sectoral rates of profit in the lines suggested by Adam Smith.[8] Following Dutt (1990) let us write the price equations for sectors 1 and 2 respectively as:

$$p_1 X_1 = wN_1 + rp_{k_1} K_1 \tag{1}$$

$$p_{k_1} X_{k_1} = wL_{k_1} + rp_{k_1} K_{k_1} \tag{2}$$

where p_1 and p_{k_1} stand for the prices for the consumption and capital goods respectively, X_1 and X_{k_1} stand for the production of the consumption and capital goods respectively, w is the nominal wage rate, L_1 and L_{k_1} are the level of labour employment in sectors 1 and k_1 respectively, r is the rate of profit and K_1 and K_{k_1} are the stock of capital in both sectors.

Now define $l_{k_1} = \dfrac{L_{k_1}}{X_{k_1}}$ as the labour per unit of output, $v_{k_1} = \dfrac{K_{k_1}}{X_{k_1 fe}}$ as the capital-output ratio and $u_{k_1} = \dfrac{X_{k1}}{X_{k1 fe}}$ as the rate of capacity utili-

7 This view is confirmed by Steedman (1992, p. 136) for whom "Kaleckian writings frequently appeal to vertically integrated representations of the economy."

8 But this is just a tendency that may not be confirmed in the real economies due to a number to barrier to capital flows from one sector to another. The existence of monopoly – or oligopoly – in some sectors may be a good explanation for the existence of a particular rate of profit in that sector.

zation, where $X_{k_1 fe}$ stands for the full capacity output in the capital goods sector. By using this notation and assuming that v_{k_1} is constant and normalized to one we can rewrite Expression (2) as:

$$p_1 = wl_1 + rp_{k_1} u_1^{-1} \tag{2'}$$

Let us assume that prices are given by a mark-up rule over wage according to:

$$p_{k_1} = (1 + \tau_{k_1}) wl_{k_1} \tag{3}$$

Where τ_{k_1} is the mark-up rate in the capital goods sector. By consi dering that $\pi_{k_1} = \frac{rp_{k1} K_{k1}}{p_{k1} X_{k1}}$ is the profit share in the capital goods sector it is possible to show after some algebraic manipulation that:

$$r = \pi_{k_1} u_{k_1} \tag{4}$$

From Expressions (2)' and (3) it is possible to show that:

$$\pi_{k_1} = \frac{\tau_{k_1}}{(1 + \tau_{k_1})} \tag{5}$$

By following the same procedure in relation to the consumption goods sector and considering that where X_{1fe} stands for the full capacity output in the consumption goods sector, $l_1 = \frac{L_1}{X_1}$ is the labour coefficient, $v_1 = \frac{K_1}{X_{1fe}}$ is the capital-output ratio and $u_1 = \frac{X_1}{X_{1fe}}$ measures the capacity utilization, Expression (2) may be rewritten as:

$$p_{k_1} = wl_{k_1} + rp_{k_1} u_{k_1} \tag{2'}$$

Let us assume a mark-up rule for prices in the consumption goods sector

$$p_1 = (1 + \tau_1) wl_1 \tag{6}$$

Where τ_1 is the mark-up rate in the consumption goods sector.

The profit share in this sector is given by: $\pi_1 = \dfrac{rp_{k_1}K_1}{p_1X_1}$. After some algebraic manipulation we conclude that:

$$\pi_1 = \frac{rv_1u_1^{-1}(1+\tau_{k1})l_{k1}}{(1+\tau_1)l_1} \tag{7}$$

This is a counterpart of expression (4) for the consumption goods sector. But we know from Expressions (2)' and (6) that:

$$\tau_1 = \frac{(1+\tau_{k_1})l_{k_1}rv_1u_1^{-1}}{l_1} \tag{8}$$

Considering Expressions (7) and (8) together we obtain:

$$\pi_1 = \frac{\tau_1}{(1+\tau_1)} \tag{9}$$

The growth rate of savings, g_S, is given by the Cambridge equation in all generations. As we are assuming that workers do not save, the savings corresponds to a marginal propensity of capitalists' income, Y_c, given by: $Y_c = rp_{k_1}(K_1 + K_{k_1})$. By considering that $K = K_1 + K_{k.}$ and $p_{k_1} = 1$ we conclude that total savings, S, are given by $S = srK$, where s is the saving propensity, with $0 \le s \le 1$. After normalizing the savings by the capital stock, we obtain:

$$g_S = sr \tag{10}$$

Note that Equation (10) does not establish the rate of profit as in the Kaldor-Pasinetti process – where the natural growth rate is given – and determines the rate of profit once the propensity to save is exogenous (See Araujo (1992-93)). Kaldor (1956) and Robinson (1956, 1962) have built models on the notion of full employment and full capacity utilization that contemplate both the supply and demand sides to determine the growth rate of a closed economy. There are some differences between the approaches developed by these authors; however, the core of their models may be described as follows. It is assumed that workers do not save and the economy operates at full capacity.[9] The growth rate of investment, g_I, is assumed to be given by:

$$g_I = \gamma + \alpha r \tag{11}$$

where $\alpha > 0$ measures the influence of the investment to the interest rate, r, and γ stands for the growth rate of autonomous investment. The positive effect of the rate of profit on investment decisions relies on the relation between actual and expected profits. In order to take into account the possibility of disequilibrium, Dutt (1984) and Rowthorn (1982), by working independently, have built what is known as the second generation of the Post-Keynesian growth model by endogenizing the rate of capacity utilization in the lines of Steindl (1952). One of the main contributions of this second generation is the possibility of a stagnationist regime in which an increase in the profit share implies a reduction in the capacity utilization. The key assumption behind this result is that the growth rate of investment is a function not only of the profit rate, as in Kaldor-Robinson but also of the rate of capacity utilization (Steindl (1952)):

$$g_I = \gamma + \alpha r + \beta u \tag{12}$$

where $\beta > 0$ measures the sensibility of the growth rate of investment to the capacity utilization, u, and captures the accelerator effect: a high rate of capacity utilization induces firms to expand capacity in order to meet anticipated demand while low utilization induces firms to contract investment. The positive effect of the rate of profit on investment decisions relies on the relation between actual and expected profits. Bhaduri and Marglin (1990) have challenged this view by considering that the growth rate of investment is a function of the rate of capacity utilization and of the profit share. According to them the rate of profit has already been implicitly considered in the equation of the growth rate of investment through the rate capacity utilization and due to the following macroeconomic relation $r = \pi.u$. Hence by substituting the rate of profit by the profit share in the expression of the growth rate of investment avoids

9 Robinson (1956, 1962) refers to a 'normal' rate of capacity utilization to express that degree of utilization of productive capacity that producers consider as ideally suited to fulfill demand requirements.

to consider twice the effects of the former on the growth rate of investment.

One of the properties of this third generation model, as it became known is the possibility of a non-stagnationist regime. In Bhaduri and Marglin (1990) the investment function now reacts positively to profits and capacity utilization, given that the profit-share is used as a measure of profitability.[10] Therefore:

$$g_I = h(\pi, u) \tag{13}$$

with partial derivatives $h_\pi(\pi, u) > 0$ and $h_u(\pi, u) > 0$.

According to Bhaduri and Marglin (1990, p. 380), influences of existing capacity on investment cannot be captured satisfactorily by simply introducing a term for capacity utilization. The investment function should also consider profit share and capacity utilization as independent and separate variables in the lines of Expression (13). Following Blecker (2002, p. 137) let us assume, for the sake of convenience only, a linear version of the investment function:

$$g_I = \gamma + \alpha\pi + \beta u \qquad (13)'$$

As we are dealing with a two-sector model let us particularize an investment function for each sector according to:

Table 1

	Kaldor-Robinson	Neo-Kaleckian	Bhaduri-Marglin
Capital goods sector	$g_{k_1}^I = \gamma_{k_1} + \alpha_{k_1} r$	$g_{k_1}^I = \gamma_{k_1} + \alpha_{k_1} r + \beta_{k_1} u_{k_1}$	$g_{k_1}^I = \gamma_{k_1} + \alpha_{k_1} \pi_{k_1} + \beta_{k_1} u_{k_1}$
Consumption goods sector	$g_1^I = \gamma_1 + \alpha_1 r$	$g_1^I = \gamma_1 + \alpha_1 r + \beta_1 u_1$	$g_1^I = \gamma_1 + \alpha_1 \pi_1 + \beta_1 u_1$

The rationale behind these specifications considers that there is a trend of equalization of profit rates between the two sectors due to the competition amongst capitalists. But it is assumed that the

[10] Bhaduri and Marglin (1990) do not linearize the investment function but some authors such as Blecker (2002) adopted a linearized version to obtain closed form solutions for the endogenous variables.

influence of the investment to the interest rate, namely $\alpha_i, i=1,k_1$, is particular to each sector. In the same vein the sensibility of the growth rate of investment to the capacity utilization, that is $\beta_i, i=1,k_1$, is also assumed to be particular to each sector. Although the parameters of the model are particular to each sector an important property of this model is that in steady state both sectors grow at the same rate, namely $g_S = g_1^I = g_{k_1}^I$. By considering that:

$\Delta = \dfrac{(1+\tau_1)l_1 v_1}{(1+\tau_{k_1})l_{k_1}}$ then it is possible to solve the model and obtain

Table 2

	Kaldor-Robinson	Neo-Kaleckian	Bhaduri-Marglin
Rate of capacity utilization	$u_{k_1}^* = 1$	$u_{k_1}^* = \dfrac{\gamma_{k_1}}{\pi_{k_1}(s-\alpha_{k_1})-\beta_{k_1}}$	$u_{k_1}^* = \dfrac{\gamma_{k_1}+\alpha_{k_1}\pi_{k_1}}{s\pi_{k_1}-\beta_{k_1}}$
Rate of utilization	$u_1^* = 1$	$u_1^* = \dfrac{\gamma_1}{\pi_1\Delta(s-\alpha_1)-\beta_1}$	$u_1^* = \dfrac{\gamma_1+\alpha_1\pi_1}{s\pi_1\Delta-\beta_1}$
Profit Rate	$r^* = \dfrac{\gamma_{k_1}}{s-\alpha_{k_1}}$	$r^* = \dfrac{\pi_{k_1}\gamma_{k_1}}{\pi_{k_1}(s-\alpha_{k_1})-\beta_{k_1}}$	$r^* = \dfrac{\pi_{k_1}(\gamma_{k_1}+\alpha_{k_1}\pi_{k_1})}{s\pi_{k_1}-\beta_{k_1}}$
Growth rate	$g^* = \dfrac{s\gamma_{k_1}}{s-\alpha_{k_1}}$	$g^* = \dfrac{s\pi_{k_1}\gamma_{k_1}}{\pi_{k_1}(s-\alpha_{k_1})-\beta_{k_1}}$	$g^* = \dfrac{s\pi_{k_1}(\gamma_{k_1}+\alpha_{k_1}\pi_{k_1})}{s\pi_{k_1}-\beta_{k_1}}$

Although the results in Table 2 refer to a two-sector set-up they keep the flavour of the original analysis of the PKGM. Taking the derivative of the rate of capacity utilization for both sectors and the rate of profit in relation to the profit share we conclude that:

Table 3

Neo-Kaleckian	Bhaduri-Marglin
$\frac{\partial u_{k_1}^*}{\partial \pi_{k_1}} = -\frac{s-\alpha_{k_1}}{[\pi_{k_1}(s-\alpha_{k_1})-\beta_{k_1}]^2} < 0$	$\frac{\partial u_{k_1}^*}{\partial \pi_{k_1}} = -\frac{s\gamma_{k_1} + \alpha_{k_1}\beta_{k_1}}{[s\pi_{k_1} - \beta_{k_1}]^2} < 0$
$\frac{\partial u_1^*}{\partial \pi_1} = -\frac{(s-\alpha_1)\Delta}{[\pi_1\Delta(s-\alpha_1)-\beta_1]^2} < 0$	$\frac{\partial u_1^*}{\partial \pi_1} = -\frac{s\Delta\gamma_1 + \alpha_1\beta_1}{[s\pi_1\Delta - \beta_1]^2} < 0$
$\frac{\partial r^*}{\partial \pi_{k_1}} = -\frac{\gamma_{k_1}\beta_{k_1}}{[\pi_{k_1}(s-\alpha_{k_1})-\beta_{k_1}]^2} < 0$	$\frac{\partial r^*}{\partial \pi_{k_1}} = -\frac{\alpha_{k_1}\pi_{k_1} - \beta_{k_1}u_{k_1}^*}{s\pi_{k_1} - \beta_{k_1}} < or > 0$

This result shows that in the Neo-Kaleckian approach a redistribution of income towards wages may result in a higher rate of capacity utilization and higher profit rate, as shown by Blecker (1989) and should then be known in the literature as the 'stagnationist'.

The main difference in the results of the Bhaduri-Marglin (1990) and the neo-Kaleckian approach is that in the former, the derivative of the profit rate in relation to the profit share may be positive or negative. Now there may be a positive capacity effect and a negative profit share effect on investment. Thus, two regimes are possible, depending on the relative magnitudes of capacity utilization and profit share effects in the investment function. If the profit effect is stronger than the capacity effect, meaning that $\alpha_{k_1}\pi_{k_1} - \beta_{k_1}u_{k_1}^* > 0$, growth is under a wage-led regime. Otherwise, if $\alpha_{k_1}\pi_{k_1} - \beta_{k_1}u_{k_1}^* < 0$, we have a profit-led regime. These results are consistent with the one sector version of the model. Now we are in a position of connecting the PKGM approach with the F-M framework that will be presented in the next section.

3. The Rate of Investment Allocation

In the F-M model investment sector grows at:

$$\frac{I}{K} = \frac{X_{k_1}}{K} \tag{14}$$

where X_{k_1} is the production of capital goods, which is described by Leontief production functions and the limiting factor of production is the stock of capital goods. Hence:

$$X_{k_1} = \min\left[\frac{K_{k_1}}{v_{k_1}}, \frac{L_{k_1}}{\upsilon_{k_1}}\right] \Rightarrow X_{k_1} = \frac{K_{k_1}}{v_{k_1}} \tag{15}$$

where K_{k_1} refers to the stock of investment goods and v_{k_1} stands for the capital-output ratio in the capital goods sector while L_{k_1} and υ_{k_1} are the quantity of employed working force and the labour coefficients respectively. By substituting (15) into (14) we obtain:

$$\frac{I}{K} = \frac{K_{k_1}}{v_{k_1} K} \tag{16}$$

For the sake of convenience only, it is assumed that there is no depreciation of capital goods, the investment goods cannot be imported and the production of capital goods does not depend on the production of consumption goods sector. Now it is possible to establish the growth rate of investment. The change in investment is given by:

$$\dot{X}_{k_1} = \dot{K}_{k_1} / v_{k_1} \tag{17}$$

But the variation in stock of capital in sector k_1 depends only on the proportion of the total output of this sector that is allocated to itself. We assume that a proportion λ of the current production of the investment sector is allocated to itself while the remaining, $1 - \lambda$, is allocated to sector 1 ($1 \geq \lambda \geq 0$). Hence:

$$\dot{K}_{K_1} = \lambda X_{k_1} \tag{18}$$

Substituting (18) into (17) leads to the growth rate of the investment sector:

$$\frac{\dot{X}_{k_1}}{X_{k_1}} = \frac{\lambda}{v_{k_1}} \tag{19}$$

Let us assume that the production in the consumption sector is also described by Leontief production function with the limiting factor of production the stock of capital goods.

$$X_1 = \min\left[\frac{K_1}{v_1}, \frac{L_1}{\upsilon_1}\right] \Rightarrow X_1 = \frac{K_1}{\upsilon_1} \tag{20}$$

where K_1 refers to the stock of investment goods and v_1 stands for the capital-output ratio in the consumption goods sector while L_1 and υ_1 are the quantity of employed working force and the labour coefficients respectively. Adopting the same procedure in relation to the consumption sector and considering that $\dot{K}_1 = (1-\lambda)X_{k_1}$,we establish its growth rate:

$$\frac{\dot{X}_1}{X_1} = \frac{(1-\lambda)X_{k_1}}{v_1 X_1} \tag{21}$$

Taking limits of both sides of Expression (21) when t tends to in-

finity and applying the L'Hôpital rule lead us to conclude that the growth rate of consumption depends on the growth rate of investment and, in the long run, the former converges to the later, which will be the growth rate of the economy as a whole.

$$\lim_{t\to\infty} \frac{\dot{X}_1}{X_1} = \frac{\lambda}{v_{k_1}} \tag{22}$$

Besides the composition of capital goods in this economy will be given by:

$$\frac{K_{k_1}}{K_1} = \frac{\lambda}{1-\lambda} \tag{23}$$

The results in the third line of Table 1 yield the investment in equilibrium normalized by the stock of capital. Table 2 shows this outcome:

Table 4

	Kaldor-Robinson	Neo-Kaleckian	Bhaduri-Marglin
Growth rate	$\left(\frac{I}{K}\right)^* = \frac{s\gamma_{k_1}}{s-\alpha_{k_1}}$	$\left(\frac{I}{K}\right)^* = \frac{s\pi_{k_1}\gamma_{k_1}}{\pi_{k_1}(s-\alpha_{k_1})-\beta_{k_1}}$	$\left(\frac{I}{K}\right)^* = \frac{s\pi(\gamma_{k_1}+\alpha_{k_1}\pi_{k_1})}{s\pi_{k_1}-\beta_{k_1}}$

By equalizing these results with Expression (16), we obtain for each case the following share for the stock of capital goods of sectors k_1 and 1 in total stock of capital. This is shown is Table 3:

Table 5

Kaldor-Robinson	Neo-Kaleckian	Bhaduri-Marglin
$\left(\frac{K_{k_1}}{K_1}\right)^* = \frac{v_{k_1}s\gamma_{k_1}}{s-\alpha_{k_1}-v_{k_1}s\gamma_{k_1}}$	$\left(\frac{K_{k_1}}{K_1}\right)^* = \frac{v_{k_1}s\pi_{k_1}\gamma_{k_1}}{\pi_{k_1}(s-\alpha_{k_1})-\beta_{k_1}-v_{k_1}s\pi_{k_1}\gamma_{k_1}}$	$\left(\frac{K_{k_1}}{K_1}\right)^* = \frac{v_{k_1}s\pi_{k_1}(\gamma_{k_1}+\alpha_{k_1}\pi_{k_1})}{s\pi_{k_1}-\beta_{k_1}-v_{k_1}s\pi_{k_1}(\gamma_{k_1}+\alpha_{k_1}\pi_{k_1})}$
$\left(\frac{K_{k_1}}{K}\right)^* = \frac{v_{k_1}s\gamma_{k_1}}{s-\alpha_{k_1}}$	$\left(\frac{K_{k_1}}{K}\right)^* = \frac{v_{k_1}s\pi_{k_1}\gamma_{k_1}}{\pi_{k_1}(s-\alpha_{k_1})-\beta_{k_1}}$	$\left(\frac{K_{k_1}}{K}\right)^* = \frac{v_{k_1}s\pi_{k_1}(\gamma_{k_1}+\alpha_{k_1}\pi_{k_1})}{s\pi_{k_1}-\beta_{k_1}}$
$\left(\frac{K_1}{K}\right)^* = \frac{s-\alpha_{k_1}-v_{k_1}s\gamma_{k_1}}{s-\alpha_{k_1}}$	$\left(\frac{K_1}{K}\right)^* = \frac{(s-\alpha_{k_1})\pi_{k_1}-\beta_{k_1}-v_{k_1}s\pi_{k_1}\gamma_{k_1}}{\pi_{k_1}(s-\alpha_{k_1})-\beta_{k_1}}$	$\left(\frac{K_1}{K}\right)^* = \frac{s\pi_{k_1}-\beta_{k_1}-v_{k_1}s\pi_{k_1}(\gamma_{k_1}+\alpha_{k_1}\pi_{k_1})}{s\pi_{k_1}-\beta_{k_1}}$

By equalizing these results to (23) we obtain:

Table 6

Kaldor-Robinson	Neo-Kaleckian	Bhaduri-Marglin
$\lambda^* = \frac{v_{k_1}s\gamma_{k_1}}{s-\alpha_{k_1}}$	$\lambda^* = \frac{v_{k_1}s\pi_{k_1}\gamma_{k_1}}{\pi_{k_1}(s-\alpha_{k_1})-\beta_{k_1}}$	$\lambda^* = \frac{v_{k_1}s\pi_{k_1}(\gamma_{k_1}+\alpha_{k_1}\pi_{k_1})}{s\pi_{k_1}-\beta_{k_1}}$

The procedure adopted here ensures that the economic system will be endowed with the capital goods required to fulfil the requirements expressed by the equalization of savings and investment decisions in the PKGM. In order to proceed to capital accumulation it is necessary to build the background in terms of the expansion of the production of the capital sector to meet the demand requirements. In fact, we learn from this analysis that the actual structural dynamics depends ultimately on the distributive features of the economy and not only on the evolution patterns of demand and technological progress as in the Pasinettian view.

4. Towards a more Disaggregated Economy

The analysis of the previous section may be extended to an arbitrary number of sectors. As shown by Araujo and Teixeira (2002), the F-M model is built under the notion of vertical integration and may be seen as a particular case of Pasinetti's model of structural change and economic growth. Hence it is possible to consider the analysis of investment allocation in a multi-sector economy in each every sector is subject to a particular rate of growth of demand and technological progress. In this case the sectoral rate of rate of investment allocation is given by:

$$\lambda_i^* = v_{k_i}(n + \theta_i) \tag{24}$$

Where θ_i is the growth rate of demand for the consumption good i and v_{k_i} is the capital-output ratio for the i-th sector. As shown by Araujo and Teixeira (2011) it is also possible to consider a multi--sector version of the PKGM and in this vein to consider sector expressions for the investment and savings according to the rationale to the generations of this model. According to them it is possible because the PKGM is also build on the notion of vertical integration. In this case the analysis of the previous sections may be extended to a multi-sector economy and each sector and the actual rate of investment allocation for each sector will be given by the following table according to each generation:

Table 7

Kaldor-Robinson	Neo-Kaleckian	Bhaduri-Marglin
$\lambda_i^* = \dfrac{v_{k_i} s \gamma_{k_i}}{s - \alpha_{k_i}}$	$\lambda_i^* = \dfrac{v_{k_i} s \pi_{k_i} \gamma_{k_i}}{\pi_{k_i}(s - \alpha_{k_i}) - \beta_{k_i}}$	$\lambda_i^* = \dfrac{v_{k_i} s \pi_{k_i}(\gamma_{k_i} + \alpha_{k_i}\pi_{k_i})}{s\pi_{k_i} - \beta_{k_i}}$

Now $\alpha_i > 0$ measures the influence of the investment to the interest rate in the i-th sector, π_i stands for the profit share in i-th sector and γ_i stands for the growth rate of autonomous investment. By adopting the approach of the previous section, it is possible to understand now that each sector should have its own growth rate compatible with the correct allocation of capital goods according to the evolution of preferences. Hence by particularizing a saving rate for each sector we obtain:

Table 8

Kaldor-Robinson	Neo-Kaleckian	Bhaduri-Marglin
$s_i^* = \frac{n+\theta_i - \gamma_{k_i}}{\alpha_{k_i}(n+\theta_i)}$	$s_i^* = \frac{(n+\theta_i)(\pi_{k_i}\alpha_{k_i} + \beta_{k_i})}{\pi_{k_i}(\gamma_{k_i} + n + \theta_i)}$	$s_i^* = \frac{(n+\theta_i)\beta_{k_i}}{n+\theta_i - \gamma_{k_i} - \alpha_{k_i}\pi_{k_i}}$

These results show that the fulfilment of the capital accumulation conditions in each sector requires the existence of particular saving rates for each sector. Besides, Pasinetti (1981) shows that in fact each sector has to be a particular rate of profit in order to fulfil the demand requirements. He has called this profit rate as natural ones and has showed that for each sector the natural rate of profit is given by:

$$r_i^* = g + \theta_i \tag{25}$$

Note that if $r_i < g + \theta_i$ then capitalists in the *i*-th sector will not have the necessary amount of resources to invest in such sector in order to meet the expansion of demand. If $r_i > g + \theta_i$ then capitalist will overinvest in the *i*-th sector leading to excess of productive capacity. Araujo and Teixeira (2011) have shown that the multi-sectoral version of the PKGM also entails the derivation of the profit rate, which is in fact an actual profit rate.

Table 9

	Kaldor-Robinson	Neo-Kaleckian	Bhaduri-Marglin
Profit Rate	$r_i^* = \frac{\gamma_{k_i}}{s_i - \alpha_{k_i}}$	$r_i^* = \frac{\pi_{k_i}\gamma_{k_i}}{\pi_{k_i}(s_i - \alpha_{k_i}) - \beta_{k_i}}$	$r_i^* = \frac{\pi_{k_i}(\gamma_{k_i} + \alpha_{k_i}\pi_{k_i})}{s_i\pi_{k_i} - \beta_{k_i}}$

By equalizing the natural profit rate with the actual profit rate it is also possible to obtain the saving rate for each sector that fulfils the capital accumulation condition, namely:

	Kaldor-Robinson	Neo-Kaleckian	Bhaduri-Marglin
Saving rates	$s_i = \frac{\gamma_{k_i}}{n+\theta_i} + \alpha_{k_i}$	$s_i = \frac{\beta_{k_i}}{\pi_{k_i}} + \frac{\gamma_{k_i}}{n+\theta_i} + \alpha_{k_i}$	$s_i = \frac{\beta_{k_i}}{\pi_{k_i}} + \frac{\gamma_{k_i} + \alpha_{k_i}\pi_{k_i}}{n+\theta_i}$

It is important to note that the results of the table above are different from the one obtained to guarantee the equalization of the actual rate of investment allocation with the natural rate of investment allocation. Hence in general it is not possible to establish sectoral saving rates compatible with two different goals: endow vertically integrated sectors with the right composition of capital goods and give capitalists the warranted rate of profit.

5. Concluding Remarks

One of the key distinctions between the orthodox view and the Post-Keynesian growth models is the importance given to the supply and demand determination of economic growth. While the later focuses on demand the former stresses the supply side as determinant of the process of economic growth. Emphasis upon demand composition offers a significant qualitative improvement vis–a–vis traditional, aggregated models that fail to adequately consider composition of consumption demand. Note that what is known as the original PKGM is actually subject to the same criticism – aggregation hypothesis – as the Neoclassical one sector model, calling the PKGM approach into question.

In this article, in order to overcome this limitation of the PKGM its analysis was extended to a multi-sector framework by treating it initially as a particular case of the two sector F-M model of investment allocation. The standpoint of the analysis is the concept of vertical integration which allows us to establish a correspondence between the two approaches. Then it was possible to study how the demand side, portrayed by the decisions of savings and investment, may affect the decisions of investment allocation. The influence of these factors on the investment allocation between capital and consumption goods sectors were analysed in order to establish the rate of investment allocation subject to the equilibrium in the credit market. This rate was determined by taking into account the structure of consumer preferences. This fact shows that the structural economic dynamics is conditioned not only by patterns of evolution of demand and diffusion of technological progress but also by the different regimes of economic growth.

Accordingly, a multi-sector version of the PKGM allows us to consider that while one sector is operating in a wage led regime others could be operating in a profit led regime. This gives rise to important structural economic dynamics that relies not only to the dynamics of human preferences and technology but also on distributive features of the economy. It was also shown that when dealing with the most general version of the PKGM, where capital goods are considered, there is an additional expression in the system of equations that characterize the economic system to be verified. This condition was referred here as the investment allocation condition.

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