



Universidade de Brasília

Instituto de Geociências

Programa de Pós-Graduação em Geociências Aplicadas e Geodinâmica

**PROCESSAMENTO SÍSMICO DE DIFRAÇÕES:
UM PROGRAMA PARA IMAGEAMENTO DE ALTA
RESOLUÇÃO**

Guilherme Zakarewicz de Aguiar

Orientadora: Susanne Tainá Ramalho Maciel

Dissertação de mestrado nº 217

Brasília, Junho de 2023

Guilherme Zakarewicz de Aguiar

**Processamento sísmico de difrações:
um programa para imageamento de alta resolução**

Dissertação submetida ao Programa de Pós Graduação em Geociências Aplicadas e Geodinâmica da Universidade de Brasília como requisito para obtenção do Título de Mestre em Geofísica Aplicada.

Orientadora: Susanne Tainá Ramalho Maciel

Brasília, 30 de Junho de 2023

Ficha catalográfica elaborada automaticamente,
com os dados fornecidos pelo(a) autor(a)

Zp	Zakarewicz de Aguiar, Guilherme Processamento sísmico de difrações: um programa para imageamento de alta resolução / Guilherme Zakarewicz de Aguiar; orientador Susanne Tainá Ramalho Maciel. -- Brasília, 2023. 103 p. Dissertação(Mestrado em Geociências Aplicadas) -- Universidade de Brasília, 2023. 1. Programa Aberto. 2. Imageamento de Difrações. 3. Difrações Sísmicas. 4. Modelagem Sintética. 5. Revisão Sistemática. I. Tainá Ramalho Maciel, Susanne, orient. II. Título.
----	--

Agradecimentos

Durante o período de elaboração deste trabalho, tive o privilégio de aprender que a gratidão é um dos sentimentos mais sublimes, forma humilde de demonstrar sabedoria. Desta forma, não posso deixar de expressar meus agradecimentos a todos que me acompanharam nesta jornada. Acima de tudo, agradeço a Deus e a todo o maior que me acompanha pela oportunidade de viver. Seus sussurros suaves de intuição e presença constante na minha vida foram fundamentais para me guiar.

Agradeço à minha família, pilar da minha vida, fonte inesgotável de amor ao longo dessa jornada. Pude contar com seu apoio incondicional em todas as minhas escolhas. À minha mãe, Beth, por ter me ensinado os mais preciosos valores de honestidade, humildade, compreensão e amor a Deus. Ao meu pai, Farid, por ter me incentivado sempre a trilhar o caminho baseado nos estudos e no trabalho honesto. Agradeço por ambos terem aberto mão de muitos sonhos pelos seus filhos. Ao meu irmão, Mateus, sempre presente em minhas conquistas independentemente da distância.

Por todo o acolhimento, carinho e afeto recebido por Beth e Edson. Obrigado por sempre me receberem de braços abertos! Também a Gabriel, Carol e Lara por toda a amizade, descontração e companheirismo, tornando essa etapa mais leve.

Agradeço à Luísa, companheira de vida que ilumina minha caminhada. É uma grande dádiva compartilhar sonhos e planos com você, o destino ganha novos significados ao seu lado. Te amo!

À Susanne, grande mentora da minha carreira acadêmica. É inspirador ver sua dedicação, comprometimento e paixão pelo trabalho proposto. Sua capacidade de transmitir conhecimento e despertar o melhor em seus alunos sempre me surpreende. Agradeço pelos conselhos, pela amizade e pela orientação, é um privilégio ser seu aluno.

Ao professor Luciano Cunha, por todo o auxílio com a realização do artigo de revisão. Suas incontáveis sugestões e contribuições foram fundamentais para elaborar este capítulo, ideia que surgiu no decorrer de sua disciplina. Agradeço também ao professor Ricardo Biloti por todo o suporte e auxílio fornecido no desenvolvimento do código principal.

À banca examinadora composta pelos professores Ellen e Marco. O processo de revisão por pares, muitas vezes anônimo e voluntário, é fundamental para a construção crítica da ciência. Certamente seus apontamentos melhoraram imensamente a qualidade deste trabalho. Agradeço sinceramente pelo tempo e interesse despendidos na análise deste texto.

Ao Observatório Sismológico, por toda a infraestrutura disponibilizada durante a realização de minha graduação e mestrado. Também a todos que ali tive o prazer de conhecer: Jaque, Násser, Débora, Daniel, seu Antônio, Cida, Cláudia, Abadia e dona Val.

A todas as amigas que pude cultivar neste período: Victor, Yellinson, Moara, Brayner, Tabosa, Arthur, Marcos, Gabriel, Pedro Simon, Lucas Ribeiro, Lucas Fantinatti e tantos outros.

À Universidade de Brasília, espaço inspirador que oferece uma educação pública e de qualidade. Lugar que me mostrou a importância de proporcionar o acesso democrático às instituições de ensino. Espero que todos possam ter a mesma possibilidade de buscar seus sonhos por meio do acesso ao ensino superior.

Agradeço aos professores que marcaram meu aprendizado. Em especial, à professora Mônica, que compartilha sua paixão pela geofísica com seus alunos desde o início do curso; e ao professor George Sand, que me ajudou em diversas ocasiões com seus inúmeros conselhos e todos os momentos descontraídos no Obsis.

Ao fomento disponibilizado pela CAPES. O investimento na educação superior no Brasil é fundamental para o desenvolvimento pleno da nossa sociedade, e o apoio financeiro fornecido pela CAPES permite a realização de pesquisas como essa. Agradeço pelo reconhecimento da importância da educação superior e pelo compromisso em investir no progresso acadêmico e científico do país. O presente trabalho foi realizado com o apoio da Coordenação de Aperfeiçoamento de Pessoal de Nível Superior - Brasil (CAPES) - Código de Financiamento 001.

A todos, muito obrigado! Que venham novos desafios.



*“Understand well as I may, my comprehension can only
be an infinitesimal fraction of all I want to understand.”*

Ada Lovelace

Resumo

As difrações sísmicas possuem valiosas informações sobre estruturas em subsuperfície como quinas, falhas, fraturas, cunhas e canais. No entanto, são frequentemente consideradas como ruído nos métodos convencionais de processamento sísmico. A componente difratada do campo de ondas oferece o potencial de alcançar a super resolução, ultrapassando o limite sísmico de imageamento. Essa capacidade possui implicações significativas em diversos campos e aplicações industriais. Por meio de uma revisão sistemática da literatura, apresentamos uma visão abrangente das técnicas disponíveis de imageamento sísmico de difrações. Nossa análise inclui uma avaliação quantitativa dos artigos selecionados com o objetivo de identificar técnicas emergentes, lacunas de conhecimento e explorar possíveis temas para pesquisas futuras. Notamos um interesse crescente em aplicações práticas, com foco especial na demonstração da eficácia do processamento de difrações na delimitação de estruturas baseadas exclusivamente em dados de campo. Neste estudo, apresentamos o DiffraPy, um programa gratuito implementado em Python especificamente para realizar o processamento de difrações. Nosso código oferece uma funcionalidade que inclui capacidade de modelagem sintética, permitindo testes e novas experimentações. Demonstramos a aplicação do código utilizando um modelo simplificado e avaliamos seu desempenho em um modelo sintético de maior complexidade, avaliando tanto suas limitações quanto seu potencial. Em ambos os casos, o programa suprimiu a maior parte da energia refletida, o que resulta em um produto de maior resolução em comparação com a migração convencional. O processamento de difrações destacou as respostas de pequenas lentes, descontinuidades e falhas, o que fornece importantes informações sobre a subsuperfície. Esperamos que nosso código ofereça à comunidade geofísica uma alternativa gratuita e documentada para testar, reproduzir e explorar vários cenários de imageamento de difrações.

Palavras-chave:

Programa Aberto; Python; Imageamento de Difrações; Difrações Sísmicas; Modelagem Sintética; Descontinuidades; Revisão Sistemática.

Abstract

Seismic diffractions contain valuable high-resolution information about subsurface structures, but they are frequently disregarded as noise in conventional seismic processing methods. The diffracted component of the wavefield offers the potential to achieve superresolution, surpassing the limitations of the Rayleigh limit and enabling the recovery of subsurface details. This capability has significant implications across diverse fields and industrial applications. Through a systematic literature review, we present a comprehensive overview of the available techniques in seismic diffractions imaging. Our analysis encompasses a quantitative assessment of the selected articles, aiming to identify emerging techniques, address knowledge gaps, and explore potential avenues for future research. We observed an increasing interest in practical applications, with a particular focus on demonstrating the effectiveness of diffractions imaging in delineating structures exclusively based on field data. In this study, we introduce DiffraPy, an freely available Python software specifically developed for performing a diffraction-oriented migration. Our code offers functionalities that include forward modeling, allowing for thorough testing and experimentation. To illustrate the functionality of DiffraPy, we demonstrate its workflow using a toy model and evaluate its performance on a synthetic model, discussing both its limitations and potential. In both cases, the program successfully suppressed the majority of reflected energy in the diffraction-oriented image, resulting in a higher-resolution product compared to the conventional migration. The diffraction processing prominently highlighted the responses of small lenses, discontinuities, and faults, providing valuable information about the subsurface. We expect our code to offer the geosciences community a freely accessible and well-documented alternative for testing, reproducing, and exploring various diffraction imaging examples scenarios.

Keywords:

Open-software; Python; Diffraction Imaging; Seismic Diffractions; Forward Modeling; Discontinuities; Systematic Review.

Lista de Figuras

1	(a) Bloco tridimensional ilustrando a propagação de uma onda P: a partícula se move na direção da propagação da onda. (b) Na onda S, a partícula se movimenta em um plano perpendicular à direção de propagação.	23
2	(a) Bloco tridimensional ilustrando a propagação das ondas Rayleigh: a partícula apresenta um movimento elíptico retrógrado em um plano perpendicular à superfície. (b) Nas ondas Love, a partícula se movimenta paralelamente à superfície e perpendicularmente à direção de propagação.	24
3	Comportamento de raios sísmicos (a) refletidos e (b) difratados em uma superfície. Modificado de Moser (2012).	25
4	Diferença geométrica entre reflexões e difrações em um meio de velocidade constante. S e R são as posições da fonte e dos receptores; A o ápice da hipérbole; S' a projeção da reflexão em subsuperfície; e D o ponto difrator. (a) Geometria de reflexões para uma interface planar horizontal; (b) Geometria de difrações para um ponto difrator. Note que a reflexão pode ser compreendida como originada por um ponto S' espelhado em relação à posição do refletor. Modificado de Khaidukov et al. (2004).	26
5	Modelos sintéticos de refletividade (a-c) e seções <i>common-shot</i> resultantes a partir da modelagem Kirchhoff (d-f).	27
6	Diferentes situações de interação entre pulsos sísmicos de diferentes eventos. À esquerda, a distância entre os pulsos torna possível distingui-los separadamente no pulso resultante. No meio, ocorre o limite de Rayleigh, em que ainda é possível distinguir os pulsos individualmente. À direita, a distância não permite a distinção, em que a resultante é um único pulso.	28
7	Exemplo de migração sísmica. (a) Modelo sintético de uma seção de tiro comum com um ponto difrator; (b) traços sísmicos resultantes da modelagem Kirchhoff; (c) imagem recuperada após a migração Kirchhoff.	29
8	Construção geométrica do princípio de Huygens. A fonte A emite a frente de onda B . Cada ponto D desta frente de onda atua como uma nova fonte emissora, gerando as frentes de onda como em E . O envelope dessas curvas resulta na nova frente de onda C	30
9	(a) Modelo sintético de velocidade com três pontos difratores; (b) Decomposição de uma superfície refletora em um número finito de pontos difratores; (c) Superfície refletora suave e contínua, com duas quinas. (d-f) Seções <i>common-offset</i> resultantes a partir da modelagem Kirchhoff.	31

10	Princípio da condição de imagem $w(s, x, r)$. (a) A migração convencional considera somente as reflexões especulares provenientes da zona de Fresnel. (b) A modificação de w por $w_d(s, x, r)$ faz com que a migração considere somente a informação não especular proveniente de fora da zona de Fresnel. Modificado de Moser e Howard (2008).	38
11	Esquema para cálculo do raio da zona de Fresnel em meios isotrópicos. Modificado de Xu e Stovas (2017).	38
12	(a) Synthetic model consisting of a single scatterer and a horizontal reflector. (b) Resulting common-shot gather. Note that the diffraction hyperbola show weaker amplitudes compared to the reflection hyperbola.	41
13	Illustration of the interaction of different seismic pulses and the Rayleigh's criterion. (a) When the distance between events is sufficiently large, each event can be distinguished individually in the resultant pulse. (b) The Rayleigh's criterion represents the maximum limit at which it remains possible to observe each pulse individually. (c) Beyond the Rayleigh limit, it is not possible to distinguish each event, as the resultant appears as a single pulse.	42
14	Simplified graphical representation of scale separation in seismic imaging. The black line represents the wavelength ranges and the current imaging gaps. The industry has been making efforts to widen the data frequency band at the high and low ends (blue, green and red lines).	42
15	Flowchart for primary studies selection according to the PRISMA statement.	43
16	Variation of publications (blue histograms) and quantity of citations (black line) considering the WoS and the Scopus databases.	44
17	Co-authorship map using countries as the unit of analysis, normalized citation as weight, and year of publication as score. The relative distance of the cluster's nuclei indicates the proximity of the research groups, the thickness of lines indicates the collaboration's intensity, and the number of citations determines the node's size. (a) Network for the WoS database. Although the USA research groups represent the central leadership with a more extensive network of members (10 countries), China already occupies a prominent role for appearing with the highest number of citations in recent years in the period used to elaborate the map. (b) Network for the Scopus database. China and the United States share a leading role, connected with Australia.	47
18	Bibliographic Coupling map for the 61 works published in the last 5 years (2018-2022) from the WoS database using the documents as the unit of analysis and normalized citations as weight.	47

19	Bibliographic Coupling map for the 21 works published in the last 5 years (2018-2022) from the Scopus database using the documents as the unit of analysis and normalized citations as weight.	48
20	World map with the number of published articles per country (according to the first author's affiliation).	49
21	Co-authorship map for the 150 reviewed works using authors as the unit of analysis and normalized citation as weight. There are three main clusters (a, b, and c), where the relative distance of each author's nucleus indicates their proximity. . . .	51
22	Distribution of the reviewed articles according to (a) dimension of the dataset; (b) synthetic and/or field data applications; (c) first author's affiliation type; (d) use of GPR data.	52
23	Investigation depths of the examples shown in the analyzed works. The near surface depth range from a few meters to 1 km, and the oil exploration depth ranges from 1 km to 20 km.	52
24	Principle of the weight function $w(s,x,r)$. (a) Conventional migration only considers the specular reflections from the Fresnel zone. (b) The modification of w by $w_d(s,x,r)$ causes migration to only consider non-specular information from outside the Fresnel zone.	57
25	Geometric difference between reflections and diffractions in a medium with constant velocity. S and R are the positions of the source and receivers; A is the apex of the hyperbola; S' is the projection of the reflection on the subsurface; and D is the diffracting point. (a) Reflection geometry for a horizontal planar interface; (b) Diffraction geometry for a diffracting point. Note that the reflection can be understood as originating from a virtual point S' mirrored relative to the position of the reflector.	58
26	Schematic illustration for geometry of the multifocusing method for a circular reflector Σ . S and R are the source and receiver locations, ΔX^+ and ΔX^- are the offsets from the central point, X_0 is the image point, Σ^+ and Σ^- are the virtual wavefronts emitted by P , P is the intersection point between the normal and the paraxial rays, R^+ and R^- are the radii of curvature of Σ^+ and Σ^- , and β is the incident angle of the normal incident ray.	58
27	Illustration of a specular reflector Σ and a scatterer point D illuminated by a plane source wave P_S . (a) Depth section; (b) Time section. The reflected wave Σ^+ generates a planar response R , while the diffracted response H is hyperbolic.	59

28	Illustration of the CRS parameters. (a) R_{NIP} is the curvature radius of a virtual wavefront emitted by a source at NIP. (b) R_{N} is the curvature radius of a virtual wavefront emitted by the exploding reflector element CRS. The emergence angle is denoted as β_0	61
29	Illustration of the (a) zero-offset reflection and (b) zero-offset diffraction.	62
30	General flow chart diagram showing the main steps for generating the diffraction image.	76
31	Travel-time field for shot position $sx = 120$ m resulting from the <i>MRay</i> and <i>raymodel</i> functions for the toy model.	77
32	Reflectivity of the simple toy model.	77
33	Semblance and dip field obtained by the <i>slant local</i> function for the toy model. (a) Maximum semblance values for each image point; (b) vertical component of reflector normal (<i>dipY</i>).	78
34	Synthetic common-shot gather resulting from the <i>kirchhoffModeling</i> function for the toy model (source position $sx = 75$ m).	78
35	Comparison of conventional and diffraction-oriented prestack-depth Kirchhoff migration images generated by the <i>kirchhoffMigration</i> function for the toy model. (a) Migrated image using the conventional weighted Kirchhoff migration. The specular reflections are not suppressed, but the position of the scatterers is not well defined. (b) Migrated image using the diffraction-oriented weight ($w_d = 1 - w$), where diffractions are enhanced and specular reflections are suppressed, resulting in better resolution of scatterer positions.	79
36	Combined image of the conventional and diffraction-oriented prestack-depth Kirchhoff migrations. The image was generated by summing the conventional migration image with the diffraction migration image. The resulting image highlights both the specular reflections and non-specular diffractions, allowing for a more comprehensive interpretation of the subsurface structures.	79
37	(a) Velocity model of the synthetic example. (b) Reflectivity model with five small lenses and white noise.	79
38	Semblance and dip field obtained by the <i>slant local</i> function for the synthetic example. (a) Maximum semblance values for each image point; (b) horizontal component of reflector normal (<i>dipX</i>).	80
39	Synthetic common-shot gather resulting from the <i>kirchhoffModeling</i> function for the synthetic example (source position $sx = 600$ m).	80

40	Comparison of conventional and diffraction-oriented prestack-depth Kirchhoff migration images generated by the <i>kirchhoffMigration</i> function for the synthetic example. (a) Migrated image using the conventional weighted Kirchhoff migration. (b) Migrated image using the diffraction-oriented weight ($w_d = 1 - w$), where diffractions are enhanced. The arrows indicate the main diffractions positions.	81
41	Combined image of the conventional and diffraction-oriented prestack-depth Kirchhoff migrations for the synthetic model.	81
42	Comparative analysis of diffractions-oriented migration using different velocity models. (a) Migration image with the correct velocity model. (b) Migration image with a velocity model 5% higher than the correct velocity. (c) Migration image with a velocity model 5% lower than the correct velocity.	82

Lista de Tabelas

1	Research parameters applied in the database query.	44
2	Main contributions of the most cited articles from both databases (WoS and Scopus).	45
3	Main references from the co-citation analysis in descending order of total citations. To obtain these ten main references, a minimum limit of 34 citations was adopted for each reference pair within each article analyzed in WoS, and three citations for each reference pair within each article analyzed in Scopus (Total analysed referen- ces from WoS: 2209, and Scopus: 999).	46
4	30 most frequent words in the titles, abstracts, and keywords from the 150 articles of both databases (WoS and Scopus).	47
5	30 most frequent words in the titles, abstracts, and keywords from the 82 articles published in the last 5 years (2018-2022) of both databases (WoS and Scopus).	49
6	Main applications of the seismic diffractions imaging in the literature.	50
7	Parameters applied in the toy model simulation.	77
8	Parameters applied in the realistic synthetic example.	80

Lista de Símbolos e Abreviações

λ - Comprimento de onda sísmica

V_p - Velocidade da onda sísmica P

k - Módulo de compressibilidade

μ - Módulo de cisalhamento

ρ - Densidade do meio

V_s - Velocidade da onda sísmica S

Z - Impedância acústica

c - Velocidade sísmica

WEM - Wave equation migration

δ - Delta de Dirac

Ψ - Amplitude do campo escalar

PWD - Plane-wave destruction

MSSA - Multichannel singular-spectral analysis

SVD - Single value decomposition

SVMF - Space-varying media filter

CRS - Common-reflection surface

kNN - k-nearest neighbors

SVM - Support vector machine

GPR - Ground-penetrating radar

FWI - Full-waveform inversion

SLR - Systematic literature review

PRISMA - Preferred reporting items for systematic reviews and meta-analyzes

WoS - Web of Science database

ML - Machine learning

CNN - Convolutional neural network

NIP - Normal-incident-point

MT - Magnetotellurics

Sumário

Lista de Figuras	9
Lista de Tabelas	14
Sumário	17
1 Introdução	18
1.1 Objetivo Geral	20
1.2 Objetivos Específicos	20
1.3 Justificativa	20
1.4 Hipótese	21
1.5 Organização da Dissertação	21
2 Fundamentação Teórica	22
2.1 Ondas Sísmicas	22
2.2 Difrações Sísmicas	25
2.3 Migração Sísmica	28
2.3.1 Princípio de Huygens	30
2.3.2 Migração Kirchhoff	32
2.4 Separação e Imageamento de Difrações	34
2.4.1 Alteração da Condição de Imagem	37
3 Systematic literature review on seismic diffractions imaging	40
4 DiffraPy: an open-source Python software for seismic diffractions imaging	72
5 Conclusões e Considerações Finais	87
Referências Bibliográficas	89
A Principais Funções do Código DiffraPy	97

1 Introdução

A sismica de exploração é largamente utilizada para o estudo não invasivo da subsuperfície por meio de imagens geradas após a aquisição de sismogramas. O uso das ondas refletidas para obter imagens de estruturas rasas do meio se tornou um dos principais métodos geofísicos de exploração (Fehler e Huang, 2002). As chamadas reflexões especulares são geradas por estruturas contínuas, com dimensões maiores que o comprimento de onda. O termo “especularidade” se refere ao fato de que estas ondas obedecem às leis de Snell, ou seja, a energia completa da onda incidente é refletida em um ângulo igual e oposto ao de incidência. O espalhamento da energia pode ocorrer de duas formas diferentes, a depender da dimensão do objeto em relação ao comprimento da onda incidente λ . Caso o objeto seja muito menor do que λ , ocorre a difusão Rayleigh. Já no caso em que o objeto possui dimensões próximas ou superiores a λ , ocorre a difusão Mie (Lockwood, 2016). Estruturas geológicas como falhas, canais, cunhas e fraturas interagem com as ondas sísmicas e espalham a energia sísmica, o que resulta nas difrações. Esse tipo de onda é um fenômeno físico fundamentalmente diferente das reflexões, além de possuir amplitudes comparativamente menores e ser mais rapidamente atenuado pelo espalhamento geométrico (Klem-Musatov, 1994; Zhao et al., 2015). Trorey (1970) discute que a difração não é somente produzida pela borda de um refletor descontínuo, mas depende de toda a superfície refletora. Por este motivo, as amplitudes das difrações decaem mais rapidamente do que o previsto por um simples espalhamento inverso. As técnicas tradicionais de processamento sísmico ressaltam o campo de ondas proveniente de reflexões, consequentemente suprimindo as difrações (Khaidukov et al., 2004; Lin et al., 2021; Moser, 2012). Com isso, a informação de estruturas importantes para a interpretação de seções como heterogeneidades e descontinuidades é perdida durante o imageamento do meio (Lin et al., 2022b).

As diferenças dinâmicas e cinemáticas entre difrações e reflexões podem ser utilizadas para a separação dos dois campos de onda antes de se gerar a imagem final (Schwarz, 2019a). Para isso, é necessário utilizar processos que propositalmente ressaltam a energia não especular. Sob certas condições, é possível recuperar informações de objetos abaixo do tradicional limite de Rayleigh (metade do comprimento de onda sísmica) a partir das técnicas de processamento de difrações, o que permite a “super resolução” (Khaidukov et al., 2004; Sturzu et al., 2014; Zhang e Zhang, 2014; Zhao et al., 2019). Contudo, a capacidade de gerar imagens com detalhes abaixo do limite tradicional de reflexão ainda é uma questão em aberto, uma vez que são necessárias circunstâncias idealizadas (Khaidukov et al., 2004). Situações práticas dificultam a obtenção de tais níveis de resolução, como a presença de ruído de campos distantes nos dados. Assim, a disponibilidade de novas ferramentas para testar e reproduzir exemplos de imageamento de difrações são uma solução para que os pesquisadores investiguem os parâmetros que interferem na obtenção de altas

resoluções.

A aplicação de difrações como ferramenta para facilitar a interpretação de seções sísmicas é um tópico relativamente pouco explorado pela comunidade geofísica (Decker e Fomel, 2021). No entanto, sua relevância e contribuição para a área estão cada vez mais consolidadas, conforme demonstrado pelas referências na literatura científica. Trabalhos pioneiros se concentraram em separar este tipo de onda para identificar e delinear zonas de falhas (Hagedoorn, 1954; Kanasewich e Phadke, 1988; Krey, 1952; Kunz, 1960). Atualmente, as principais aplicações das técnicas de imageamento de difrações são a caracterização de falhas, fraturas e outras estruturas em subsuperfície (Decker e Fomel, 2021; Li et al., 2020a; Lowney et al., 2021a; Prosatov et al., 2021; Zhang et al., 2020), imageamentos ligados à indústria de óleo e gás (Decker et al., 2014; Lin et al., 2018b; Merzlikin e Fomel, 2017; Tyiasning et al., 2016; Zhao et al., 2019, 2015, 2020), investigações rasas (De Figueiredo et al., 2013; Delf et al., 2022; Maciel e Biloti, 2020) e análise de velocidades necessárias para diferentes técnicas de migração (Decker et al., 2017; Fomel et al., 2007; Santos et al., 2020). Apesar de algumas empresas oferecerem soluções comerciais para o processamento de imagens de difrações, a comunidade geofísica possui poucas opções de programas de código aberto disponíveis para essa finalidade. Um exemplo de programa aberto é o Madagascar, que inclui funções específicas para o processamento e análise de dados de difrações sísmicas (Fomel et al., 2013).

A revisão sistemática da literatura desempenha um papel fundamental no avanço do conhecimento científico ao proporcionar uma abordagem transparente e replicável para a busca, seleção e análise de artigos primários (Tranfield et al., 2003). Por meio de um processo rigoroso e padronizado, é possível responder a perguntas específicas e apresentar novos métodos. Uma das vantagens da revisão sistemática é a capacidade de identificar lacunas teóricas e sintetizar estudos relevantes relacionados a um tópico específico (Kitchenham et al., 2009). É possível realizar uma análise abrangente e comparativa, o que permite a identificação de tendências, convergências e discrepâncias nos resultados publicados (Castro e Cunha, 2021). Recentemente, a revisão sistemática da literatura tem sido aplicada em diversas áreas das geociências, como radar de penetração no solo (Castro e Cunha, 2021; Rasol et al., 2022), estudos de afloramentos relacionados à exploração de hidrocarbonetos (Marques Jr et al., 2020) e aplicação de técnicas de aprendizado de máquinas para a modelagem de propriedades físicas de reservatórios (Syed et al., 2022). Algumas publicações apresentam uma revisão do estado da arte no campo do processamento de difrações, como Kim et al. (2021); Li et al. (2021); Lin et al. (2021); Schwarz (2019a); Wang et al. (2020); Zhao et al. (2019). No entanto, até o momento, nenhuma dessas pesquisas utilizou a abordagem sistemática para investigar e sintetizar o conhecimento disponível sobre esse tema.

O objetivo deste trabalho é apresentar um programa desenvolvido em Python dedicado ao processamento de difrações sísmicas. Implementamos o método de imageamento apresentado por

Moser e Howard (2008), que envolve a modificação da condição de imagem da migração Kirchhoff por meio do uso de um peso isotrópico anti-estacionário. O programa possui funções pré-definidas que permitem executar as etapas necessárias para gerar a migração direcionada a difrações. O produto final do código consiste em duas imagens geradas simultaneamente: a migração Kirchhoff convencional e a migração com o peso modificado, em que as reflexões são suprimidas e o campo de ondas difratadas é enfatizado. Além disso, o programa inclui um modelador sintético que facilita a reprodução de testes e experimentos relacionados à técnica de processamento de difrações.

Adicionalmente, este trabalho também tem como objetivo realizar uma revisão abrangente do estado da arte no campo do imageamento de difrações sísmicas. Identificamos lacunas no conhecimento e na pesquisa relacionados ao tema e fornecemos uma visão geral dos diferentes métodos de processamento de difrações disponíveis. Realizamos uma análise quantitativa das publicações selecionadas e identificamos áreas potenciais para futuras pesquisas.

1.1 Objetivo Geral

Avaliar o uso da técnica de imageamento de difrações para a obtenção de imagens de alta resolução por meio do desenvolvimento de um programa aberto para a comunidade geofísica.

1.2 Objetivos Específicos

1. Realizar uma revisão sistemática da literatura sobre o imageamento sísmico de difrações;
2. Desenvolver um programa em Python que possua a modelagem e a migração sísmica convencional;
3. Implementar o imageamento de difrações no código a partir do método da alteração da condição de imagem da integral de Kirchhoff;
4. Testar a aplicação do programa em dados sintéticos para verificar a capacidade de recuperação de informações de estruturas espalhadoras.

1.3 Justificativa

O processamento sísmico tradicional ressalta a informação contida nas ondas refletidas e suprime as difrações para gerar imagens da subsuperfície. No entanto, as difrações possuem informações relevantes sobre a interação das ondas sísmicas com estruturas como quinas, falhas, fraturas ou pequenas variações na refletividade sísmica causadas pela presença de fluidos. As ondas difratadas são utilizadas principalmente para gerar imagens da subsuperfície e para a análise de velocidades no processos de migração.

Reflexões e difrações são fenômenos físicos fundamentalmente diferentes, fato que permite a sua separação em seções sísmicas antes da imagem final. Com isso, é possível gerar uma imagem da subsuperfície em que as difrações são ressaltadas. O imageamento de difrações, em tese, é capaz de recuperar detalhes abaixo do tradicional limite de Rayleigh; ou seja, metade do comprimento de onda. Entretanto, esse fenômeno só é alcançado sob certas condições. É preciso explorar melhor a aplicação das difrações para a recuperação de detalhes em diferentes condições para melhor compreender as limitações e potencialidades da técnica.

Separar as difrações do campo de ondas total e imageá-las separadamente não é um processo computacionalmente intensivo, e pode ser alcançado a partir de modificações nos passos comuns do processamento sísmico. Atualmente, as difrações são estudadas pela indústria de óleo e gás com a finalidade da exploração em áreas fraturadas, e algumas soluções comerciais já foram apresentadas a partir dessa técnica. No entanto, ainda são poucas as opções de imageamento de difrações oferecidas para a comunidade geofísica, como os pacotes disponíveis no programa Madagascar (Fomel et al., 2013).

1.4 Hipótese

A integração do imageamento de difrações no processamento sísmico convencional pode ser feito a partir de pequenas modificações em algoritmos tradicionais e é capaz de recuperar informações de objetos abaixo do limite de resolução sísmico. Isso pode ser feito a partir da alteração da condição de imagem na migração Kirchhoff.

1.5 Organização da Dissertação

Após a introdução ao tema e contextualização da pesquisa apresentadas no Capítulo 1, abordamos os conceitos básicos sobre difrações sísmicas no Capítulo 2, assim como a teoria sobre a migração Kirchhoff e o método de imageamento proposto por Moser e Howard (2008).

No Capítulo 3, apresentamos os resultados de uma revisão sistemática de literatura sobre o imageamento de difrações sísmicas no formato de artigo científico. Realizamos uma análise quantitativa dos trabalhos selecionados para fornecer uma visão abrangente dos métodos propostos.

O Capítulo 4, também no formato de artigo científico, introduz o programa de processamento de difrações desenvolvido nesta pesquisa. Apresentamos as funcionalidades do código e testamos sua aplicação em dados sintéticos. A partir de um modelo sintético mais complexo, discutimos as aplicações, limitações e potencialidades do programa.

Por fim, no Capítulo 5, realizamos uma síntese dos resultados obtidos nos artigos, além de apresentar as conclusões e considerações finais.

2 Fundamentação Teórica

2.1 Ondas Sísmicas

Um dos principais objetivos da geofísica é obter imagens da subsuperfície e caracterizar o meio de forma indireta e não invasiva. O início da utilização de fontes artificiais para o estudo dos efeitos das ondas geradas pode ser atribuído a Robert Mallet, em 1846 (Dean, 1991). No entanto, os maiores avanços na instrumentação e aplicação dos métodos sísmicos ocorreram durante a revolução digital, em 1960. De acordo com Fehler e Huang (2002), o uso de ondas refletidas para obter informações de porções rasas da estrutura da Terra se tornou um dos principais métodos de exploração, com uma grande quantidade de aplicações e técnicas estudadas.

A base dos estudos sísmicos consiste no entendimento da propagação das ondas no interior da Terra e sua visualização em sismogramas. Ondas sísmicas são pacotes de energia de deformação liberados por uma fonte, natural ou artificial, cuja velocidade depende de parâmetros como o módulo de elasticidade e densidade do meio. A interação dessas ondas com as estruturas em subsuperfície é registrada por receptores e resultam em sismogramas que podem ser transformados em modelos realísticos do meio.

São dois os tipos de ondas que podem se propagar através do volume interno de meios elásticos. O primeiro tipo é o das ondas P, também conhecidas como compressionais, primárias, de dilatação ou longitudinais. A outra categoria de ondas de corpo são as ondas S, também conhecidas como secundárias, de cisalhamento ou transversais. O movimento das partículas associadas às ondas P envolve a oscilação de um ponto fixo na direção da propagação da onda (Figura 1a). Já o movimento associado às ondas S envolve a oscilação de um ponto fixo em um plano perpendicular à direção de propagação (Figura 1b). Essa propagação das ondas S ao longo de um plano permite sua polarização, dividindo-a em uma componente polarizada vertical (S_v) e uma polarizada horizontal (S_h).

A velocidade de propagação de uma onda de corpo em qualquer material homogêneo e isotrópico pode ser descrita pela relação entre suas propriedades elásticas e sua densidade. A velocidade da onda P é dada por:

$$V_p = \left(\frac{k + \frac{4}{3}\mu}{\rho} \right)^{1/2}, \quad (1)$$

em que k é o módulo de compressibilidade, μ é o módulo de cisalhamento, um dos coeficientes de Lamé, e ρ é a densidade do meio. Como a onda S envolve somente cisalhamento, a velocidade V_s é dada por:

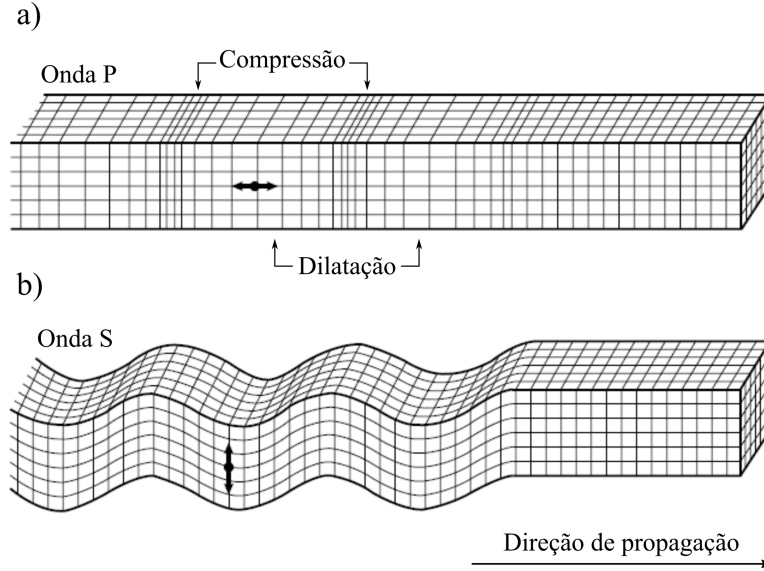


Figura 1: (a) Bloco tridimensional ilustrando a propagação de uma onda P: a partícula se move na direção da propagação da onda. (b) Na onda S, a partícula se movimenta em um plano perpendicular à direção de propagação.

$$V_s = \left(\frac{\mu}{\rho} \right)^{1/2}. \quad (2)$$

Já as ondas de superfície são praticamente restritas à propagação na superfície de meios elásticos, possuem menor velocidade que as ondas de corpo e guardam grande quantidade de energia da fonte. Possuem essa nomenclatura pelo fato de não penetrarem muito profundamente em subsuperfície. As ondas Rayleigh viajam ao longo da superfície da Terra como um pacote, com amplitudes que decaem exponencialmente com a profundidade. O movimento de partículas associado é elíptico retrógrado (Figura 2a), resultado da interferência entre as ondas P e Sv. Há uma dispersão da onda, em que a vibração ocorre em diferentes profundidades e frequências. Caso exista uma camada de baixa velocidade sobreposta a uma camada de alta velocidade, o segundo pacote de ondas é gerado, resultante da interferência das ondas S polarizadas horizontalmente (Sh), chamado de ondas Love. Se propagam paralelamente à superfície e perpendicularmente à direção da onda (Figura 2b).

Um pulso sísmico emitido por uma fonte natural ou artificial se propaga em velocidades determinadas pelas propriedades físicas do meio em que se encontra. Por exemplo, caso o pulso se propague em um meio homogêneo e isotrópico, terá a mesma velocidade em todas as direções. Para que uma onda seja refletida, é preciso que interaja com uma interface ou objeto que apresente contraste de impedância acústica Z . A impedância acústica é o produto da velocidade sísmica c

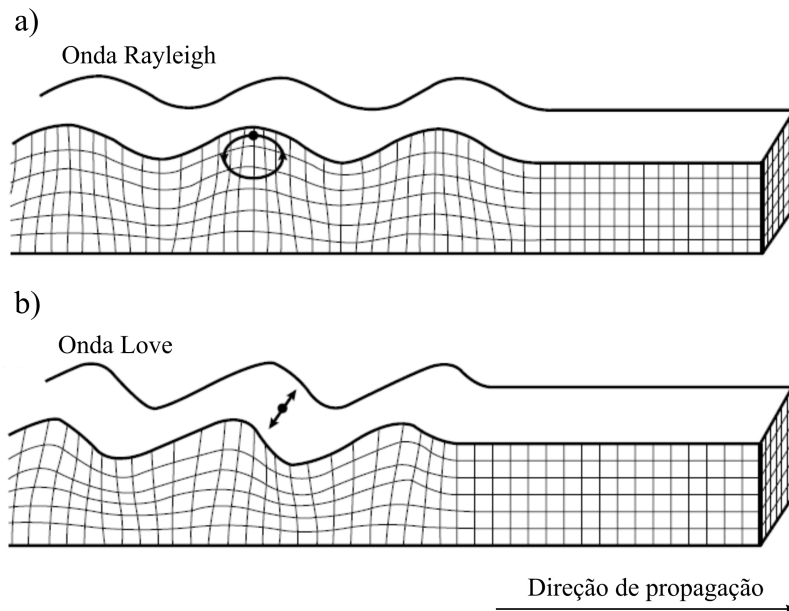


Figura 2: (a) Bloco tridimensional ilustrando a propagação das ondas Rayleigh: a partícula apresenta um movimento elíptico retrógrado em um plano perpendicular à superfície. (b) Nas ondas Love, a partícula se movimenta paralelamente à superfície e perpendicularmente à direção de propagação.

pela densidade ρ de determinada camada ou interface. A amplitude da onda refletida é descrita pelo coeficiente de refletividade.

Se a velocidade for maior em uma camada subjacente, ocorrerá um ângulo de incidência específico conhecido como ângulo crítico θ_c . Quando o ângulo de incidência é igual a θ_c , o ângulo de refração é de 90° . Consequentemente, o raio refratado irá viajar ao longo da interface entre as duas camadas. A passagem do raio refratado provoca uma perturbação na camada superior, o que resulta na geração de ondas de choque. No caso sísmico, essas ondas são denominadas “ondas frontais” e se propagam de maneira oblíqua pela camada superior em direção à superfície. A sismica de refração explora essas propriedades para investigar a geometria e a velocidade das camadas subsuperficiais.

A difusão é um fenômeno que resulta na mudança da trajetória da propagação da onda causada por obstáculos ou pequenas variações do meio. A difração é um caso particular da difusão, relacionado a perturbações com dimensões comparáveis ao comprimento de onda (Weill, 2014). Neste trabalho, utilizaremos o termo “difração” para se referir ao espalhamento da onda sísmica causado por discontinuidades na subsuperfície.

Até o momento, assumimos que as superfícies são suficientemente contínuas, suaves e planas para discutir os fenômenos de reflexão e refração. No entanto, quando as ondas sísmicas interagem

com objetos, descontinuidades, bordas, quinas ou estruturas com raio de curvatura menor que o comprimento de onda incidente, as leis de Snell não são mais válidas. Isso resulta em fenômenos conhecidos como difrações sísmicas.

2.2 Difrações Sísmicas

A interação das ondas sísmicas com diferentes interfaces ou obstáculos resulta em eventos distintos. Difrações e reflexões são fenômenos fundamentalmente diferentes (Khaidukov et al., 2004; Klem-Musatov, 1994). Enquanto as reflexões são geradas por interfaces suaves e contínuas, difrações são resultado da interação da onda com pequenas estruturas e não satisfazem as condições gerais de validação da teoria de raios apresentadas por Červeny (2001). Segundo Moser (2012), as ondas refletidas podem ser referidas como “ondas especulares” pelo fato de obedecerem à lei de Snell, o que não ocorre com as ondas difratadas. A reflexão resultante da incidência de uma onda em uma superfície suave é compreendida a partir da aplicação da lei de Snell, em que o ângulo de incidência é equivalente ao ângulo do raio refletido (Figura 3a). Assim, este raio é definido e parametrizado por um único critério, como o tempo de percurso da onda. Como as difrações não especulares são um espalhamento da energia da onda incidente em vários raios, é necessária mais de uma propriedade para parametrizar os raios resultantes, e.g., o tempo de percurso da onda e os ângulos de saída (Figura 3b).

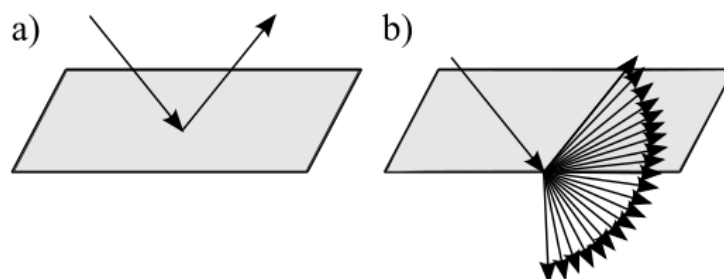


Figura 3: Comportamento de raios sísmicos (a) refletidos e (b) difratados em uma superfície. Modificado de Moser (2012).

Portanto, o comportamento cinemático dos dois tipos de onda é diferente (Torey, 1970). Reflexões e difrações provenientes de pontos com a mesma profundidade irão se manifestar de forma diferente em seções. Além disso, a posição do ápice da hipérbole também sofrerá alterações em cada conjunto de tiros (*shot gather*) (Khaidukov et al., 2004). Para exemplificar, consideramos o simples caso de um refletor horizontal e velocidades constantes, como na Figura 4a. Deste modo, o ápice da hipérbole de reflexão sempre será na posição de *offset* igual a zero, i.e., logo abaixo da posição da fonte. Se a fonte muda de posição, o mesmo ocorre com o ápice da hipérbole, acompa-

nhando a fonte. Já a hipérbole gerada por um ponto difrator em um meio de velocidade constante possui o ápice fixado acima deste ponto; a hipérbole de difração não se move lateralmente (Figura 4b). Outra diferença fundamental é que a energia de reflexão pode ser compreendida como gerada por uma fonte secundária em subsuperfície (pontos S' , Figura 4a), fato que não ocorre com as difrações (Keller, 1962).

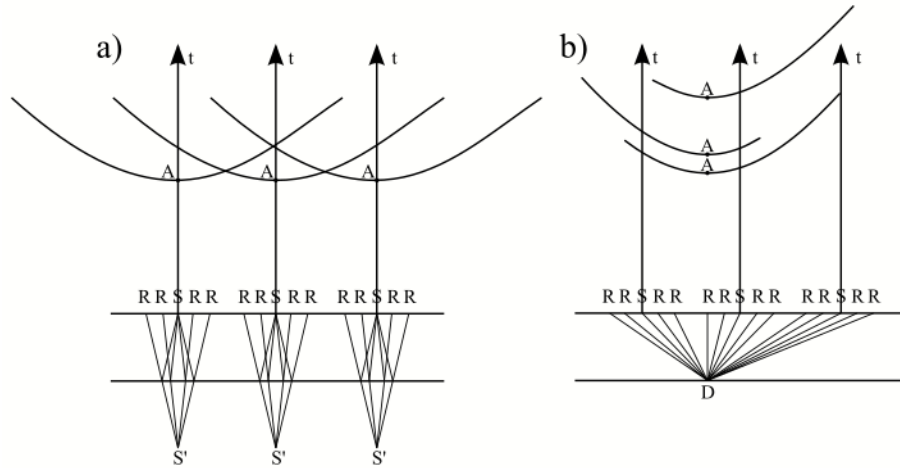


Figura 4: Diferença geométrica entre reflexões e difrações em um meio de velocidade constante. S e R são as posições da fonte e dos receptores; A o ápice da hipérbole; S' a projeção da reflexão em subsuperfície; e D o ponto difrator. (a) Geometria de reflexões para uma interface planar horizontal; (b) Geometria de difrações para um ponto difrator. Note que a reflexão pode ser compreendida como originada por um ponto S' espelhado em relação à posição do refletor. Modificado de Khaidukov et al. (2004).

Além disso, a energia das difrações é consideravelmente menor que a de reflexões, que pode chegar a uma ou duas ordens de diferença de magnitude (Klem-Musatov, 1994). A Figura 5 demonstra essa propriedade a partir de modelos sintéticos simples, com velocidade constante $v=1000$ m/s (Figuras 5a a 5c), e as seções de tiro comum (*common-shot*) resultantes (Figuras 5d a 5f). Geramos três diferentes modelos de refletividade: um refletor plano horizontal na posição $z=1000$ m; um ponto difrator na posição $x, z=1200, 500$ m; e a combinação entre esses modelos. Simulamos a reposta esperada de uma fonte explosiva no centro do modelo a partir da modelagem Kirchhoff. Ao comparar as amplitudes das hipérboles resultantes (Figuras 5d e 5e), notamos a diferença de uma ordem de magnitude (0.005 e 0.0005). Para a seção apresentada na Figura 5f, observamos que a hipérbole gerada pelo ponto difrator é pouco visível, devido às menores amplitudes comparadas à reflexão. Dessa forma, essas propriedades fazem com que seja possível separar o campo de ondas total em uma componente composta por reflexões e uma composta por difrações.

As reflexões só recuperam informações de refletores acima do tradicional limite de Rayleigh, separação mínima entre eventos sísmicos que permite sua distinção (Figura 6). Este limite é nor-

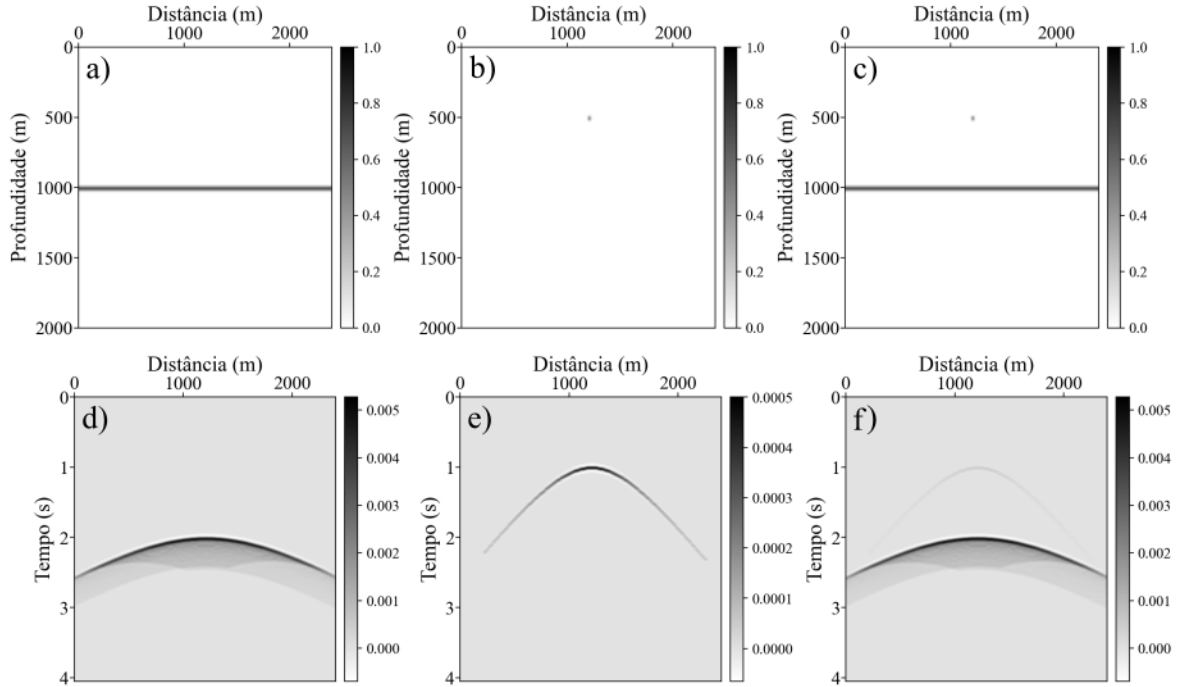


Figura 5: Modelos sintéticos de refletividade (a-c) e seções *common-shot* resultantes a partir da modelagem Kirchhoff (d-f).

malmente definido como metade do comprimento de onda sísmico, ou seja, possui relação com a frequência de aquisição ou a resolução temporal do sinal. As difrações, em teoria, são capazes de recuperar informações de estruturas abaixo deste limite, o que permite a “super resolução” (Neidell, 1997; Zhang e Zhang, 2014; Zhao et al., 2019). O processamento de difrações para aumentar a resolução de imagens sísmicas é um tópico de pesquisa relativamente novo, porém já consolidado na literatura. Os esforços iniciais do uso de difrações se concentraram na identificação e delimitação de falhas, como feito por Kanasewich e Phadke (1988), Krey (1952), Kunz (1960) e Trorey (1970). Berryhill (1977) determinou o comportamento dinâmico de difrações de borda. Harlan et al. (1984) estimaram velocidades do meio usando modelagens sintéticas e introduziram técnicas de análise de velocidade a partir de difrações. Vries e Berkhout (1984) usaram o conceito de entropia mínima para avaliar o método de focar respostas difratoras e extrair velocidades. Landa et al. (1987) propuseram seções *common-diffraction point* (*D-sections*) por meio do cálculo de coerência ao longo de hipérboles geradas por estruturas espalhadoras. Neidell (1997) diferenciou as contribuições dos campos de ondas difratado e refletido e utilizou a componente difratada para estabelecer maiores resoluções. Landa e Keydar (1998) extraíram informações de seções sísmicas para imagear difrações e identificar heterogeneidades do meio geradas por atividades humanas ou rápidos processos geológicos. Papziner e Nick (1998) também aplicaram difrações para detectar

heterogeneidades em porções rasas da subsuperfície. Bansal e Imhof (2005) estudaram diferentes módulos para separar o campo de ondas difratados em diferentes domínios.

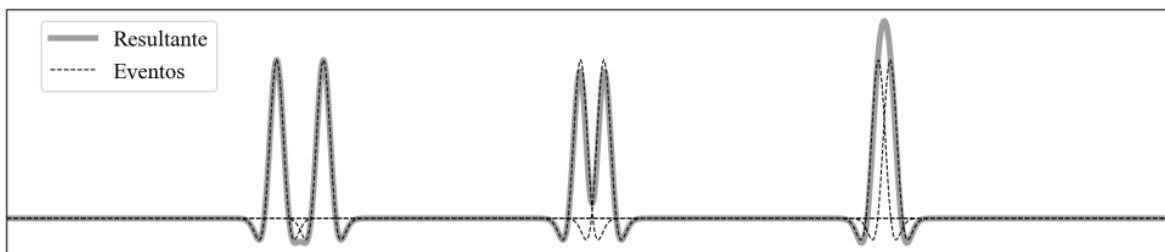


Figura 6: Diferentes situações de interação entre pulsos sísmicos de diferentes eventos. À esquerda, a distância entre os pulsos torna possível distingui-los separadamente no pulso resultante. No meio, ocorre o limite de Rayleigh, em que ainda é possível distinguir os pulsos individualmente. À direita, a distância não permite a distinção, em que a resultante é um único pulso.

As atuais aplicações das difrações estão principalmente ligadas à incorporação de análise de velocidades associada a técnicas de migrações sísmicas (Fomel et al., 2007; Lin et al., 2020, 2018a; Reshef e Landa, 2009; Sava et al., 2005) e ao imageamento de pequenas estruturas em sismogramas (Bauer et al., 2019; Decker et al., 2014; Khoshnavaz et al., 2016; Li et al., 2020b; Maciel e Biloti, 2020; Schwarz, 2019a; Tyiasning et al., 2016), especialmente ligadas a reservatórios de óleo e gás (Merzlikin e Fomel, 2017; Schwarz e Krawczyk, 2020; Tyiasning et al., 2016). Estudos também foram direcionados ao imageamento de complexos de massa de transporte (*mass-transport complexes*, depósitos gerados por deslizamentos de terra subaquáticos) (Ford et al., 2021), redes complexas de falhas e fraturas (Schwarz e Krawczyk, 2020), inversões relacionadas à tomografia sísmica (Bauer et al., 2017; Santos et al., 2012), detecção de estruturas relacionadas à mineração de carvão (Zhou et al., 2017), monitoramento de captura de CO₂ (Alonaizi et al., 2014) e estudos de estruturas internas de geleiras (Delf et al., 2022).

2.3 Migração Sísmica

A migração sísmica é a etapa do processamento que permite o imageamento da subsuperfície a partir do colapso da energia espalhada ao longo das hipérboles ao seu ponto de origem (Yilmaz, 2008). Esta etapa converte a informação de reflexões e outros eventos contida nos traços em uma imagem que representa o meio. Um dos principais objetivos do imageamento sísmico é obter imagens realísticas do meio a partir de processamentos aplicados aos dados adquiridos. Uma seção sísmica convencional não representa a imagem de subsuperfície, pois é composta pelo registro do tempo de chegada das ondas nos receptores. As amplitudes geradas por um ponto difrator captadas por um hipotético conjunto de receptores irão formar uma hipérbole (Figura 7b). Assim,

é necessário empilhar a energia que está espalhada ao longo da hipérbole no ponto em que foi gerada. A migração é a operação que retorna a imagem esperada a partir de uma seção sísmica, pois empilha a energia das difrações elementares aos seus respectivos pontos emissores (Figura 7c). Dessa forma, os eventos passam do domínio do tempo (traços sísmicos) para o domínio da profundidade (Tygel et al., 1994).

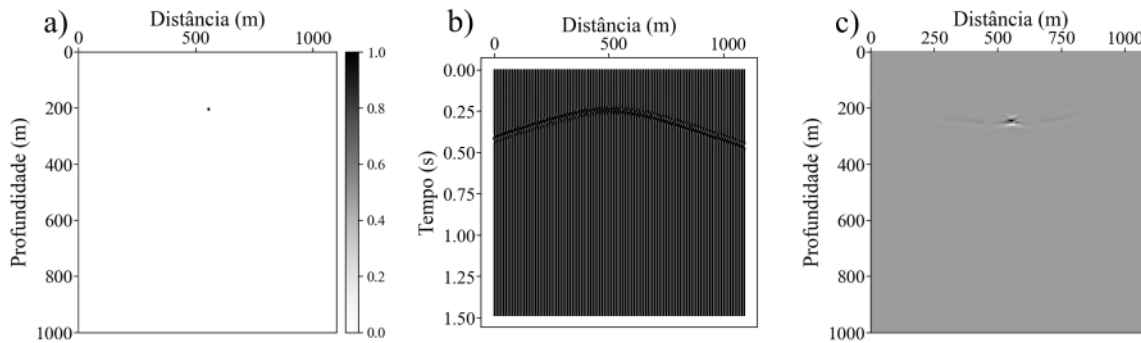


Figura 7: Exemplo de migração sísmica. (a) Modelo sintético de uma seção de tiro comum com um ponto difrator; (b) traços sísmicos resultantes da modelagem Kirchhoff; (c) imagem recuperada após a migração Kirchhoff.

Há duas classes gerais de algoritmos utilizadas para a migração de dados sísmicos (Bednar, 2005). A primeira é baseada na solução da equação da onda em sua forma completa, conhecida como “migração da equação de onda” (*wave equation migration* - WEM). O objetivo é obter uma solução que satisfaça a equação completa da onda. Isso pode ser feito no domínio do tempo, por meio do método de diferenças finitas (Mulder e Plessix, 2004), ou no domínio da frequência, a partir da extrapolação do campo de ondas usando a equação do tempo de ida (*one-way*) ou de ida e volta da onda (*two-way*) (Fomel e Kaur, 2021; Gazdag, 1978; Stolt, 1978).

Mulder e Plessix (2004) ressaltam que apesar de resultarem em imagens que representam a subsuperfície com maior acurácia, os algoritmos da migração da equação de onda possuem um alto custo computacional. São necessárias elevadas taxas de tempo de processamento que podem ser irrealísticas para aplicações em dados de larga escala (Zhu e Lines, 1998).

A segunda categoria de algoritmos é a migração Kirchhoff, que utiliza o princípio de Huygens (Berryhill, 1979; Schneider, 1978). Além de considerar o meio como uma malha de pontos espalhadores, esta categoria faz uma aproximação da integral da equação da onda a partir de duas suposições. A primeira é que os dados devem satisfazer a suposição de alta frequência, e a segunda é que o campo de ondas espalhado deve estar linearmente relacionado ao campo de ondas incidente por meio de um fator denominado “refletividade”.

2.3.1 Princípio de Huygens

O princípio de Huygens foi proposto por Christiaan Huygens em seu estudo “*Traité de la Lumière*” (Tratado da Luz), em que o autor apresenta discussões sobre a propagação ondulatória da luz e fenômenos como reflexão e refração (Huygens, 1690).

Quando qualquer tipo de onda se propaga, cada ponto da frente de onda age como um novo ponto emissor de energia (Anderson, 2021), como ilustrado na Figura 8. Ou seja, a frente de onda é compreendida como o envelope da energia de fontes secundárias; a energia se interfere de maneira construtiva ao longo da frente de onda e destrutiva fora dela (Enders, 1996). A partir disso, é possível compreender uma superfície refletora como composta por infinitesimais pontos emissores de frentes de onda, chamadas de “difrações elementares”.

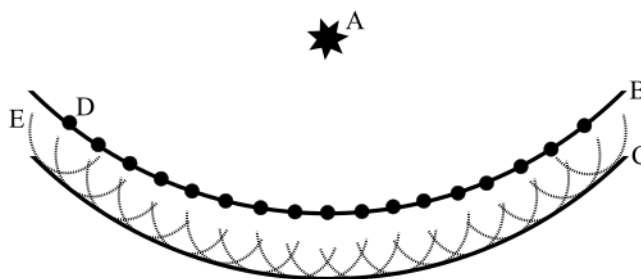


Figura 8: Construção geométrica do princípio de Huygens. A fonte A emite a frente de onda B. Cada ponto D desta frente de onda atua como uma nova fonte emissora, gerando as frentes de onda como em E. O envelope dessas curvas resulta na nova frente de onda C.

A teoria proposta por Huygens descreve a hipérbole de reflexão sísmica como o envelope de difrações elementares, também chamadas “ondas de Huygens” (Hubral et al., 1996b), que se interferem construtivamente ao longo da frente de onda. Esta propriedade é ilustrada na Figura 9. A Figura 9a é um modelo sintético com três pontos difratores. Abaixo, a Figura 9d mostra a seção sintética resultante de uma hipotética aquisição *common-offset*, em que as respectivas hipérboles de difração são visíveis. Ao aumentar o número de pontos difratores do modelo (Figura 9b), a imagem do refletor começa a se tornar nítida, mas ainda é possível observar as hipérboles de difrações reais individualmente (Figura 9e). No caso de uma superfície contínua (Figura 9c), a seção resultante mostra o envelope das difrações elementares, que dá origem à reflexão (Figura 9f).

A teoria de Huygens aplicada para o imageamento sísmico é válida tanto para refletores suaves como para quinas e discontinuidades. No entanto, conforme apontado por Moser e Howard (2008), há importantes diferenças entre as difrações elementares previstas na teoria e difrações reais. As difrações elementares são idealizações matemáticas que se interferem construtivamente ao longo da frente de ondas. Já as difrações reais não são somente conceitos matemáticos, mas o resultado da interação da onda sísmica com pequenos objetos ou variações do meio. Ao contrário de difrações

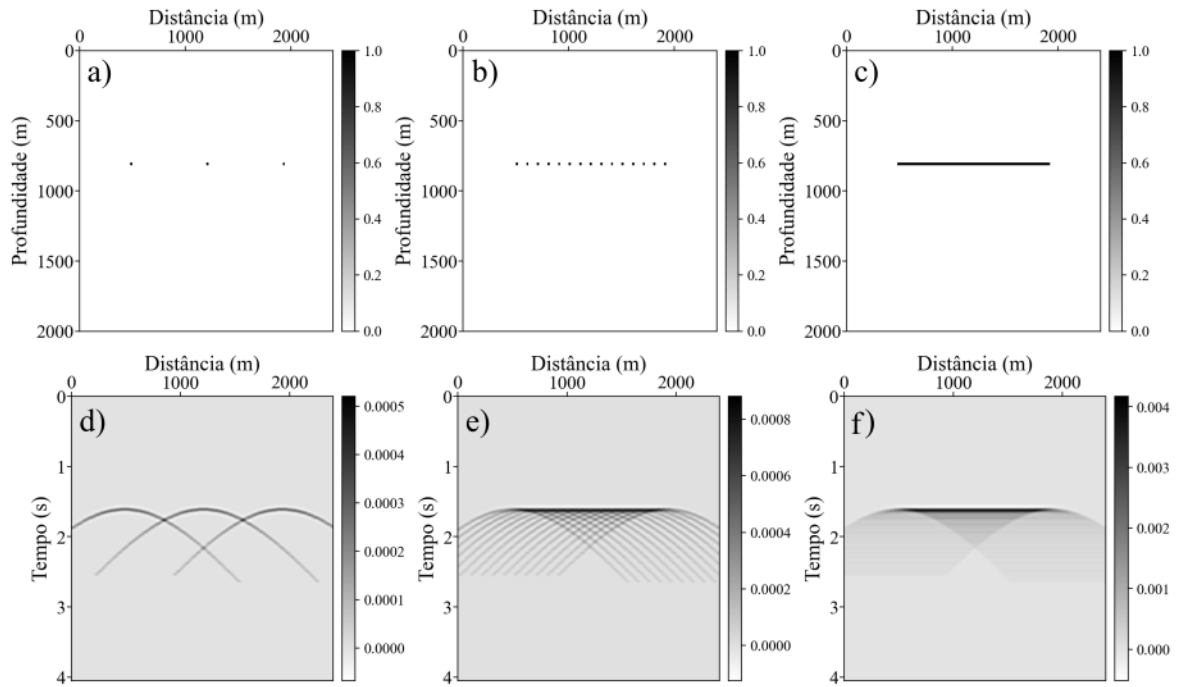


Figura 9: (a) Modelo sintético de velocidade com três pontos difratores; (b) Decomposição de uma superfície refletora em um número finito de pontos difratores; (c) Superfície refletora suave e contínua, com duas quinas. (d-f) Seções *common-offset* resultantes a partir da modelagem Kirchhoff.

elementares, a energia individual das difrações reais é observável em seções sísmicas.

2.3.2 Migração Kirchhoff

O princípio de Huygens explica a formulação proposta por Hagedoorn na década de 1950 para compreender reflexões sísmicas (Hagedoorn, 1954). O autor apresentou um método que utiliza régua e compassos para definir reflexões como o envelope de curvas de tempo de trânsito, um dos trabalhos pioneiros para o imageamento sísmico a partir da noção de migração (Bednar, 2005). Foi somente após a revolução digital iniciada em 1960 que a implementação computacional de um algoritmo de migração se tornou possível. Com isso, as primeiras soluções comerciais de migrações baseadas no trabalho de Hagedoorn surgiram no fim desta década (Schneider, 1978). Apesar da aplicação do método ser distinta da migração Kirchhoff, as duas abordagens possuem semelhanças por se basearem no princípio de Huygens para compreender as reflexões sísmicas.

A integral de Kirchhoff utilizada na migração sísmica é baseada na solução da equação da onda em sua forma simplificada para o caso 3D (Enders, 1996):

$$\Delta\Psi - \frac{1}{c^2} \frac{\partial^2 \Psi}{\partial t^2} = -q(r, t), \quad (3)$$

em que $\Psi(r, t)$ é a amplitude do campo escalar, $c = c(x)$ é a velocidade acústica do meio e $q(r, t)$ é a densidade da fonte. A Equação 3 também é chamada de equação da onda acústica e descreve a propagação de ondas compressionais.

A função de Green para a Equação 3 é a sua solução para fontes unitárias:

$$\Delta g - \frac{1}{c^2} \frac{\partial^2 g}{\partial t^2} = -\delta(r - r_0)\delta(t - t_0). \quad (4)$$

Uma possível solução para a Equação 4 é a expansão da onda esférica impulsiva em um espaço livre (Schneider, 1978):

$$g_f(r, t | r_0, t_0) = \frac{\delta(R/c - \tau)}{4\pi R}; \quad (5)$$

$$R \equiv |r_0 - r|; \quad \tau = t - t_0.$$

Ao utilizar a Função de Green determinada na Equação 5, a solução geral para a Equação 4 pode ser escrita como:

$$\begin{aligned}
\Psi(r, t) = & \int_0^t dt_0 \iiint g_f(r, t | r_0, t_0) q(r_0, t_0) dV_0 \\
& - \frac{1}{c^2} \iiint \left[\left(\frac{\partial g_f}{\partial t_0} \right)_{t_0=0} \Psi(r_0, 0) - g_f(r, t | r_0, 0) \frac{\partial \Psi}{\partial t}(r_0, 0) \right] dV_0 \\
& + \int_0^t dt_0 \iint (g_f(r, t | r_0, t_0) \nabla_0 \Psi(r_0, t_0) - \Psi(r_0, t_0) \nabla_0 g_f(r, t | r_0, t_0)) dS_0.
\end{aligned} \tag{6}$$

O primeiro termo da Equação 6 descreve a propagação de partes do campo de amplitudes que se originam de fontes externas, enquanto o segundo e o terceiro item descrevem a propagação de campos originados das condições iniciais e de borda, respectivamente (Enders, 2009). Ao terceiro item é dado o nome de “integral de Kirchhoff”, que resulta na informação espacial da fonte emissora.

No caso da aplicação da integral de Kirchhoff para a migração de dados sísmicos, é possível simplificar o terceiro termo da Equação 6 na seguinte forma (Tygel et al., 1996):

$$V(x) = \int d\xi \int dt U(\xi, t) \delta(t - t_d(\xi, x)), \tag{7}$$

em que $U(\xi, t)$ é o dado sísmico parametrizado pelo vetor ξ ; δ é distribuição delta de Dirac; e t_d é a curva do tempo de trânsito, ou seja, o tempo de propagação de uma difração elementar no ponto x .

Por compreender as superfícies refletoras como compostas por diversos pontos difratores, a integral de Kirchhoff também é chamada de *stack* de difrações, uma vez que faz o empilhamento da energia de difrações elementares. Moser e Howard (2008) ressaltam que, ao contrário do que o nome indica, o *stack* de difrações resalta a informação das reflexões em detrimento às difrações reais. Dessa forma, a integral de Kirchhoff possui um viés para a energia de reflexões; há uma perda da energia de difrações após esta operação.

2.4 Separação e Imageamento de Difrações

Wang et al. (2020) discutem que as ondas difratadas são tratadas como ruído em fluxos tradicionais de processamento, fato que dificulta a obtenção de imagens de alta resolução da subsuperfície. O campo difratado é mascarado pelas reflexões devido à diferença de magnitude entre os dois tipos de ondas (Klem-Musatov, 1994). Além disso, como discutido por Moser e Howard (2008), técnicas usuais de imageamento sísmico, como a migração Kirchhoff, possuem um viés para reflexões, suprimindo a energia das difrações não especulares. A partir das diferenças do comportamento dinâmico e cinemático entre reflexões e difrações, é possível separar o campo total de ondas em duas componentes. Diferentes técnicas foram apresentadas pelos pesquisadores para obter imagens em que as difrações não especulares são destacadas e reflexões especulares são suprimidas. Autores classificam os métodos de imageamento de difrações por meio da separação de reflexões em duas (Kim et al., 2021; Lin et al., 2021; Schwarz, 2019a; Wang et al., 2020) ou três (Li et al., 2021; Zhao et al., 2019) diferentes categorias ao considerar as diferentes abordagens matemáticas utilizadas. Neste trabalho, iremos considerar duas categorias distintas.

A primeira inclui os métodos que separam as difrações das reflexões antes ou após o processo de migração, no domínio do dado. Faz parte desse grupo o método de separação *local slant-slack* apresentado por Harlan et al. (1984). Diversos estudos exploram filtros de destruição de onda plana (*plane-wave destruction* ou PWD), como Decker et al. (2017), Fomel (2002), Fomel et al. (2007), Li et al. (2022a), Li et al. (2022b), Lin et al. (2018c), Lowney et al. (2020), Taner (2006) e Yuan et al. (2019). O método de subtração coerente realiza a soma das amplitudes provenientes de reflexões e as separa das difrações a partir de filtros de frentes de onda (Schwarz, 2019b; Schwarz e Gajewski, 2017). Lin et al. (2020) apresentam o método da análise multicanal de espectro (MSSA, *multichannel singular-spectral analysis*), em que as reflexões são suprimidas no domínio da frequência-espaco ($f-x$) a partir da análise das características lineares da energia de reflexões (Lin et al., 2022a; Zhou et al., 2023); posteriormente, o estudo de Shen et al. (2020) aprimorou o método por meio da técnica *singular value decomposition* (SVD), em que os autores utilizam um filtro baseado nas diferenças de coerência e dos tempos de trânsito entre os campos de ondas. Jiang et al. (2022) propuseram uma melhoria para o método SVD ao utilizar uma estratégia de espectro de diferença de segunda ordem. Outra técnica que utiliza o domínio $f-x$ é o *variational mode decomposition* (Lin et al., 2021), que considera os aspectos esparsos da distribuição da energia dos eventos. Estudos também utilizam filtros no domínio da frequência - número de onda ($f-k$) para separar as difrações. Dell et al. (2018) utilizam filtros no domínio $f-k$ para separar o modelo de eventos de ondas planas e posteriormente subtrai-lo do campos de onda total. Bashir et al. (2018, 2020) também usam filtros no domínio $f-k$, porém com posterior filtro PWD no domínio do tempo-espaco ($t-x$) para separar as difrações. Zhao et al. (2019) realizam a separação dos eventos no domínio do tiro por meio do filtro de variação no espaco (*space-varying media*

filter ou SVMF); o domínio no tiro também é conhecido como *common virtual source gather* (Li et al., 2021). Decker e Fomel (2021) apresentaram uma técnica para ser aplicada em conjuntos de dados em que a energia de reflexões já foi suprimida. Neste estudo, é feita a soma dos possíveis sinais gerados por diferentes velocidades de migração (*path integral seismic imaging*) (Merzlikin e Fomel, 2017; Merzlikin et al., 2019, 2020) de forma a acentuar a energia gerada por difratores em subsuperfície.

A segunda categoria inclui os métodos que separam as difrações diretamente durante o processo de migração. A vantagem desta categoria é que grande parte das técnicas apresentadas podem ser implementadas a partir de pequenas modificações na equação de migração. Além disso, a implementação representa um baixo custo computacional, pois não representa uma alteração brusca nos algoritmos de processamento convencionais. Landa et al. (1987) criaram seções de difrações (*D-sections*) a partir de correlações de fase; Kanasevich e Phadke (1988) construíram gathers de *common-fault-points* a partir da distribuição de amplitudes de difrações; Landa e Keydar (1998) criaram seções de *common-diffraction-point* para ressaltar a amplitude das difrações e detectar heterogeneidades. Khaidukov et al. (2004) modificaram o termo do tempo de trânsito de difrações elementares no algoritmo de migração para criar um domínio em que é possível focar a energia de reflexões e posteriormente mutá-las do campo de ondas difratado. Berkovitch et al. (2009) também realizaram pequenas modificações na equação de migração para criar o método *multifocus* que, analogamente ao método anterior, foca a energia de difrações baseado na correção do tempo de trânsito da integral de migração. Moser e Howard (2008) desenvolveram um filtro anti-estacionário a partir da modificação da função de peso na integral de Kirchhoff para suprimir a energia de reflexões e ressaltar as difrações. Li et al. (2020b) modificaram este filtro por meio da análise das fases dos sinais e posterior reversão de polaridade de modo a reduzir a energia residual de reflexões. Zhang e Zhang (2014) propuseram mutar a energia de reflexões da zona de Fresnel nos *gathers* de tiro e ângulos de abertura. Li et al. (2020c), Zhao et al. (2016) e Zhao et al. (2020) utilizaram a distância Mahalanobis para incluir uma função exponencial na integral de Kirchhoff de forma a suprimir as amplitudes de reflexões. Dell e Gajewski (2011), Rad et al. (2018) e Schwarz et al. (2014) propuseram o método *common-reflection-surface*, baseado na diferença de curvatura entre respostas de reflexões e difrações. Os métodos baseados na transformada de Radon (Klokov e Fomel, 2012, 2013; Li et al., 2018; Liu et al., 2016; Ogiesoba e Klokov, 2017) realizam a separação das diferentes componentes do campo de ondas no domínio *dip-angle* após a migração; no entanto, como são baseados em derivações de algoritmos de migração, entendemos que se encaixam melhor nesta categoria. É importante frisar que a separação de eventos no domínio *dip-angle* não é restrita à migração Kirchhoff tradicional (Dafni e Symes, 2017; Silvestrov et al., 2016; Zhu e Wu, 2010).

Adicionalmente, técnicas de aprendizado de máquinas também se mostraram efetivas para a

separação e imageamento de difrações. De Figueiredo et al. (2013) apresentaram um dos estudos pioneiros no uso de algoritmos de *machine learning* para a separação e imageamento de difrações. Os autores classificaram imagens a partir da análise das amplitudes de difrações elementares. Para isso, aplicaram a técnica *k-nearest neighbors* (kNN) a fim de distinguir imagens contendo difrações, reflexões ou ruído pela análise da distribuição de amplitudes ao longo das curvas de tempo de trânsito de difrações. Esta abordagem foi posteriormente aplicada e aperfeiçoada por Maciel e Biloti (2020). Neste estudo, os autores desenvolveram um descritor estatístico baseado na diferença entre aspectos dinâmicos e cinemáticos dos campos de onda. Este método apresenta a classificação correta de eventos utilizando a técnica de aprendizado de máquinas conhecida como máquina de vetores de suporte (*support vector machines* ou SVMs) (Cortes e Vapnik, 1995). Shustak e Landa (2018) empregaram o princípio do foco de reflexões em fontes secundárias (Figura 4) para analisar os padrões gerados após a migração dos dados. Para isso, utilizaram o classificador gaussiano *naive Bayes*, técnica de aprendizagem supervisionada. Markovic et al. (2022) aplicaram um modelo de segmentação para reconhecer padrões de difração em conjuntos de dados sísmicos e GPR. Tschannen et al. (2020) aplicaram redes neurais convolucionais para diferenciar imagens contendo difrações e reflexões. O algoritmo foi treinado a partir de seções sintéticas baseadas em modelagens da equação da onda. A aplicação de redes neurais, classe de algoritmos de *deep learning*, também foi estudada por Kim et al. (2021), Lowney et al. (2021a), Lowney et al. (2021b), Sheng e Zhao (2022) e Sun et al. (2022).

Dentre os diferentes métodos utilizados para separar a componente de difrações do campo de ondas total, destacamos aqui a alteração da função peso da integral de migração proposta por Moser e Howard (2008), que envolve manipulações da integral de Kirchhoff, utilizada na migração de seções sísmicas (Equação 7). A Equação 7 também é chamada de *stack* de difrações, pois considera a hipérbole de reflexão como o envelope das curvas de muitas difrações elementares, como definido pelo princípio de Huygens (Baker e Copson, 2003). Como discutido anteriormente, na migração e na modelagem Kirchhoff, um refletor suave é entendido como a soma de vários pontos difratores; cada ponto emite uma difração elementar que, somada a todas as outras, forma a curva de reflexão observada nos sismogramas. As difrações elementares são idealizações matemáticas que se interferem construtivamente ao longo da curva de reflexão e destrutivamente fora dela; assim, não podem ser observadas individualmente em sismogramas. No entanto, as difrações reais, que carregam informações de espalhadores em subsuperfície, não são somente idealizações e são observáveis em seções sísmicas.

2.4.1 Alteração da Condição de Imagem

É possível incorporar um fator de peso isotrópico na Equação 7 (Hubral et al., 1996a), também chamado de “condição de imagem” ou “peso anti-estacionário”. Se considerarmos o vetor de parametrização ξ como a posição da fonte s e do receptor r , podemos reescrever a integral de Kirchhoff (Li et al., 2020b):

$$V^D(x) = \int dt ds dr w(s, x, r) U(t, s, r) \delta(t - t_d(s, x, r)), \quad (8)$$

em que $w(s, x, r)$ é o fator peso. O tempo de trânsito das difrações elementares $t_d(s, x, r)$ pode ser considerado como:

$$t_d(s, x, r) = T(s, x) + T(x, r), \quad (9)$$

em que $T(s, x)$ é o tempo de propagação da fonte ao ponto difrator x e $T(x, r)$ é o tempo do ponto difrator ao receptor. Para os vetores $p_s = \nabla_x T(s, x)$ e $p_r = \nabla_x T(x, r)$ referentes à fonte e ao receptor, e considerando um vetor unitário normal ao refletor n , a condição de imagem pode ser definida como

$$w(s, x, r) = n^T \frac{(p_s + p_r)}{\|p_s + p_r\|}. \quad (10)$$

Isso pode ser compreendido como o produto interno entre a normal unitária do refletor e a bissetriz dos raios incidentes e refletidos em x . A condição possui valor máximo quando a normal e a bissetriz são colineares, que é o caso das reflexões especulares. Essa definição faz com que as contribuições de reflexões especulares sejam enfatizadas no processo de migração, o que atenua as difrações advindas de fora da zona de Fresnel (Figura 10). A zona de Fresnel é definida como $w(s, x, r) > 1 - \varepsilon$, em que ε é um pequeno número dependente da frequência (Moser e Howard, 2008); também pode ser compreendida como a porção do refletor que apresenta interferência construtiva e cuja energia refletida é detectada por sensores (Sheriff, 1980).

Os raios sísmicos são descritos como vetores perpendiculares à frente de onda. Apesar de não possuírem significado físico, são conceitos úteis para a compreensão do percurso das energia sísmica em subsuperfície. Ao representar a trajetória da frente de ondas por meio de raios, assumimos que a reflexão ocorre em um único ponto. No entanto, a energia é refletida em uma área mais ampla, definida como a zona de Fresnel. O cálculo do raio da zona de Fresnel R_F para meios isotrópicos pode ser realizado a partir do teorema de Pitágoras (Figura 11) (Sheriff, 1980):

$$R_F^2 = (Z + \lambda/4)^2 - Z^2, \quad (11)$$

em que Z é a profundidade do refletor e λ é o comprimento da onda sísmica. Se considerarmos

$\lambda = V/f$, podemos reescrever a Equação 11 a partir dos parâmetros de frequência e velocidade (Xu e Stovas, 2017):

$$R_F = \sqrt{\left(Z + \frac{V}{4f}\right)^2 - Z^2}. \quad (12)$$

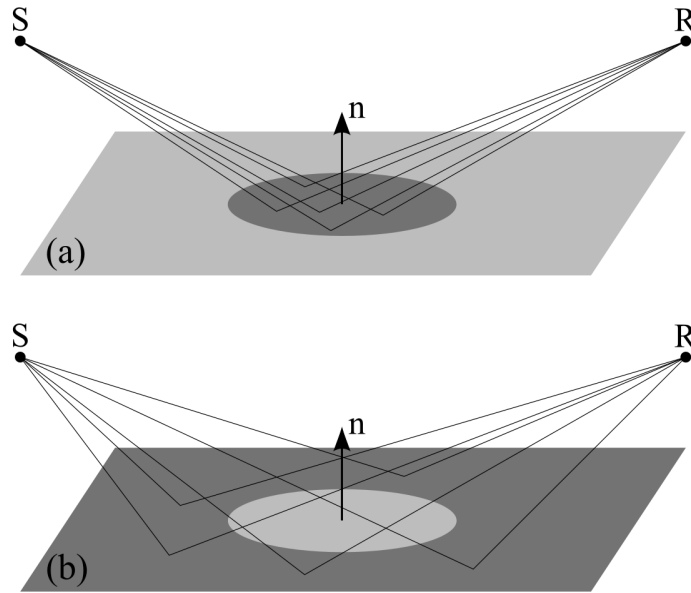


Figura 10: Princípio da condição de imagem $w(s, x, r)$. (a) A migração convencional considera somente as reflexões especulares provenientes da zona de Fresnel. (b) A modificação de w por $w_d(s, x, r)$ faz com que a migração considere somente a informação não especular proveniente de fora da zona de Fresnel. Modificado de Moser e Howard (2008).

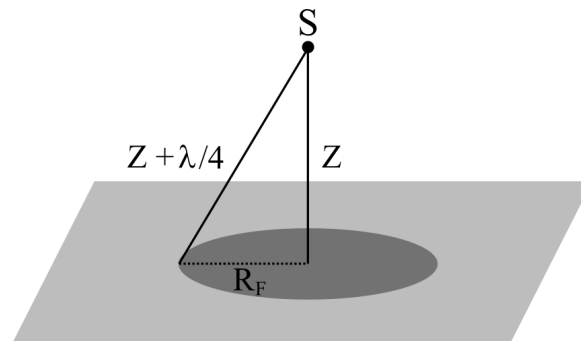


Figura 11: Esquema para cálculo do raio da zona de Fresnel em meios isotrópicos. Modificado de Xu e Stovas (2017).

É possível utilizar a Equação 10 para definir uma nova condição de imagem que explicitamente suprime as reflexões especulares e enfatiza as difrações ao considerar somente a energia não especular:

$$w_d(s, x, r) = 1 - n^T \frac{(p_s + p_r)}{\|p_s + p_r\|}. \quad (13)$$

Dessa forma, a técnica apresentada por Moser e Howard (2008) substitui $w(s, x, r)$ por $w_d(s, x, r)$ na Equação 8 para atenuar a resposta de reflexões e ressaltar a energia das difrações (Figura 10b), o que resulta em uma imagem com informações de pequenas estruturas espalhadoras do meio.

Systematic Literature Review on Seismic Diffractions Imaging

GUILHERME ZAKAREWICZ¹, SUSANNE TAINÁ RAMALHO MACIEL²,
LUCIANO SOARES DA CUNHA¹

¹Universidade de Brasília (UnB), Instituto de Geociências

²Faculdade UnB Planaltina (FUP)

Resumo. O campo de ondas difratadas muitas vezes é tratado como ruído durante os fluxos convencionais de pré-processamento sísmico. No entanto, ele possui informações valiosas para a visualização de falhas, fraturas, cunhas e outras estruturas em pequena escala que apresentam implicações significativas em diversos campos e aplicações industriais. Além disso, os métodos de imageamento de difrações sísmicas têm o potencial de alcançar a super resolução, o que permite recuperar detalhes menores que o limite de Rayleigh. Neste estudo, realizamos uma revisão sistemática da literatura para fornecer uma visão abrangente das técnicas de imageamento de difrações sísmicas. Nossa análise inclui uma avaliação quantitativa dos artigos selecionados com o objetivo de identificar técnicas emergentes, lacunas de conhecimento e potenciais áreas para pesquisas futuras. A maioria dos estudos propõem a aplicação dos métodos em seções sísmicas para delinear falhas, fraturas e outras heterogeneidades. Essas técnicas possuem amplas aplicações na indústria de petróleo e gás, investigações rasas e análise de velocidade. Observamos uma demanda crescente por aplicações práticas, com ênfase em demonstrar a capacidade das imagens de difração em delinear e interpretar estruturas usando exclusivamente dados de campo. Além disso, há uma tendência crescente em estudos relacionados à mineração de carvão e à utilização de dados de radar de penetração no solo. Pesquisas adicionais são necessárias para avaliar o potencial e as limitações destas técnicas.

Palavras-chaves: Difrações sísmicas; Imageamento de difrações; Revisão; Sistemática; Análise quantitativa; Iluminação de subsuperfície; Heterogeneidades.

Abstract. The diffractive component of the wavefield is often treated as noise during traditional pre-processing workflows. However, it possesses valuable information for imaging faults, fractures, wedges, and other small-scale structures, which have significant implications across diverse fields and industrial applications. Furthermore, the seismic diffractions imaging methods have the potential to achieve the superresolution, enabling the recovery of subsurface details beyond the Rayleigh limit. In this study, we conducted a systematic literature review to provide a comprehensive overview of the state-of-the-art techniques in seismic diffraction imaging. Our analysis includes a quantitative assessment of selected articles, aimed at identifying emerging techniques, knowledge gaps, and potential areas for future research. The majority of studies focus on the application of proposed methods in seismic sections to delineate faults, fractures, and other heterogeneities, thereby enhancing subsurface illumination. Notably, these techniques find wide-ranging applications in the oil and gas industry, near-surface investigations, and velocity analysis for migration kernels. We observed a growing demand for practical applications and interpretation, with a specific emphasis on showcasing the ability of diffraction imaging to delineate structures using exclusively field data. Additionally, there is a growing trend in studies related to coal mining and the utilization of ground-penetrating radar data. Further research is necessary to fully assess the potential and limitations of seismic diffraction imaging in these emerging topics.

Keywords: Seismic diffractions; Diffractions imaging; Review; Systematic; Quantitative analysis; Subsurface illumination; Heterogeneities.

1 Introduction

1.1 Seismic Diffractions

The interaction of seismic waves with different interfaces/obstacles results in different events. The seismic diffractions are fundamentally different physical phenomena from reflections and have comparatively smaller amplitudes (Khaidukov et al., 2004; Klem-Musatov, 1994), as illustrated in Figure 1. While smooth and continuous interfaces generate reflections, diffractions result from wave interaction with small structures and do not satisfy the general conditions of validation of the ray theory presented by Červeny (2001). According to Moser (2012), the reflected waves can be referred to as “specular waves” because they obey Snell’s law, which is not the case for diffracted waves.

The specular reflections are generated by continuous structures with dimensions greater than the seismic wavelength. Geological structures such as faults, channels, wedges, and fractures interact with waves and scatter the seismic energy, which results in diffractions. Traditional seismic processing techniques enhance reflections and suppress diffractions (Khaidukov et al., 2004; Moser, 2012). As a result, information on important structures for interpreting sections as small heterogeneities, scatterers, and discontinuities is lost during the imaging process. Furthermore, diffractions imaging can theoretically lead to superresolution, i.e., the retrieval of information smaller than the Rayleigh limit (half the seismic wavelength) (Khaidukov et al., 2004; Moser and Howard, 2008; Zhang and Zhang, 2014) (Figure 2).

The application of diffractions to facilitate the interpretation of seismic sections is a relatively underexplored topic within the geophysical community, but it has already established itself in the literature. Pioneering works used this type of wave to identify fault zones (Kanasewich and Phadke, 1988; Krey, 1952; Kunz, 1960; Trorey, 1970). Berryhill (1977) studied the behavior of edge diffractions responses and determined their dynamic characteristics. Harlan et al. (1984) introduced techniques for analyzing velocities from diffractions in simple geological settings, such as a constant velocity medium. Vries and Berkhout (1984) developed a minimum-entropy based technique for focusing diffracted energy and subsequent velocity analysis. Landa et al. (1987) introduced the concept of common-diffraction point sections, which involves computing coherence along hyperbolic diffraction events generated by scattering structures. Neidell (1997) differentiated the contribution of diffractions and reflections to the total wavefield and used the diffraction component to establish higher resolutions. Landa and Keydar (1998) and Papziner and Nick (1998) used diffractions’ energy

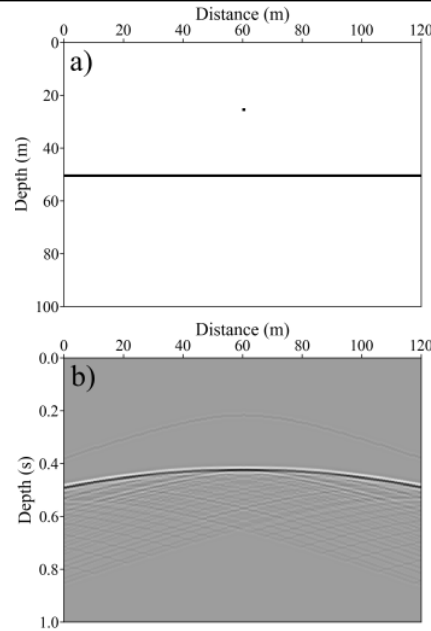


Figure 1: (a) Synthetic model consisting of a single scatterer and a horizontal reflector. (b) Resulting common-shot gather. Note that the diffraction hyperbola show weaker amplitudes compared to the reflection hyperbola.

for detecting near surface heterogeneities. Bansal and Imhof (2005) investigated the combination of processing modules to enhance diffractions in different domains.

Presently, the retrieval of information contained in diffractions is being used for velocity analysis necessary for different migration techniques (Delf et al., 2022; Fomel et al., 2007; Lin et al., 2020, 2018a; Reshef and Landa, 2009; Sava et al., 2005) and to obtain detailed images of the subsurface (Bauer et al., 2019; Li et al., 2020c; Maciel and Biloti, 2020; Tyiasning et al., 2016). Studies have also focused on imaging mass-transport complexes (Ford et al., 2021), complex fault and fracture networks (Schwarz and Krawczyk, 2020), seismic tomography (Bauer et al., 2017; Santos et al., 2012), detection of structures related to coal mining (Zhou et al., 2017), and monitoring of CO₂ capture (Alonaizi et al., 2014).

Figure 3 illustrates the concept of scale separation in seismic imaging, showing the temporal-frequency bands (black line) representing different wavelength ranges. The acoustic impedance is usually seen through the spectral window from 10 to 100 Hz, while velocity is seen through a window ranging from zero to 2 Hz. Claerbout (1985) has discussed the existent informa-

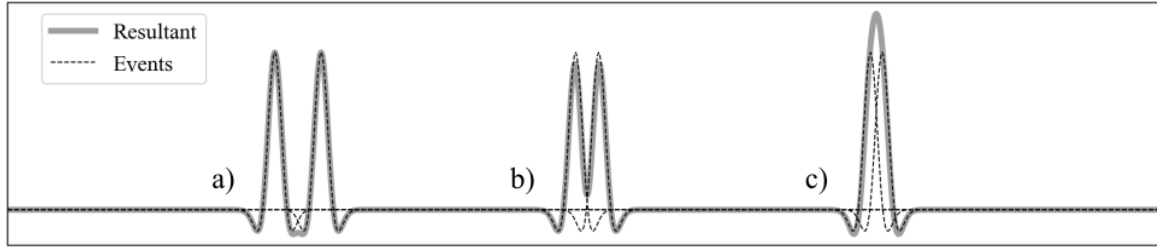


Figure 2: Illustration of the interaction of different seismic pulses and the Rayleigh's criterion. (a) When the distance between events is sufficiently large, each event can be distinguished individually in the resultant pulse. (b) The Rayleigh's criterion represents the maximum limit at which it remains possible to observe each pulse individually. (c) Beyond the Rayleigh limit, it is not possible to distinguish each event, as the resultant appears as a single pulse. Modified from Kallweit and Wood (1982).

tion gap in traditional reflection seismology, between 2-10Hz. As discussed by Biondi and Almomin (2014), the industry has made efforts to broaden the frequency band at both high and low ends, as shown by the blue (tomographic models) and green (Full-Waveform Inversion - FWI for improved velocity estimation) lines. Seismic diffraction imaging can extend high-frequency imaging (red line) by retrieving information about small subsurface structures and scatterers. Diffractions have about twice the seismic resolution of the reflections. Huang et al. (2015) show that when we compute the radius of the Fresnel zone as a function of wavelength and ray path length, the radius for the diffraction is about half of the radius for the reflection. This implies that velocity analysis using diffractions will be more accurate, improving imaging resolution in high-frequency bands.

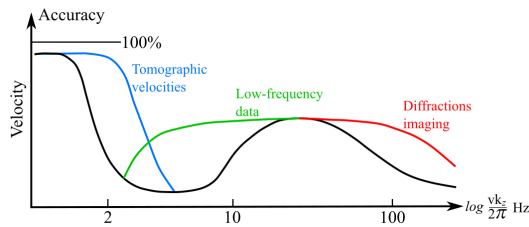


Figure 3: Simplified graphical representation of scale separation in seismic imaging. The black line represents the wavelength ranges and the current imaging gaps. The industry has been making efforts to widen the data frequency band at the high and low ends (blue, green and red lines). Modified from Biondi and Almomin (2014).

1.2 Systematic Literature Review

The systematic literature review (SLR) consists of a replicable, scientific, and transparent process (Tranfield et al., 2003) to minimize bias for qualitative analysis. Kitchenham et al. (2009) define the SLR as a rigorous,

methodological, and repeatable review of primary studies, i.e., empirical articles and publications conducted to answer specific questions or present new methods. Through the SLR, it is possible to identify gaps in a particular theme/study field, summarize relevant studies in a topic, and perform a qualitative analysis of the extracted data (Rasol et al., 2022). It differs from the conventional literature review by applying pre-established and repeatable protocols on a specific topic (Castro and Cunha, 2021).

The SLR comprises three main stages: planning, conducting, and reporting/documenting (Wahono, 2015). In the first step, we define the main parameters and protocols to be applied in the query. We apply the defined protocols in the second step: search for primary studies, assess quality/extract selected publications, synthesize data, and perform qualitative analysis. The final stage involves elaborating the written report document, with the discussion and conclusion (Porritt et al., 2014). Recently, the SLR has been applied in earth sciences research, such as GPR (ground-penetrating radar) (Castro and Cunha, 2021; Rasol et al., 2022), outcrops in petroleum exploration (Marques Jr et al., 2020), and the application of machine learning to model physical properties of shale reservoirs (Syed et al., 2022).

SLRs are useful in identifying knowledge gaps, summarizing existing information, and providing a comprehensive overview of the research topic. The method is also known to minimize bias and increase the reliability of the results. Given the broad scope of geosciences, incorporating systematic reviews can help in synthesizing the available information, revealing research gaps, and developing a research agenda. Hence, conducting the systematic research in geosciences can offer valuable contributions to the field and help in the development of new research ideas. Some articles re-

view the state-of-the-art of seismic diffractions separation and imaging topic, e.g., Kim et al. (2021); Li et al. (2021a); Lin et al. (2021b); Schwarz (2019a); Wang et al. (2020b); Zhao et al. (2019a), but none of them have applied the methodology of a systematic review.

The meta-analysis is a subset of the SLR, defined as the quantitative study applied for assessing the results of primary studies (Haidich, 2010) to draw a more robust conclusion. It aims to provide a comprehensive overview of the evidence on a specific research question by pooling together the results of many individual studies. The process of meta-analysis involves a systematic and comprehensive search for relevant studies, a critical evaluation of the quality of the studies, and a quantitative synthesis of the results. The PRISMA (Preferred Reporting Items for Systematic reviews and Meta-analyses) statement (Moher et al., 2009) is a workflow that guides the application of systematic reviews and meta-analyses. The statement includes the identification, screening, and eligibility of the works in the database.

Hence, the objective of this study is to apply a SLR and meta-analysis to present a comprehensive overview of the current state-of-the-art in seismic diffraction imaging. The study aims to identify and analyze the gaps in knowledge and research related to the topic and provide an overview of the different methods used for seismic diffraction imaging. Furthermore, we present a quantitative analysis on the selected publications. Additionally, this study proposes potential areas for future research in the field.

2 Materials and Methods

2.1 Quantitative Analysis

The first step is organizing the metadata from the articles available on the Web of Science (web of science.com) and Scopus (scopus.com) platforms. It is possible to determine keywords and terms for database queries. We defined the terms “seismic diffraction” and “imaging” for the query in the paper’s title, abstract, or keywords. The Web of Science database returned 612 works from January 1st, 1990, to August 5th, 2022. We applied the PRISMA statement (Moher et al., 2009) for articles’ selection (Figure 4). We manually selected the studies based on the inclusion/exclusion criteria defined in Table 1. A total of 116 articles were selected for the quantitative analysis from the Web of Science platform. For the same period, the Scopus database returned 924 works. After the manual selection and the duplicate removal, 34 articles were selected. In total, we analyzed 150 articles in this work.

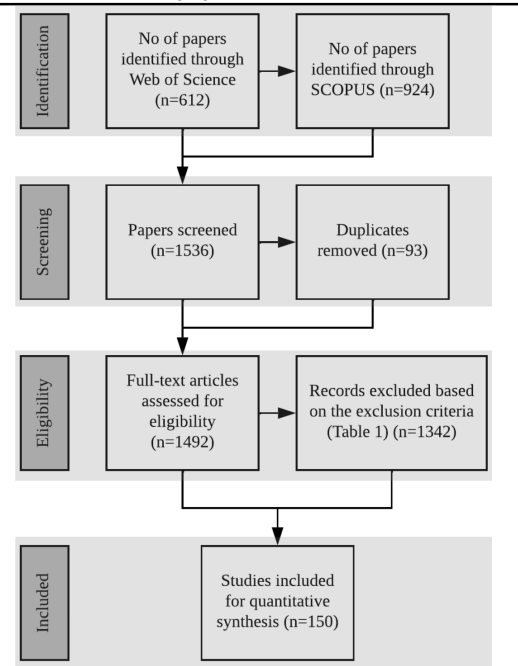


Figure 4: Flowchart for primary studies selection according to the PRISMA statement.

Each article’s title, abstract, and keywords were exported from the Web of Science (WoS) and Scopus databases in a .txt format. We opened the files in the VOS-Viewer software (version 1.6.18), which provides constructing and visualizing bibliometric networks (Van Eck and Waltman, 2010). We generated networks based on instances such as co-citation (the relatedness of items is determined based on the number of times they are cited together), co-authorship (based on the number of co-authored documents), and bibliographic coupling (based on the number of references shared by the papers). We also evaluated the countries that publish the most on the topic.

3 Results

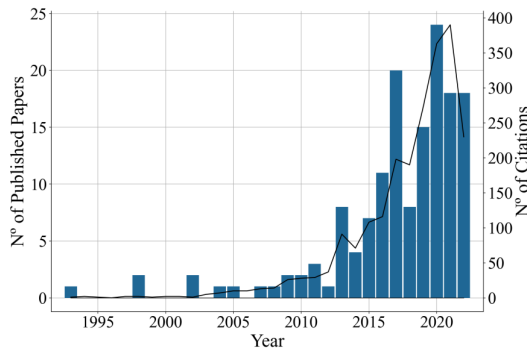
3.1 Number of Publications

Figure 5 shows the number of published works and the direct citations over the years. The first publication dates from 1993, and from 1990 to 2000, only three articles are returned from the query. Although some works focus on the applications of the seismic diffractions separation before the decade 1990, Schwarz (2019a) emphasizes that it was only in the late 1990s that diffraction processing methods presented reliability to detect small-scale heterogeneities, e.g., Landa and Keydar

Table 1: Research parameters applied in the database query.

Parameter	Applied in Query
Keyword / Term	(seismic diffraction AND imaging)
Topic	Title, Abstract and Keywords
Publication Year	January 1st, 1990 to August 5th, 2022
Publication Type	Article / Review Article
Language	English
Area of Research	All (unrestricted)
Inclusion Criteria	Diffraction separation / enhancement for subsurface imaging; Diffraction separation for migration velocity analysis; New techniques for diffractions imaging; Study cases of diffractions imaging.
Exclusion Criteria	Book / Book's Chapter / Conference Abstract / Theses; Papers not related with seismic diffractions separation / enhancement; Abstract / full text not found; Not peer reviewed.

(1998). There has been an increase in publication and citation from 2010. After this year, the number of publications (n=137) represents 91.33% of the total analyzed works.

**Figure 5:** Variation of publications (blue histograms) and quantity of citations (black line) considering the WoS and the Scopus databases.

3.2 Most Cited Articles

The most cited papers are Khaidukov et al. (2004) (n=149) and Moser and Howard (2008) (n=122), which present the focusing-muting-defocusing and the anti-stationary phase filter methods, respectively. Following, the most cited articles (over 90 citations) are Dell and Gajewski (2011) (n=105), Klovov and Fomel (2012) (n=104), Fomel et al. (2007) (n=103), Landa and Keydar (1998) (n=101) and Berkovitch et al. (2009) (n=95). The main contributions of each publication are presented in Table 2.

3.3 Co-authorship (countries)

Figure 6 shows the co-authorship network using countries as the unit of analysis. It indicates the scientific collaboration between the countries based on the authors' affiliations. Different collaborative groups tend to form a decentralized network, which contributes to solve more complex scientific problems and broad research (Fagan et al., 2018). Figure 6a shows the collaboration map considering the WoS database, where a group of 10 countries interacts when publishing new papers led by the USA. In Figure 6b, referring to the Scopus database, only three countries present expressive scientific collaborations: China, Australia, and the USA.

3.4 Co-citation

The co-citation analysis shows the similarities between two documents based on the times they are cited in other works (Small, 1973). Two publications cited together frequently are relatively closer on the map, indicating proximity between them (White and Griffith, 1981). We present a table based on the primary references obtained from the network. Table 3 shows the most cited articles after the co-citation network analysis, where mapping the most relevant authors and articles on the topic is possible. We noticed a similarity between the references retrieved by the two databases since eight articles appear in both lists.

3.5 Wordcloud (full record)

To visualize the words with higher occurrences in the publications' titles, abstract, and keywords, we used the

Table 2: Main contributions of the most cited articles from both databases (WoS and Scopus)

Authors, Year, Journal	Title	Citations	Main Contributions
Khaidukov, V.; Landa, E.; Moser, T. 2004, Geophysics	Diffraction imaging by focusing-defocusing: An outlook on seismic superresolution	149	Introduction of the focusing-muting-defocusing method
Moser, T.; Howard, C. 2008, Geophysical Prospecting	Diffraction imaging in depth	122	Presented the reverse stationary phase migration method; further developed the focusing-muting-defocusing method
Dell, S.; Gajewski, D. 2011, Geophysics	Common-reflection-surface-based workflow for diffraction imaging	105	Presented the common-reflection-surface operator for diffraction imaging and poststack time migration velocity analysis
Klokov, A.; Fomel, S. 2012, Geophysics	Separation and imaging of seismic diffractions using migrated dip-angle gathers	104	Presented the diffraction separation in the dip-angle domain after the Radon transform
Fomel, S.; Landa, E.; Taner, T. 2007, Geophysics	Poststack velocity analysis by separation and imaging of seismic diffractions	103	Introduced the plane-wave destruction filter to separate diffractions for imaging in the time domain and velocity analysis in the poststack domain
Landa, E.; Keydar, S. 1998, Geophysics	Seismic monitoring of diffraction images for detection of local heterogeneities	101	Proposed the common-diffraction-point sections (D-sections) to enhance the diffractions amplitudes and detect heterogeneities; presented a time correction for diffraction curves based on the wavefronts' curvature radius.
Berkovitch, A.; Belfer, I.; Hassin, Y.; Landa, E. 2009, Geophysics	Diffraction imaging by multifocusing	95	Introduced the diffraction multifocusing stack method to image local heterogeneities

Python library wordcloud (Oesper et al., 2011). Table 4 shows the 30 most frequent words in the selected papers. The words with the highest occurrences are “diffraction” (n=440), “diffractions” (n=259), “seismic” (n=388), “imaging” (n=233), and “method” (n=204), all directly related to the main research topic explored in this study. The word “migration” has 118 occurrences, related to publications where the migration has a fundamental role in diffractions imaging, e.g., Berkovitch et al. (2009); Li et al. (2020c); Moser and Howard (2008); Zhang and Zhang (2014).

The word “synthetic,” with 82 occurrences, is related to works with applications in synthetic seismic data to test and validate the proposed methods, e.g., Ber-

kovitch et al. (2009); Dell and Gajewski (2011); Fomel et al. (2007); Ford et al. (2021); Khaidukov et al. (2004); Klokov and Fomel (2012); Landa and Keydar (1998); Li et al. (2021a); Lin et al. (2022c, 2021c); Maciel and Biloti (2020); Merzlikin et al. (2020); Moser and Howard (2008); Prosatov et al. (2021); Rad et al. (2018); Tschannen et al. (2020); Wang et al. (2020b). Note that all most cited publications use synthetic data to validate their methods.

“Dip-angle” (n=52) is associated with works that use the dip-domain to separate the diffracted energy (Dafni and Symes, 2017; Klokov and Fomel, 2012, 2013; Li et al., 2018, 2020d; Li and Zhang, 2019; Lin et al., 2018b; Liu et al., 2016; Ogiesoba and Klo-

Table 3: Main references from the co-citation analysis in descending order of total citations. To obtain these ten main references, a minimum limit of 34 citations was adopted for each reference pair within each article analyzed in WoS, and three citations for each reference pair within each article analyzed in Scopus (Total analysed references from WoS: 2209, and Scopus: 999).

Item	Reference from WoS	Item	Reference from Scopus
1	Fomel S., 2007, doi 10.1190/1.2781533	1	Harlan W., 1984, doi 10.1190/1.1441600
2	Landa E., 1998, doi 10.1190/1.1444387	2	Reshef M. and Landa E., 2009, doi 10.1111/j.1365-2478.2008.00773.x
3	Kanasewich E., 1988, doi 10.1190/1.1442467	3	Landa E., 1987, doi 10.1111/j.1365-2478.1987.tb00823.x
4	Fomel S., 2002, doi 10.1190/1.1527095	4	Fomel S., 2002, doi 10.1190/1.1527095
5	Berkovitch A., 2009, doi 10.1190/1.3198210	5	Moser T. and Howard C., 2008, doi 10.1111/j.1365-2478.2007.00718.x
6	Landa E., 1987, doi 10.1111/j.1365-2478.1987.tb00823.x	6	Berkovitch A., 2009, doi 10.1190/1.3198210
7	Dell S. and Gajewski D., 2011, doi 10.1190/geo2010-0229.1	7	Khaidukov V., 2004, doi 10.1190/1.1836821
8	Klokov A. and Fomel S., 2012, doi 10.1190/geo2012-0017.1	8	Dell S. and Gajewski D., 2011, doi 10.1190/geo2010-0229.1
9	Khaidukov V., 2004, doi 10.1190/1.1836821	9	Klokov A. and Fomel S., 2012, doi 10.1190/geo2012-0017.1
10	Moser T. and Howard C., 2008, doi 10.1111/j.1365-2478.2007.00718.x	10	Fomel S., 2007, doi 10.1190/1.2781533

kov, 2017; Reshef and Landa, 2009), and “plane-wave” (n=44) is related with works that apply the plane-wave destruction (PWD) filter for diffractions separation and enhancement (Decker et al., 2017; Fomel, 2002; Fomel et al., 2007; Kong et al., 2017; Li et al., 2022b; Lin et al., 2022c, 2018a,c; Lowney et al., 2020; Taner, 2006). The word “3d” (n=49) indicates works with applications in 3D seismic data (Alonazi et al., 2014; Klokov and Fomel, 2013; Li and Zhang, 2019; Lin et al., 2021c; Prosatov et al., 2021; Schoepp et al., 2015; Zhao et al., 2020; Zhou et al., 2017).

“Velocity” (n=126) combined with “analysis” (n=62) refers to works that focus on the application of the seismic diffractions for velocity analysis in migration kernels, e.g., Fomel et al. (2007); Landa and Keydar (1998); Lin et al. (2018a); Reshef and Landa (2009); Reshef and Rüger (2008); Sava et al. (2005).

3.6 Bibliographic Coupling

The bibliographic coupling links researchers based on the number of referenced documents they share (Boyack and Klavans, 2010; Kessler, 1963). Consequently, it connects papers from their similarity using bibliographic coupling counts. The circles’ size is determined by the number of citations, i.e., how many ti-

mes the document was cited. The distance between the two circles and the thickness of the lines represent the relatedness of the papers in terms of bibliographic coupling, i.e., the number of cited references two publications have in common (Van Eck and Waltman, 2010). The authors commonly acknowledge co-citation analysis as a dynamic approach, in contrast to the static nature of bibliographic coupling (Vogel and Güttel, 2013). Co-citation analysis is useful to identify fundamental works within a specific research topic, while bibliographic coupling is effective in detecting current trends and future directions (Aria and Cuccurullo, 2017). Therefore, both methods complement each other, with co-citation analysis focusing on foundational works and bibliographic coupling highlighting emerging trends (Ferreira, 2018). For this reason, we conduct both analyzes in our work to present a comprehensive understanding of the historical development of the topic as well as its current/future trajectories.

Figures 7 and 8 show the bibliographic coupling network for the WoS and Scopus databases, respectively, when considering a minimum of 5 citations for each paper. We considered only the articles published in the last five years. In Figure 7, two connected clusters are marked in red and green. Cluster 1 (red) has the

Table 4: 30 most frequent words in the titles, abstracts, and keywords from the 150 articles of both databases (WoS and Scopus).

Word	Number of Occurrences	Word	Number of Occurrences	Word	Number of Occurrences
diffraction	440	subsurface	89	analysis	62
seismic	388	reflections	86	results	61
diffractions	259	diffracted	83	events	56
imaging	233	synthetic	82	wavefield	54
data	204	wave	80	dip-angle	52
method	204	separation	79	conventional	52
velocity	126	field	73	3d	49
reflection	119	energy	65	prestack	48
migration	118	time	63	plane-wave	44
image	116	small-scale	63	model	42

following works: Bauer et al. (2019); Li et al. (2020b); Lin et al. (2020, 2018b); Merzlikin et al. (2019); Wang et al. (2020b); Zhao et al. (2020), while cluster 2 (green) is composed by Tschannen et al. (2020) and Zhang et al. (2019).

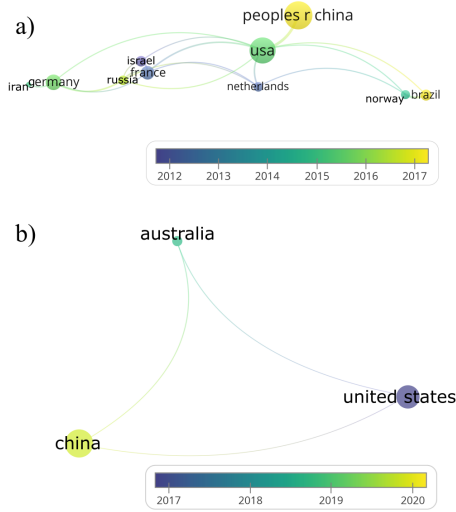


Figure 6: Co-authorship map using countries as the unit of analysis, normalized citation as weight, and year of publication as score. The relative distance of the cluster's nuclei indicates the proximity of the research groups, the thickness of lines indicates the collaboration's intensity, and the number of citations determines the node's size. (a) Network for the WoS database. Although the USA research groups represent the central leadership with a more extensive network of members (10 countries), China already occupies a prominent role for appearing with the highest number of citations in recent years in the period used to elaborate the map. (b) Network for the Scopus database. China and the United States share a leading role, connected with Australia.

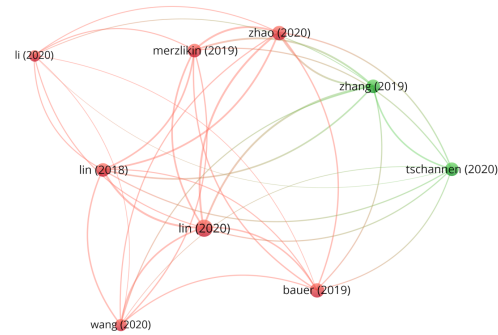


Figure 7: Bibliographic Coupling map for the 61 works published in the last 5 years (2018-2022) from the WoS database using the documents as the unit of analysis and normalized citations as weight.

Figure 8 has three connected clusters (red, green, and blue). Cluster 1 (red) is composed of Burnett et al. (2021); Decker and Fomel (2021); Khoshnavaz et al. (2018); Li et al. (2021b); Mu et al. (2021); Zhao et al. (2019a); cluster 2 (green) is formed by Pedersen and Marcy (2022); Wang et al. (2020a); and cluster 3 (blue) is composed of Economou et al. (2020); Xiang and Landa (2019). The first two clusters are more closely related than the third one.

After analysing the articles present in the networks of Figures 7 and 8, we noted that the clusters lack a distinct theme or specific topic that interconnects them. All works present in the clusters propose innovative and improved methods for seismic diffractions separa-

tion and imaging, but there is an absence of a common theme or shared application within the publications.

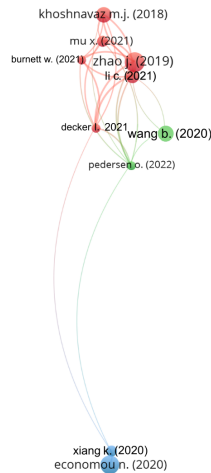


Figure 8: Bibliographic Coupling map for the 21 works published in the last 5 years (2018-2022) from the Scopus database using the documents as the unit of analysis and normalized citations as weight.

3.7 Wordcloud (last 5 years)

Table 5 shows the 30 words with the highest occurrences in the articles' titles, abstracts, and keywords in the past five years. Similarly to Table 4, the words with the highest occurrences are related to the main topic ("diffraction," $n=390$; "diffractions," $n=238$; "imaging," $n=248$; "seismic," $n=249$). "Velocity" ($n=72$) and "analysis" ($n=78$) refer to articles that study the application of diffractions for velocity analysis in migration kernels (Santos et al., 2020). "Migration" ($n=96$) indicates studies and methods in which the migration process is essential (Li et al., 2020c). "Plane-wave" has 37 occurrences (Lin et al., 2022c, 2018a,c; Lowney et al., 2020; Mu et al., 2021), while "3d" has 41 occurrences (Li and Zhang, 2019; Zhao et al., 2020). "Reservoir" has 36 occurrences related to works with applications in oil and gas exploration (Schwarz and Krawczyk, 2020; Zhao et al., 2019a, 2020). "Faults" ($n=60$) and "discontinuities" ($n=43$) are related to works applied to faults, fractures, and small-scale discontinuities characterization (Decker and Fomel, 2021; Li et al., 2020b; Lowney et al., 2021a; Tschannen et al., 2020).

3.8 World Map

Most papers are published by institutions/researchers from China (29.68%, $n=46$) and United States (21.29%, $n=33$), followed by Germany (9.03%, $n=14$), Israel (5.81%, $n=9$), Russia (4.52%, $n=7$), Brazil (3.87%, $n=6$), Iran (3.87%, $n=6$), Australia (3.23%, $n=5$), Norway (2.58%, $n=4$), Ireland (1.94%, $n=3$), Malaysia (1.94%, $n=3$), Saudi Arabia (1.42%, $n=3$), Italy (1.29%, $n=2$), and Netherlands (1.29%, $n=2$). Canada, Denmark, England, France, Greece, Japan, and South Korea represent 0.65% each ($n=1$). Figure 9 shows the geographical distribution of the publications according to the first author's affiliation. In summary, the data shows that most of the research on seismic diffractions imaging is being conducted by researchers from China and the United States, while other countries, such as Germany, Israel, Russia, and Brazil, also have a significant presence in the field. The data also highlights the contributions of other countries with a smaller number of papers, as well as the limited but important contribution of some countries, each with less than five papers.

3.9 Co-authorship (authors)

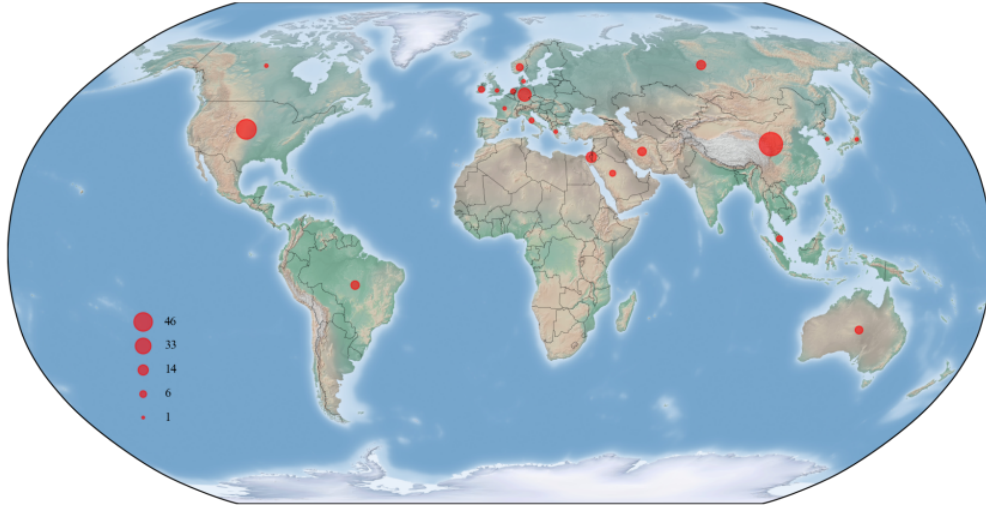
The network of co-authorship is presented in Figure 10. The clusters indicate the authors who publish the most relevant works together. The distance between the two nodes and the thickness of the lines represent the number of publications co-authored by the authors, and the circles' size indicates the number of times the author's publications are cited. We identified three main disconnected clusters: Figure 10a indicates the first cluster, composed of researchers of the University of Hamburg: Dell, S., Schwarz, B., and the core, Gajewski, D. (Dell and Gajewski, 2011; Rad et al., 2018; Schwarz and Gajewski, 2017; Schwarz et al., 2014). Figure 10b shows the second cluster, composed by researchers from the China University of Mining and Technology and the Chinese Academy of Sciences: Wang, Y., Yu, C., Zhao, J. (core), Li, C., Peng, S., Lin, P., and Cui, X. (Li et al., 2022b, 2020c; Lin et al., 2020, 2018c; Zhao et al., 2020). This cluster indicates a more recent contribution to literature. The third cluster (Figure 10c) is composed of the following authors: Klovov, A. (University of Texas at Austin - UT), Merzlikin, D. (UT), Fomel, S. (core, UT), and Landa, E. (Tel Aviv University) (Fomel et al., 2007; Klovov and Fomel, 2012, 2013; Merzlikin and Fomel, 2017; Merzlikin et al., 2017).

3.10 Applications

Table 6 indicates the main categories of seismic diffractions' applications after reading and analyzing the 150

Table 5: 30 most frequent words in the titles, abstracts, and keywords from the 82 articles published in the last 5 years (2018-2022) of both databases (WoS and Scopus).

Word	Number of Occurrences	Word	Number of Occurrences	Word	Number of Occurrences
diffraction	390	image	79	diffracted	49
seismic	249	analysis	78	waves	48
imaging	248	velocity	72	time	47
diffractions	238	images	70	discontinuities	43
data	194	small-scale	66	model	43
method	173	subsurface	62	plane-wave	42
separation	113	faults	60	3d	41
reflections	97	energy	59	resolution	37
reflection	96	field	57	reservoir	36
migration	96	synthetic	53	high-resolution	35

**Figure 9:** World map with the number of published articles per country (according to the first author's affiliation).

articles. The primary application is the characterization of faults, fractures, and other small-scale discontinuities to enhance the subsurface illumination, with 75 papers. Following, the application of seismic diffractions imaging in oil and gas exploration has 42 papers that suggest to apply their methods on the identification of migration paths and fracture corridors. 14 articles concern the near-surface applications such as the detection of pipes (Keydar and Landa, 2019; Rad and Hickey, 2021; Zhao et al., 2019b) and archaeological investigations (Liu et al., 2021; Wechsler et al., 2020). Six papers focus on applying velocity analysis necessary for migration processes (Decker et al., 2017; Fomel et al., 2007; Santos et al., 2020). Six papers explore the appli-

cation of diffractions imaging in coal mining, and seven articles explore other applications listed in Table 6.

3.11 Documents Analysis

Figure 11a shows the distribution of papers that use 2D, 3D, or both 2D and 3D data to illustrate their applications. Most of the analyzed works uses only 2D data ($n=101$) (Bauer et al., 2016; Khaidukov et al., 2004; Klovov and Fomel, 2012; Lin et al., 2022c, 2021b; Merzlikin et al., 2019; Moser and Howard, 2008; Reshef and Landa, 2009; Zhao et al., 2015), followed by papers that combine 2D and 3D examples ($n=35$) (Bansal and Imhof, 2005; Bauer et al., 2019; Chen et al., 2022; Decker and Fomel, 2021; Fomel

Table 6: Main applications of the seismic diffractions imaging in the literature.

Application	Papers
Oil and gas exploration	Bashir et al. (2018b); Burnett et al. (2015); Decker et al. (2014); Grasmueck et al. (2015); Klovov et al. (2017); Lin et al. (2018b); Merzlikin and Fomel (2017); Nosjean et al. (2020); Schoepp et al. (2015); Schwarz and Krawczyk (2020); Tyiasning et al. (2016); Yu et al. (2016); Zhao et al. (2019a, 2015, 2020)
Near surface (tunnels, tumbs, cables and other utilities identification)	Belfer et al. (1998); De Figueiredo et al. (2013); Economou et al. (2020); Keydar et al. (2010); Landa and Keydar (1998); Lellouch and Reshef (2016); Liu et al. (2021); Maciel and Biloti (2020); Peterie and Miller (2014); Peterie et al. (2020); Rad and Hickey (2021); Wechsler et al. (2020)
Coal mining	Li et al. (2020c,d); Zhao et al. (2021); Wang et al. (2020a); Zhou et al. (2017, 2022)
Velocity analysis	Decker et al. (2017); Dell and Gajewski (2011); Fomel et al. (2007); Klovov and Fomel (2012); Reshef and Landa (2009); Santos et al. (2020)
Fault, fractures, and small scale discontinuities characterization	Bansal and Imhof (2005); Berkovitch et al. (2009); Decker and Fomel (2021); Khaidukov et al. (2004); Li et al. (2020b); Lowney et al. (2021a); Moser and Howard (2008); Prosatov et al. (2021); Schwarz and Gajewski (2017); Tschannen et al. (2020); Zhang et al. (2020); Zhang and Zhang (2014)
Hard rock environment characterization	Ahmed et al. (2019); Khoshnavaz et al. (2016)
CO2 sequestration monitoring	Alonaizi et al. (2014)
Reef identification	Klovov et al. (2015)
Boulder detection in shallow sub-seafloor	Römer-Stange et al. (2022)
Identification and location of carbonate concretions (septaria)	Bellezza et al. (2022)
Characterization of Mass-Transport Complexes	Ford et al. (2021)

et al., 2007; Khoshnavaz et al., 2016; Li et al., 2020d; Schwarz and Krawczyk, 2020; Zhao et al., 2021). 14 works present only 3D data in their results (Decker et al., 2014; Dell et al., 2020; Klovov et al., 2017; Prosatov et al., 2021; Schoepp et al., 2015; Zhao et al., 2020). Figure 11b shows synthetic, field data distribution or both applications in the analyzed papers. 93 articles use synthetic and field data to discuss their proposed method (Bauer et al., 2017, 2019; Berkovitch et al., 2009; Dafni and Symes, 2017; Ford et al., 2021; Gelius et al., 2013; Li et al., 2020b; Lin et al., 2020; Merzlikin et al., 2020; Rad and Hickey, 2021; Tschannen et al., 2020; Wang et al., 2020b; Yu et al., 2016; Zhang et al., 2019), while 33 articles use only synthetic data (Bansal and Imhof, 2005; Bashir et al., 2020; Dell et al., 2019; Gelius and Asgedom, 2011; Jiang et al., 2022; Khosh-

navaz et al., 2018; Kong et al., 2017; Sturzu et al., 2014; Zhu and Wu, 2010), and 24 articles use only field data (Burnett et al., 2015; Dell et al., 2018, 2020; Klovov et al., 2017; Meng et al., 2021; Montazeri et al., 2019; Ogiesoba and Klovov, 2017; Ogiesoba et al., 2015; Schwarz and Gajewski, 2017; Tyiasning et al., 2016). The use of both synthetic and field data allows for a comprehensive evaluation of the proposed methods, as synthetic data can be used to test the method in controlled and well-understood conditions, while field data provides a real-world evaluation of the method's performance and limitations. Studies that use only synthetic data might be focused on developing or testing new techniques or algorithms, while studies that use only field data might be focused on applying existing methods to real-world situations.

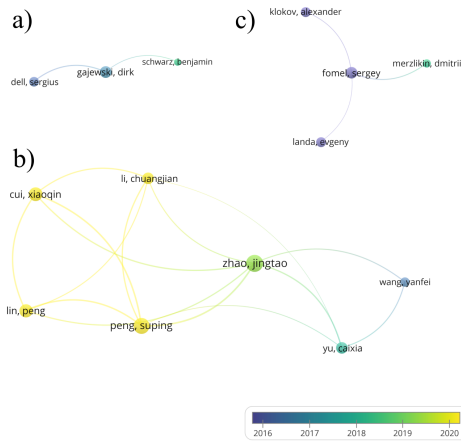


Figure 10: Co-authorship map for the 150 reviewed works using authors as the unit of analysis and normalized citation as weight. There are three main clusters (a, b, and c), where the relative distance of each author's nucleus indicates their proximity.

Figure 11c shows the distribution of the first authors' affiliations regarding academic or industrial institutions. 130 articles (86.7%) were written by authors from academic institutions, while authors wrote 20 articles (13.3%) from industrial institutions, most related to oil and gas exploration, e.g., Burnett et al. (2015, 2021); Liu et al. (2022); Nosjean et al. (2020); Schoepp et al. (2015); Zhang et al. (2019, 2020). For comparison, we did a quick query with the terms “seismic” and “full-waveform inversion,” a well-established and widely-used technique in the industry (Virieux and Operto, 2009); 12.2% of the articles were written by authors affiliated with industrial institutions. This suggests that the field of seismic diffractions imaging has a level of engagement from industrial institutions similar to that seen in the well-established technique of seismic full-waveform inversion. This partnership between academia and industry has the potential to drive the creation of innovative techniques and approaches that are both scientifically rigorous and practically relevant to the challenges faced in oil and gas exploration. The integration of academic and industrial perspectives can result in more robust and effective solutions to the challenges faced in the industry. It indicates that the participation of the industrial sector in this seismic research field is important, as they can bring their expertise and resources in the oil and gas exploration industry to the development of the techniques.

Figure 11d shows the difference between articles that apply only seismic data ($n=143$) and articles that combine seismic with GPR (ground-penetrating radar)

data ($n=7$) (De Figueiredo et al., 2013; Economou et al., 2020; Grasmueck et al., 2015, 2013; Maciel and Biloti, 2020; Schwarz and Krawczyk, 2020; Zhao et al., 2019b). GPR is a non-destructive geophysical imaging technique commonly used in various applications, including civil engineering (Orlando and Slob, 2009; Pérez-Gracia and Solla, 2015), archaeology (Conyers, 2013; Conyers and Leckebusch, 2010), environmental assessments (Knight, 2001), forensics investigations (Barone and Maggio, 2019; Castro and Cunha, 2021; Fiedler et al., 2009), estimation of water content (Parsekian, 2018), and tree roots imaging (de Aguiar et al., 2021; Guo et al., 2013). It works by emitting electromagnetic waves into the subsurface and recording the return signals. The ability of GPR to image subsurface features and anomalies has made it a valuable tool for various applications, including the study of diffractions imaging. The use of GPR for diffractions imaging is relatively limited compared to seismic techniques, but the results obtained from radargrams have shown promise in this area. It provides a complementary approach to seismic methods and offers some unique advantages, such as high-resolution and the ability to image near-surface features. Despite its potential, more research is needed to fully realize the potential of this method for diffractions imaging and to better understand the associated limitations and challenges.

Regarding the investigation depths of the analyzed papers, we considered three different data types (toy models, synthetic, and field data), and two different depth ranges (near-surface and oil exploration) (Figure 12). Many works use more than one data type to assess and demonstrate their methods, e.g., Chen et al. (2022); Li et al. (2020b, 2022c); Lowney et al. (2021b, 2020); Schwarz and Krawczyk (2020); Zhang et al. (2020). The near surface depths range from a few meters to 1 km, and the oil exploration depths range from 1 km up to 20 km. Our query did not return any work using seismic diffractions for crustal studies. We identified 98 examples of toy models, i.e., synthetic models with simple geological settings as constant velocities and few reflectors, for a general understanding of the proposed method. 32.7% ($n=32$) of the toy models are in the near-surface range, and 67.3% ($n=66$) of the toy model examples are in the oil exploration depths range. There are 56 realistic synthetic examples. Only 10.7% ($n=6$) refers to the near-surface range, while 89.3% ($n=50$) refers to the oil exploration range. The most common data type is field data, with 119 examples. 25.2% ($n=30$) of the field data examples are in the near-surface range, and 74.8% ($n=89$) are in the oil exploration range.

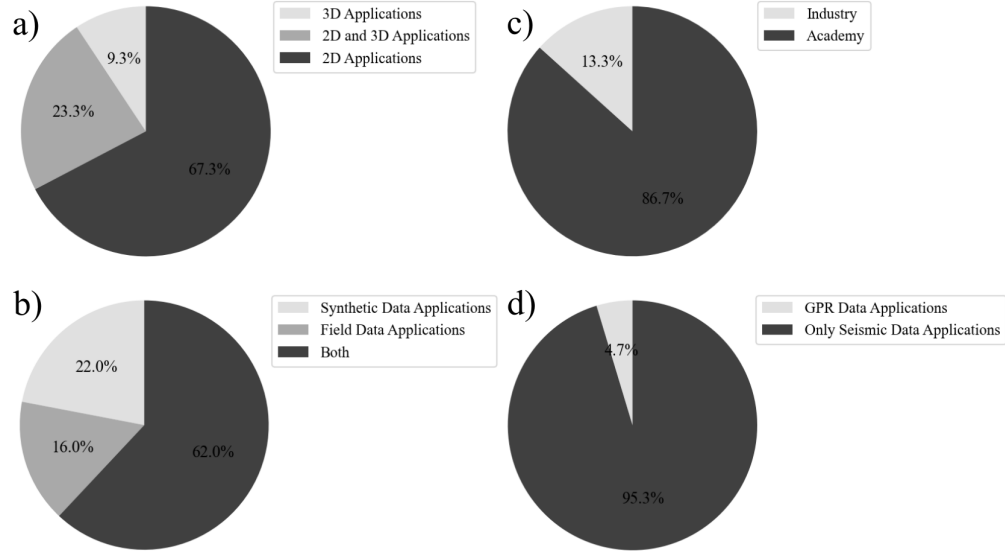


Figure 11: Distribution of the reviewed articles according to (a) dimension of the dataset; (b) synthetic and/or field data applications; (c) first author's affiliation type; (d) use of GPR data.

4 Discussion

The most used method in the analyzed works for diffractions enhancement and imaging is plane-wave destruction (PWD), applied by 15.33% ($n=23$) of the papers, e.g., Bashir et al. (2018a,b); Burnett et al. (2015); Decker et al. (2014); Fomel et al. (2007); Ford et al. (2021); Kong et al. (2017); Lin et al. (2022c, 2018a); Lowney et al. (2021b, 2020); Merzlikin and Fomel (2017); Merzlikin et al. (2019, 2020); Montazeri et al. (2019); Mu et al. (2021); Ogiesoba et al. (2015); Sturzu et al. (2014); Tyiasning et al. (2016); Zhao et al. (2019b). PWD is a powerful tool for diffraction imaging, as it can suppress strong reflections and reveal weak diffracted signals with minimal parameters tuning, thus enhancing the resolution of seismic images. Despite its effectiveness, PWD has some limitations. One of these limitations is the assumption that the reflection events have a smoothed, gradual slope. This assumption is not always accurate, especially in areas with complex geology, such as faults and fractures, where residual reflection energy may remain, potentially compromising the image quality (Chen et al., 2013). Nevertheless, its widespread use in the literature highlights its potential for improving the interpretation of seismic data and detecting important geological features. Some works compare their methods with the PWD as a state-of-the-art example (Chen et al., 2022;

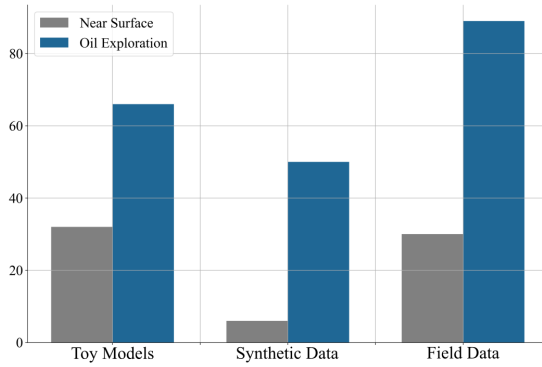


Figure 12: Investigation depths of the examples shown in the analyzed works. The near surface depth range from a few meters to 1 km, and the oil exploration depth ranges from 1 km to 20 km.

Lowney et al., 2021a; Wang et al., 2020b).

We noticed a growing trend in the number of publications over the years, according to Figure 5. It suggests that the technique is gaining more attention and recognition in the geoscience community. It is an indication of growing interest in this field of research, which could be attributed to the increasing need for more precise and accurate subsurface imaging in different applications. The increase in the number of publications could also be an indication of advances in technology and the availability of more sophisticated imaging methods, which make it easier and more effective to apply this technique. The trend suggests that there is a growing interest and momentum to develop new methods and approaches to enhance its application and effectiveness, as well as studying its limitations.

Comparing the most common words from the word clouds (Tables 4 and 5), there is a continuity in the themes and topics explored by the new publishing authors since many words are repeated in both tables (e.g., “diffraction,” “seismic,” “imaging,” “method,” “separation,” “reflections,” “velocity,” “analysis,” “migration,” “3d,” and “plane-wave”). From the 30 most common words in the word map elaborated for all the 150 articles, 23 are repeated in the word map of the 82 articles published in the last 5 years. Among the new words that have appeared in the last 5 years (Table 5), we highlight “faults,” “discontinuities,” “high-resolution,” and “reservoirs,” related to the main applications of the diffractions imaging (Table 6). The continuity in the themes and topics explored by new publishing authors in the past 5 years suggests that the field of seismic diffractions imaging has reached a level of maturity, where researchers are consistently exploring and developing the fundamental concepts. The high frequency of words such as “diffraction,” “seismic,” “imaging,” “method,” “separation,” and “reflections” in both word clouds highlights the central focus and the core concepts of the research on this topic. The emergence of new words such as “faults,” “discontinuities,” and “reservoirs” in the word map of the 82 articles published in the last 5 years suggests a shift in the focus of research towards the practical applications of diffractions imaging. This shift in focus is likely driven by the increasing demand for more accurate and efficient imaging methods in the oil and gas industry, where the diffractions imaging is used to locate and map subsurface features such as faults, fractures, and other indicators of potential reservoirs.

Considering the co-authorship maps using countries and authors as units of analysis (Figures 6 and 10, respectively), we noticed that the United States has a le-

ading role in all analysis. Nevertheless, China has occupied an important role in recent years, with significant contributions from its authors to developing methods. The dominance of the United States in the co-authorship map suggests that the country has been a major contributor to the advancement of this field. However, the increasing involvement of Chinese authors in recent years also indicates a shift in the distribution of research efforts and collaborations. The significant contributions of Chinese authors in developing methods for seismic diffractions imaging may reflect the country's growing investment in research and development, as well as its emphasis on innovation and technological advancement. This shift may also reflect the increasing globalization and internationalization of research, as more countries and authors from diverse regions become involved in this field. This finding highlights the dynamic and evolving nature of seismic diffractions imaging research topic and the importance of collaborations across borders and cultures.

Regarding the contributions of the Chinese researchers to the theme, we highlight the emergent contributions on the coal mining topic (Table 6), as the application of diffraction imaging has shown to be highly beneficial for various reasons. Firstly, it enables the interpretation of minor faults and fractures, which are often related to the presence of coal seams (Zhou et al., 2022). This information can be used to identify potential areas for coal mining and to better understand the geological structures that underlie coal deposits. Additionally, diffraction imaging can also be used to prevent water inrush and coal outbursts (Li et al., 2021a; Wang et al., 2020a), which are major safety concerns in coal mines. The energy of the diffractions can provide crucial information that can aid in preventing these types of disasters. Several studies in the field have highlighted the significance of these contributions and their potential to improve the safety and efficiency of coal mining operations.

After analyzing the 10 most cited references in the co-citation analysis for both WoS and Scopus databases (Table 3), 8 references are present in both databases (Berkovitch et al., 2009; Dell and Gajewski, 2011; Fomel, 2002; Fomel et al., 2007; Khaidukov et al., 2004; Klovov and Fomel, 2012; Landa et al., 1987; Moser and Howard, 2008). A total of 3208 references were considered in the co-citation analysis and only 12 were returned as the most cited in the two databases. Despite analyzing a large number of references, the co-citation analysis revealed a limited overlap in the most cited references between the two databases. This result suggests that there is some agreement between the two da-

tabases on the most important references in the field, as well as on the citation patterns and research interests.

According to Figure 11b, most works (84%, $n=126$) use synthetic data (including toy models) to test, validate and demonstrate their method. The prevalence of synthetic data in the majority of seismic diffractions imaging research indicates a need for a controlled and reproducible environment to test and validate new methods. The use of synthetic data allows researchers to evaluate the effectiveness of their method before applying it to real-world data. Additionally, synthetic data can be more easily shared and compared across different research groups, promoting collaboration and knowledge sharing. The most common synthetic examples are the Sigsbee2A / Sigsbee2B (Bauer et al., 2016; Dafni and Symes, 2017; Dell and Gajewski, 2011; Klovov and Fomel, 2012; Li et al., 2020a,b; Liu et al., 2016; Mu et al., 2021; Sheng and Zhao, 2022; Silvestrov et al., 2016; Yin and Nakata, 2019; Zhao et al., 2021), and the Marmousi / Marmousi II models (Asgedom et al., 2013; Gelius et al., 2013; Klovov and Fomel, 2012; Li et al., 2020c; Liu et al., 2022, 2016; Zhang et al., 2019, 2020; Zhao et al., 2021). The frequent use of the Sigsbee2A/B and Marmousi/Marmousi II models as synthetic examples may suggest that these models have become standards in the field. This may make it easier for researchers to compare their results to those of other studies that have used the same models, promoting consistency and comparability. However, it is also important to recognize that these models may not accurately reflect real-world geological structures, and therefore the effectiveness of methods demonstrated on these models may not necessarily translate to real-world applications. Concerning the field data, several examples can be found in the articles: the Nankai Trough subduction zone lines (Moore et al., 1990), e.g., Decker and Fomel (2021); Decker et al. (2017); Li et al. (2022c, 2021a,b); Lin et al. (2022c, 2020); Sheng and Zhao (2022); Wang et al. (2020b); data sets from the Viking Graben (Madiba and McMechan, 2003), e.g., Dafni and Symes (2017); Kim et al. (2021); Klovov and Fomel (2012); Lin et al. (2022a); Merzlikin et al. (2019); the Gulf of Mexico data set, e.g., Fomel et al. (2007); Lin et al. (2022b, 2021b); Merzlikin et al. (2020); and the Jequitinhonha data set, e.g., Asgedom et al. (2013); Coimbra et al. (2018); Gelius et al. (2013). The greatest depths found in the field data examples (Figure 12) do not exceed 20 km or 10 s in depth: the Nankai data set (depth = 8 s); the deep-water marine examples of Lowney et al. (2020) (depth = 5.5 s), Lowney et al. (2021a) (depth = 5.4 s), and Lowney et al. (2021b) (depth = 4 s); the 3D seismic land examples of Zhao et al. (2020)

(depth = 3.7 s), Zhao et al. (2019a) (depth = 6 s), Zhuang et al. (2022) (depth = 10 km), and Ahmed et al. (2019) (depth = 8.4 km); the example of a salt diapir interpretation in the North Sea of Montazeri et al. (2019) (depth = 3.5 s); the Jequitinhonha Basin examples (Asgedom et al., 2013; Coimbra et al., 2018; Gelius et al., 2013) (depths = 4 and 7 s); and the example of Zhang (2013) (depth = 6 km). The field data examples from the 150 analyzed papers are mostly focused on shallower portions of the Earth's crust. One possible reason could be the limitations of current seismic diffraction imaging methods, which may not be able to accurately image deeper portions of the crust. Another possibility is that the authors present more interested in studying shallow geological structures and processes, such as sedimentary basins, rather than the deeper portions of the Earth's crust. These structures zones can provide information on the presence and distribution of hydrocarbon reservoirs, which is crucial for the exploration and production of oil and gas. Seismic diffractions imaging can provide high-resolution images of these shallow structures and help to identify areas with high potential for oil and gas deposits. While this focus provides valuable information on shallow geological structures, it could be beneficial to investigate deeper portions of the Earth's crust. Studying deeper portions of the Earth's crust can provide important insights into tectonic processes, the Earth's interior, and other geological phenomena that are not observable at shallower depths. For example, the Mohorovicic discontinuity is a boundary of great interest to the geology and seismology communities and studying it using seismic diffractions could provide valuable insights. Future studies could concentrate on addressing the technical difficulties in exploring deeper portions of the Earth's crust, and on exploring new methods' advancements that could lead to a better understanding of the Earth's interior using the seismic diffractions.

Figure 5 indicates a significant surge in the number of publications since 2010, with 91.33% ($n=137$) of works being published during this period. This suggests a consistent presentation and improvement of new methods in seismic imaging. However, it is noteworthy that a limited number of articles focus on practical applications with field data. Figure 11b demonstrates that only 16% ($n=24$) of the analyzed papers exclusively utilize field data in their applications. Despite many studies suggesting the potential of their techniques in identifying structures related to oil and gas presence (Decker et al., 2014; Li et al., 2020c,d; Lin et al., 2021a; Yu et al., 2017; Zhao et al., 2015), only a few papers actually provide concrete examples focused on interpreta-

tions, e.g., Burnett et al. (2015); Klovov et al. (2017); Montazeri et al. (2019); Ogiesoba et al. (2015); Schoepp et al. (2015). We noticed that this lack of practical application is further reinforced by the absence of common themes or shared applications within the bibliographic coupling networks (Figures 7 and 8). The works within the clusters mainly present new, relevant, and improved methods for separating and imaging seismic diffractions without a specific application or theme connecting them. These findings indicate a clear demand and open horizons for new studies that focus on the application, analysis, and interpretation of diffraction images. There is a need to bridge the gap between theoretical advancements and practical implementation, providing concrete examples and addressing real-world challenges. Exploring and exploiting the potential of diffraction images in various applications can lead to significant advancements in the field and contribute to a more comprehensive understanding of subsurface structures.

5 Categorization of Seismic Diffractions Processing Methods

Authors separate the diffractions processing methods in two (Kim et al., 2021; Li et al., 2022c; Li and Qu, 2022; Lin et al., 2021b; Liu et al., 2022; Schwarz, 2019a; Wang et al., 2020b; Zhou et al., 2023) or three (Li et al., 2021a; Zhao et al., 2019a) categories based on the different mathematical approaches employed. In this work, we will consider two distinct categories. It is important to note that in addition to the works analyzed in our previous discussions, we have included additional papers in this section that were not initially included in our database. This allows us to broaden the scope of our review and incorporate other significant and relevant contributions to the discussion on seismic diffractions imaging.

The first category includes methods that separate diffractions from reflections in the data domain before or after the migration process. This group includes the local slant-slack separation method presented by Harlan et al. (1984). Several studies explore the plane-wave destruction filter (PWD), such as Decker et al. (2017), Fomel et al. (2007), Li et al. (2022a), Li et al. (2022b), Lin et al. (2018c), Lowney et al. (2020), Taner (2006), and Yuan et al. (2019). The coherent subtraction method sums the amplitudes of reflections and separates them from diffractions based on wavefront filters (Delf et al., 2022; Schwarz, 2019b; Schwarz and Gajewski, 2017). Lin et al. (2020) present the method of multichannel singular-spectrum analysis (MSSA), in which the reflections are suppressed in the space-

frequency domain (f - x) from the analysis of the reflections' linear characteristics (Lin et al., 2022a; Zhou et al., 2023). Later, the study of Shen et al. (2020) improved the method through the singular value decomposition (SVD). The authors use a filter based on differences in coherence and travel-times between the wavefields. Jiang et al. (2022) proposed an improvement for the SVD method by using a second-order difference spectrum strategy. Another f - x domain technique is the variational mode decomposition (VMD) (Lin et al., 2021b), which considers the sparse aspects of the event's energy distribution. Zhao et al. (2019a) perform the separation of events in the shot domain through the space-varying media filter (SVMF); the shot domain is also known as common virtual source gather (Li et al., 2021a). Decker and Fomel (2021) apply a technique in datasets where the reflection energy has already been suppressed. This study summed the possible signals generated by different migration velocities (path integral seismic imaging) (Merzlikin and Fomel, 2017; Merzlikin et al., 2019, 2020) in order to accentuate the energy generated by subsurface heterogeneities.

The second category includes methods that separate diffractions directly during the migration process. Landa et al. (1987) create diffraction sections (D-sections) from phase correlations; Kanasevich and Phadke (1988) build gathers of common-fault-points from the distribution of diffraction amplitudes; Landa and Keydar (1998) create sections of common-diffraction-point to emphasize the amplitude of diffractions and detect heterogeneities. Khaidukov et al. (2004) modify the elementary diffractions' travel-time term in the migration kernel to create a domain where it is possible to focus the energy of reflections and later mute them from the diffracted wavefield. Berkovitch et al. (2009) also explore minor modifications in the migration equation to create the diffraction multifocus method. Similarly to the previous method, it focuses the diffraction energy based on the travel-time corrections in the migration integral. Moser and Howard (2008) developed an anti-stationary phase filter by modifying the Kirchhoff integral's weight function to suppress the reflections' energy. Li et al. (2020c) modified this filter by analyzing the phases of the signals and then reversing the polarity to reduce the residual energy of reflections. Zhang and Zhang (2014) propose muting the energy of reflections from the Fresnel zone in the shot gathers and aperture angles. Li et al. (2020d), Zhao et al. (2016), and Zhao et al. (2020) used the Mahalanobis distance to include an exponential function in the Kirchhoff integral to suppress the reflection amplitudes. Dell and Gajewski (2011), Rad

et al. (2018), and Schwarz et al. (2014) proposed the common-reflection-surface method based on the curvature differences between reflections and diffractions responses. Methods based on the Radon transform (Klokov and Fomel, 2012, 2013; Li et al., 2018; Liu et al., 2016; Ogiesoba and Klokov, 2017) perform the separation of the different components of the wavefield in the dip-angle domain after the migration; however, as they are based on derivations of migration algorithms, we understand that they fit better in this category. It is important to note that events separation in the dip-angle domain is not restricted to the traditional Kirchhoff migration (Dafni and Symes, 2017; Silvestrov et al., 2016; Zhu and Wu, 2010).

Additionally, machine learning techniques (ML) have also been effective in separating and imaging diffractions. De Figueiredo et al. (2013) presented one of the pioneering studies on using ML algorithms to separate diffractions. The authors classified images based on the analysis of elementary diffraction amplitudes. They applied the k-nearest neighbors technique (kNN) in order to distinguish images containing diffractions, reflections or noise by analyzing the distribution of amplitudes along the diffraction travel-time curves. Maciel and Biloti (2020) introduced a novel approach to characterize seismic events using statistical parameters. In this study, the authors developed a statistical descriptor based on the difference between dynamic and kinematic aspects of wavefields. This method presents the correct classification of events using the ML technique known as support vector machines (SVMs) (Cortes and Vapnik, 1995). Shustak and Landa (2018) employed the principle of focusing reflections on secondary sources to analyze the patterns generated after data migration. They used the Gaussian classifier naive Bayes, a supervised learning technique. Markovic et al. (2022) applied a deep segmentation model for recognizing diffraction patterns on seismic and GPR datasets. Tschannen et al. (2020) applied convolutional neural networks (CNNs) to differentiate images containing diffractions and reflections. The algorithm was trained from synthetic sections based on wave equation modeling. The application of CNNs, a class of deep learning algorithms, was also studied by Kim et al. (2021), Lowney et al. (2021a), Lowney et al. (2021b), Sheng and Zhao (2022), and Sun et al. (2022).

6 Overview of Methods' Theory

Based on our analysis, we selected six methods and provided an overview of their theories: the anti-stationary phase filter (Moser and Howard, 2008), the focusing-muting-defocusing (Khaidukov et al., 2004),

the multifocusing (Berkovitch et al., 2009), the plane-wave destruction (Fomel, 2002; Fomel et al., 2007), the common-reflection-surface-based method (Dell and Gajewski, 2011), and the dip-angle domain approach (Klokov and Fomel, 2012). This section provides a brief summary of the main methods' theories. For a more in-depth understanding of each method, we recommend consulting the original papers individually.

6.1 Anti-stationary phase filter

The Kirchhoff migration integral is defined as (Schneider, 1978; Tygel et al., 1996)

$$V(x) = \int d\xi \int dt U(\xi, t) \delta(t - t_d(\xi, x)), \quad (1)$$

where $V(x)$ is the seismic image at position x , obtained by integrating the seismic data $U(\xi, t)$ over all possible source-receiver positions ξ and times t . The travel-time trajectory t_d is given by

$$t_d(s, x, r) = T(s, x) + T(x, r), \quad (2)$$

where $T(s, x)$ is the travel-time from the source to the diffraction point, and $T(x, r)$ is the travel-time from the diffraction point to the receiver. It is possible to include an isotropic weighting term in Equation 1 and rewrite the integral as (Hubral et al., 1996)

$$V^D(x) = \int w(s, x, r) U(t, s, r) \delta(t - t_d(s, x, r)) dt ds dr. \quad (3)$$

Considering the ray vectors from the source $p_s = \nabla_x T(s, x)$ and the receiver $p_r = \nabla_x T(r, x)$, and a local unit vector normal to the reflector n , the weight function can be defined as (Moser and Howard, 2008)

$$w(s, x, r) = n^T \frac{(p_s + p_r)}{\|p_s + p_r\|}. \quad (4)$$

The weight function, also known as the stationary phase filter, is implemented to the diffraction stack with the objective of emphasizing contributions in the Fresnel zone around the specular reflection point, which minimizes the aliasing effect (Chen, 2004), as illustrated in Figure 13. Moser and Howard (2008) proposed modifying the weight function to emphasize contributions from outside the first Fresnel zone:

$$w_d(s, x, r) = 1 - n^T \frac{(p_s + p_r)}{\|p_s + p_r\|}. \quad (5)$$

The anti-stationary phase filter w_d explicitly suppresses the reflected portion of the total wavefield and enhances the diffractions in the migrated image (Li et al., 2020c; Moser and Howard, 2008). An open-source solution for performing the anti-stationary phase filter migration is available at github.com/GuilhermeZakarewicz/DiffraPy.

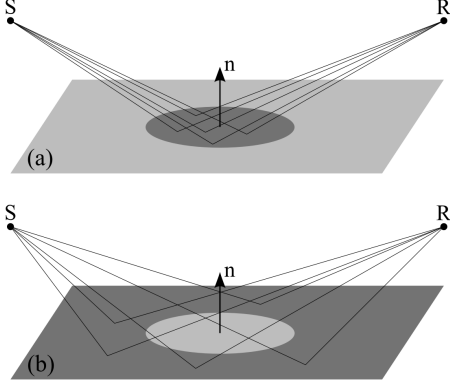


Figure 13: Principle of the weight function $w(s, x, r)$. (a) Conventional migration only considers the specular reflections from the Fresnel zone. (b) The modification of w by $w_d(s, x, r)$ causes migration to only consider non-specular information from outside the Fresnel zone. Modified from Moser and Howard (2008).

6.2 Focusing-Muting-Defocusing

The method proposed by Khaidukov et al. (2004) involves separating the diffraction component from the reflection component of the total wavefield before applying the Kirchhoff integral to the seismic data. The authors suggest replacing the diffraction propagation time t_d with the reflection propagation time $t_r = T(s, x)$ in Equation 1:

$$V^R(x) = \int d\xi \int dt U(\xi, t) \delta(t - t_r(\xi, x)). \quad (6)$$

The new operation is called the reflection stack or “reflection focusing operator”. The propagation time of a reflection can be understood as originating from a virtual source mirrored relative to the position of the reflector (Figure 14), which is not valid for real diffractions. Thus, the operator focuses the energy of the reflections to this virtual point, which usually has a greater depth than the reflector. Since the operator only acts for reflection propagation times, diffractions remain unfocused. Therefore, the image of focused reflections $V^R(x)$ has twice the depth of the diffraction stack $V(x)$

and transfers the seismic data to a domain where diffractions and reflections are separated.

The next step of the method is to mute the regions of $V^R(x)$ with a high concentration of focused energy, i.e., suppress the energy of the reflections and preserve the diffractions. The new set of data is denoted as $V_0^R(x)$. The Kirchhoff demigration integral (Santos et al., 2000) is applied to the data:

$$U^D(\xi, t) = \int dx V_0^R(x) \delta(t - t_r(\xi, x)). \quad (7)$$

U^D will mainly contain diffractions, depending on the quality of the reflection focus and the process of muting its energy. The final step of the method proposed by Khaidukov et al. (2004) is to apply the conventional diffractions stack (Equation 6) to the field of residual diffracted waves U^D obtained after Equation 7:

$$V^D(x) = \int d\xi \int dt U^D(\xi, t) \delta(t - t_d(\xi, x)), \quad (8)$$

The resulting $V^D(x)$ will be a seismic image in which the diffractions were intentionally preserved, without the traditional bias in relation to reflections.

6.3 Multifocusing

The method proposed by Berkovitch et al. (2009) is a time imaging approach based on move-out corrections and coherent summation of diffractions. The method is based on a normal ray and a paraxial ray to determine the location of the intersect point M. The normal ray starts at point X_0 with a specific angle relative to the vertical β , encounters a reflecting interface at the normal incidence point O and then returns to the point X_0 (Figure 15). On other hand, the paraxial ray emitted from an arbitrary source S intersects the central ray at point M , reflects at the interface Σ (point P) and is detected by receiver R .

Both the normal and paraxial rays determine imaginary focused wavefronts. The intersect point M can be considered as a source of two virtual wavefronts, one emitted upward to the surface (Σ^-), and the other emitted downward to the reflecting interface Σ , which reflects the wavefront and emerges at X_0 as Σ^+ . The focusing points M^- and M^+ are the center of curvature for the fictitious circular wavefronts Σ^- and Σ^+ , respectively. The method involves calculating the travel-time correction, which is the time difference between the normal and paraxial rays. For a laterally homogeneous velocity model, the equation is defined as (Berkovitch et al., 2008):

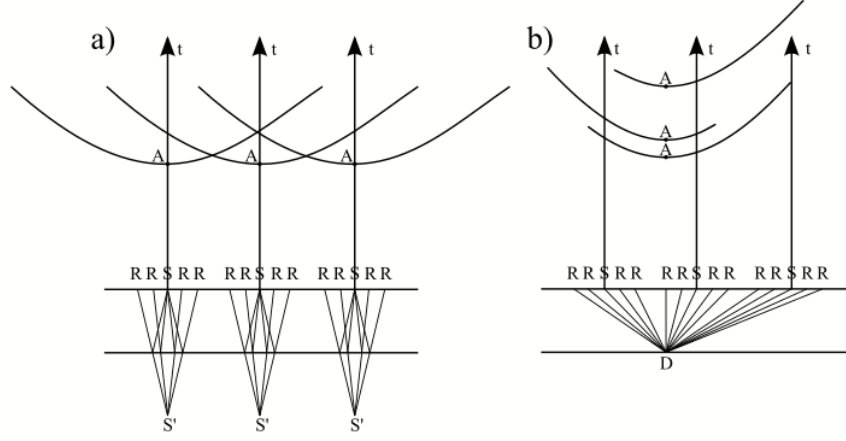


Figure 14: Geometric difference between reflections and diffractions in a medium with constant velocity. S and R are the positions of the source and receivers; A is the apex of the hyperbola; S' is the projection of the reflection on the subsurface; and D is the diffracting point. (a) Reflection geometry for a horizontal planar interface; (b) Diffraction geometry for a diffracting point. Note that the reflection can be understood as originating from a virtual point S' mirrored relative to the position of the reflector. Modified from Khaidukov et al. (2004).

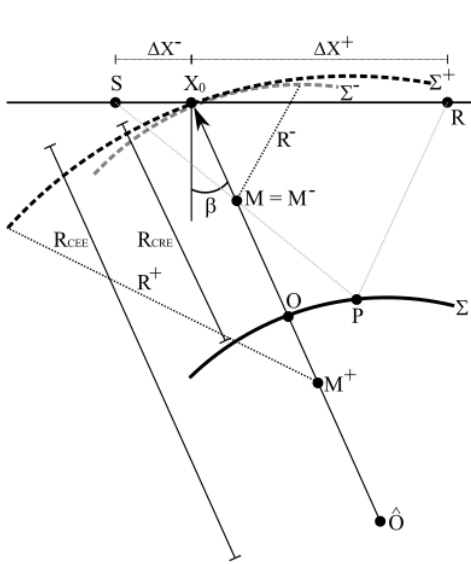


Figure 15: Schematic illustration for geometry of the multifocusing method for a circular reflector Σ . S and R are the source and receiver locations, ΔX^+ and ΔX^- are the offsets from the central point, X_0 is the image point, Σ^+ and Σ^- are the virtual wavefronts emitted by P , P is the intersection point between the normal and the paraxial rays, R^+ and R^- are the radii of curvature of Σ^+ and Σ^- , and β is the incident angle of the normal incident ray. Modified from Berkovitch et al. (2009).

$$\Delta t = \frac{\sqrt{(R^+)^2 - 2R^+\Delta X^+ \sin \beta + (\Delta X^+)^2} - R^+}{V_0} + \frac{\sqrt{(R^-)^2 - 2R^-\Delta X^- \sin \beta + (\Delta X^-)^2} - R^-}{V_0}, \quad (9)$$

where

$$R^+ = \frac{1 + \sigma}{\frac{1}{R_{CEE}} + \frac{\sigma}{R_{CRE}}}; \quad R^- = \frac{1 - \sigma}{\frac{1}{R_{CEE}} - \frac{\sigma}{R_{CRE}}}; \quad (10)$$

and

$$\sigma = \frac{\Delta X^+ - \Delta X^-}{\Delta X^+ + \Delta X^- + 2 \frac{\Delta X^+ \Delta X^-}{R_{CRE}} \sin \beta}. \quad (11)$$

In Equations 9, 10, and 11, ΔX^+ and ΔX^- are the offsets of the source and receiver, respectively, for a given specular ray with respect to the central point X_0 , R^+ and R^- are the radii of curvature of the virtual wavefronts Σ^+ and Σ^- , V_0 is the near-surface velocity, and σ is the “focusing parameter”. For a given source and receiver (S, R) in 2D situations, the multifocusing move-out formula (Equation 10) express the time shifts with respect to a zero offset trace in terms of three parameters: the emergent angle of a normal ray β (ray perpendicular to a reflector) and the curvature radii (R_{CRE} and R_{CEE}) of two fundamental circular wavefronts.

Considering the situation where the reflecting interface defined in Figure 15 shrinks to a diffraction point

O , the points P and M^+ will coincide with the position O . Furthermore, $R_{CEE} = R^+ = R_{CRE} = R^- = R_d$, where R_d represents the radius of curvature of the diffracted wavefront. Substituting it in the move-out Equation 10, we obtain the expression for multifocusing move-out correction of diffraction wavefronts:

$$\Delta t = \frac{\sqrt{(R_d)^2 - 2R_d\Delta X^+ \sin \beta + (\Delta X^+)^2} - R_d}{V_0} + \frac{\sqrt{(R_d)^2 - 2R_d\Delta X^- \sin \beta + (\Delta X^-)^2} - R_d}{V_0}, \quad (12)$$

For a diffracted wave, the multifocusing move-out expression (Equation 12) depends on only two parameters, namely R_d and β (Berkovitch et al., 2009). However, the generalization of the multifocusing method for 3D situations is not a trivial task (Schoepp et al., 2014), as there are five parameters to be estimated from data (three curvatures and two emergence angles).

6.4 Plane-wave destruction

Considering the case of a point source, as shown in Figure 14, both the reflector and the diffractor act as point sources. However, the point source of the reflector is projected onto a virtual point S' , which does not occur with the diffractor. This results in hyperbolic responses in the seismograms. On the other hand, when a plane wave source is considered, the diffractor generates a hyperbolic response similar to the point source case. However, the reflected event creates a plane wave response (Taner, 2006), as illustrated in Figure 16.

As described by Taner (2006), a way to create a synthetic trace that simulates the reflection data obtained if multiple sources exploded at the receiver location and recorded the reflected data at the source position is by summing common shot seismic traces horizontally without any time delay. By performing this process for more shots, we generate a record that simulates a plane wave source.

The local plane differential equation is defined as (Claerbout, 1992; Fomel, 2002):

$$\frac{\partial P}{\partial x} + \sigma \frac{\partial P}{\partial t} = 0, \quad (13)$$

where $P(t, x)$ is the wavefield, and $\sigma(t, x)$ is the local slope. The PWD filter uses plane wave theory to estimate the local slope of energy in seismic data, identifying events that conform to a linear pattern and suppressing them. PWD predicts and flattens strong reflections in the shot domain while retaining diffractions by

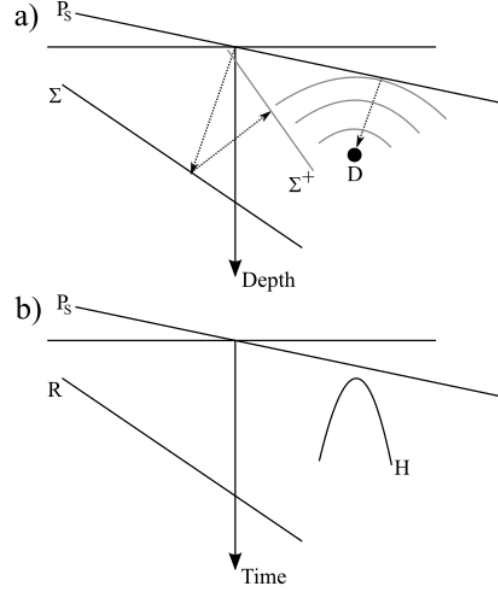


Figure 16: Illustration of a specular reflector Σ and a scatterer point D illuminated by a plane source wave P_S . (a) Depth section; (b) Time section. The reflected wave Σ^+ generates a planar response R , while the diffracted response H is hyperbolic. Modified from Taner (2006).

estimating the local slope of plane-wave energy and determining the required vertical shift to minimize residual energy not predicted by the corresponding trace. Therefore, events that exhibit linear patterns, or plane waves, will be suppressed by PWD, while diffractions are preserved.

For predicting several plane waves, we can cascade several filters similar to $A(Z_x)$:

$$A(Z_x) = \left(1 - \frac{Z_x}{Z_1}\right) \cdot \left(1 - \frac{Z_x}{Z_2}\right) \left(1 - \frac{Z_x}{Z_n}\right), \quad (14)$$

where $Z_1, Z_2, \dots, Z_n$ are the zeroes of the polynomial

$$A(Z_x) = 1 + a_1 Z_x + a_2 Z_x^2 + \dots + a_n Z_x^n. \quad (15)$$

Calculating the Taylor series for Equation 14, i.e., equalizing the coefficients of the Taylor series expansion around zero frequency, generates the following:

$$B_3(Z_t) = \frac{(1 - \sigma)(2 - \sigma)}{12} Z_t^{-1} + \frac{(2 + \sigma)(2 - \sigma)}{6} + \frac{(1 + \sigma)(2 + \sigma)}{12} Z_t, \quad (16)$$

and

$$\begin{aligned}
 B_5(Z_t) = & \frac{(1-\sigma)(2-\sigma)(3-\sigma)(4-\sigma)}{1680} Z_t^{-2} \\
 & + \frac{(4-\sigma)(3-\sigma)(2-\sigma)(4+\sigma)}{420} Z_t^{-1} \\
 & + \frac{(4-\sigma)(3-\sigma)(4+\sigma)(3+\sigma)}{280} \\
 & + \frac{(4-\sigma)(4+\sigma)(3+\sigma)(2+\sigma)}{420} Z_t \\
 & + \frac{(4+\sigma)(3+\sigma)(2+\sigma)(1+\sigma)}{1680} Z_t^2
 \end{aligned} \quad (17)$$

for a three and five-point centered filters $B_3(Z_t)$ and $B_5(Z_t)$, respectively. The detailed derivation of Equations 16 and 17 can be found in Fomel (2002). It is possible to characterize several plane waves in a modified form of Equation 14 using the filter

$$C(Z_t, Z_x) = A(Z_t, Z_x)B\left(\frac{1}{Z_t}\right) = \left(\frac{1}{Z_t}\right) - Z_x B(Z_t), \quad (18)$$

which avoids the requirement of a polynomial division. In the case of a three-point centered filter (Equation 16), the 2D filter (Equation 18) presents six coefficients comprised in two columns, where each has three coefficients and the second column is a reversed copy of the first one.

The estimation of the slope is necessary to apply the plane wave filters on real data. Considering $C(\sigma)$ as the convolution of the data with the 2D filter $C(Z_t, Z_x)$ (Equation 18), we determine the least-square goal (Fomel, 2002):

$$C(\sigma)d \approx 0, \quad (19)$$

where d is the data. Fomel (2002) discusses that the slope enters in the filter coefficients in a nonlinear way, as shown in Equations 16 and 17. Nevertheless, it is possible to apply the linear iterative optimization methods by an analytical linearization of Equation 19. The process of linearization involves the solution of a linear system:

$$C'(\sigma_0)\Delta\sigma d + C(\sigma_0)d \approx 0, \quad (20)$$

where $\Delta\sigma$ is the slope increment, σ_0 is the initial slope estimate, and C' is obtained by differentiating the filter coefficients of C with respect to σ . The number of nonlinear iterations required to achieve acceptable convergence may vary depending on the initial solution. To eliminate problems of oscillatory local slopes estimates, Fomel (2002) suggests adding a regularization goal to Equation 20:

$$\epsilon D\Delta\sigma \approx 0, \quad (21)$$

where the variable ϵ represents a scaling coefficient, while D denotes a suitable operator for roughening.

Estimating a single local slope $\sigma(t, x)$ is a more complicated task than estimating differential dual slopes σ_1 and σ_2 (Fomel, 2002). Therefore, the linearization yields

$$\begin{aligned}
 C'(\sigma_1)C(\sigma_2)\Delta\sigma d + C(\sigma_1)C'(\sigma_2)\Delta\sigma_2 d \\
 + C(\sigma_1)C(\sigma_2)d \approx 0.
 \end{aligned} \quad (22)$$

The regularization condition should be considered for both slopes:

$$\begin{aligned}
 \epsilon D\Delta\sigma_1 &\approx 0, \\
 \epsilon D\Delta\sigma_2 &\approx 0.
 \end{aligned} \quad (23)$$

The Madagascar software (Fomel et al., 2013) provides freely available functions for performing PWD filtering.

6.5 Common-reflection-surface-based method

The stacking trajectory is defined as the second-order approximation of the squared travel-time around the normal ray (Jäger et al., 2001):

$$\begin{aligned}
 t_{\text{hyp}}^2(m, h) = & (t_0 + 2p_m m)^2 \\
 & + 2t_0(M_N m^2 + M_{\text{NIP}} h^2),
 \end{aligned} \quad (24)$$

where m is the midpoint displacement with respect to the considered CMP position m_0 , h is half of the source-receiver distance, t_0 is the zero offset two way travel-time; p_m is the first-order horizontal spatial travel-time derivative to the midpoint coordinate, M_N is the second-order horizontal spatial travel-time derivative to the midpoint coordinate, and M_{NIP} is the second-order horizontal spatial travel-time derivative to the half-offset coordinate (Dell and Gajewski, 2011). p_m is related to the angle β_0 :

$$p_m = \frac{\sin \beta_0}{v_0}, \quad (25)$$

where v_0 is the near-surface velocity. M_{NIP} relates with the curvature of a wavefront emerging from m_0 when there is a point source at the normal-incident-point (NIP) of the reflector:

$$M_{\text{NIP}} = \frac{\cos^2 \beta_0}{v_0} K_{\text{NIP}}, \quad (26)$$

where K_{NIP} is the curvature of the NIP-wave. M_N relates to the curvature of a wavefront emerging from m_0

considering an exploding reflector element (the common reflection surface) centered at the NIP:

$$M_N = \frac{\cos^2 \beta_0}{v_0} K_N, \quad (27)$$

where K_N is the curvature of the normal wave.

The kinematic wavefield attributes of the CRS method are represented by the radii and emergence angles of the wavefront curvatures, which can be expressed as $K_N = 1/R_N$ and $K_{NIP} = 1/R_{NIP}$ (Figure 17). By substituting the expressions for p_m , M_N , and M_{NIP} in terms of CRS attributes into Equation 24, we obtain the CRS stacking operator:

$$t_{hyp}^2(m, h) = \left(t_0 + \frac{2 \sin \beta_0}{v_0} m \right)^2 + \frac{2 t_0 \cos^2 \beta_0}{v_0} \left(\frac{m^2}{R_N} + \frac{h^2}{R_{NIP}} \right). \quad (28)$$

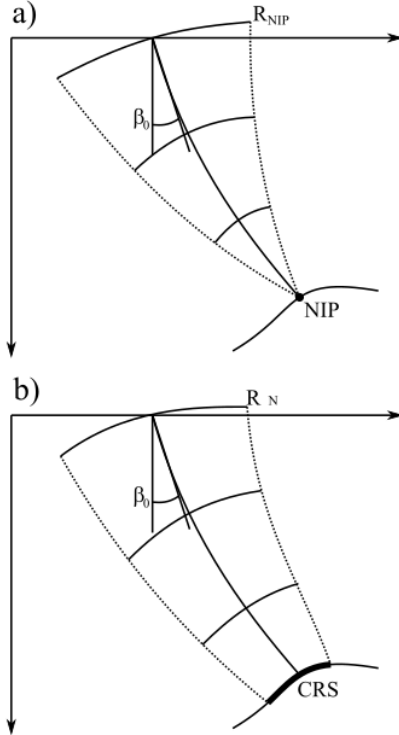


Figure 17: Illustration of the CRS parameters (Hubral, 1983). (a) R_{NIP} is the curvature radius of a virtual wavefront emitted by a source at NIP. (b) R_N is the curvature radius of a virtual wavefront emitted by the exploding reflector element CRS. The emergence angle is denoted as β_0 . Modified from Rad et al. (2018).

Following the CRS theory, we consider a diffractor as a reflector segment with a very high curvature and

undefined orientation. Therefore, the CRS shrinks to a single point and $R_N = R_{NIP}$. We apply this modification on Equation 28:

$$t_{hyp}^2(m, h) = \left(t_0 + \frac{2 \sin \beta_0}{v_0} m \right)^2 + \frac{2 t_0 \cos^2 \beta_0}{v_0 R_N} (m^2 + h^2). \quad (29)$$

As explained by Rad et al. (2018), prior to applying Equation 29 for stacking, it is necessary to separate diffractions with a criterion. This can be accomplished by comparing the values of R_N and R_{NIP} , which can be done using a specific function as a threshold (Dell and Gajewski, 2011):

$$T_f(m_0, t_0) = \exp \left(- \frac{|R_N - R_{NIP}|}{|R_N + R_{NIP}|} \right). \quad (30)$$

The value of R_N is equal to R_{NIP} for diffractions in ideal cases, but it does not occur in practice. Therefore, the $T_f(m_0, t_0)$ function can help defining a threshold. It will guarantee a soft transition for the diffraction filter, as the function approximates one for close values of R_N and R_{NIP} , and zero otherwise.

6.6 Dip-angle domain

In seismic processing, the Kirchhoff migration method relies on reflection opening and dip-angles (Klokov and Fomel, 2012; Xu et al., 2001). For a 2D scenario, the summation surface is a function of the opening and the dip-angles of the reflector, denoted by θ and α , respectively (Klokov and Fomel, 2012). By integrating a constant value of α over θ , we can generate a common-image gather (CIG) in the dip-angle domain. The CIG includes traces that represent partial images of a lateral position for various reflector dips.

Considering a 2D zero-offset constant velocity time migration, the model (x, z) and the data coordinates (y, t) can be expressed through the spatial correlations:

$$y = x + z \tan \alpha_0, \quad (31)$$

and

$$t = \frac{2z}{v \cos \alpha_0}, \quad (32)$$

where α_0 is the dip-angle (Figure 18a) and v is the constant subsurface velocity.

In other words, time migration is the process of inverting these transformations, which results in:

$$x_m = y - \frac{v_m t}{2} \sin \alpha, \quad (33)$$

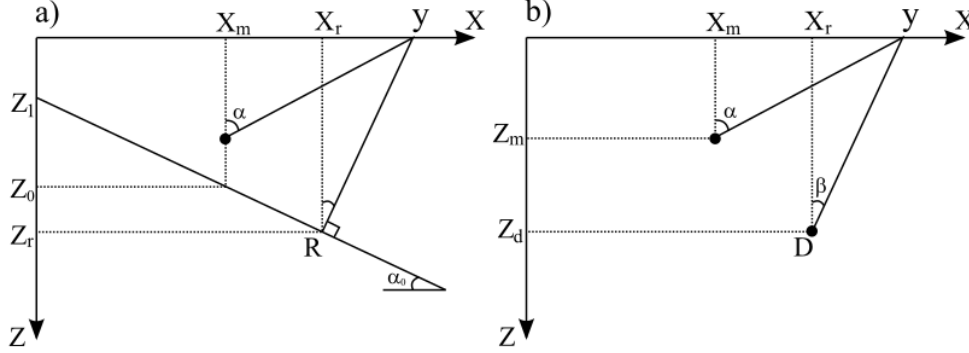


Figure 18: Illustration of the (a) zero-offset reflection and (b) zero-offset diffraction. Modified from Klovov and Fomel (2012).

$$\tau = t \cos \alpha, \quad (34)$$

where v_m is the migration constant velocity and α is the migrated angle. Considering a plane reflector, it is possible to define its response in the data domain:

$$t(y) = \frac{2(z_1 \cos \alpha_0 + y \sin \alpha_0)}{v}, \quad (35)$$

where the reflector is defined as

$$z(x) = z_1 + x \tan \alpha_0. \quad (36)$$

We can also define the reflector time image (x_m, τ):

$$x_m = -\frac{v_m}{v} z_1 \cos \alpha_0 \sin \alpha + y \left(1 - \frac{v_m}{v} \sin \alpha_0 \sin \alpha\right), \quad (37)$$

$$\tau = \frac{2}{v} \cos \alpha (z_1 \cos \alpha_0 + y \sin \alpha_0). \quad (38)$$

By removing y from Equations 37 and 38, and replacing $\gamma = v_m/v$, we generate the image of the reflector in the dip-angle coordinates (Klovov and Fomel, 2012):

$$\tau(x_m, \alpha) = \left(\frac{2 \cos \alpha}{v}\right) \frac{x_m \sin \alpha_0 + z_1 \cos \alpha_0}{1 - \gamma \sin \alpha_0 \sin \alpha}. \quad (39)$$

From Equation 36, we can derive

$$z_0 \cos \alpha_0 = x_m \sin \alpha_0 + z_1 \cos \alpha_0. \quad (40)$$

It is possible to generate the shape of a reflector in the dip-angle CIG for a lateral position by replacing Equation 40 in 39:

$$\tau(\alpha) = \tau_0 \frac{\cos \alpha_0 \cos \alpha}{1 - \gamma \sin \alpha_0 \sin \alpha}, \quad (41)$$

where the two way vertical travel-time is defined as $\tau_0 = 2z_0/v$.

Hence, for a correct migration velocity ($\gamma = 1$), the event present a shape of a smile with a stationary point (Klovov and Fomel, 2012).

Considering a diffraction point (coordinates x_d, z_d), we parameterize the data responses by the angle β :

$$y = x_d + z_d \left(\tan \beta - \frac{v_m \sin \alpha}{v \cos \beta} \right), \quad (42)$$

$$t = \tau_0 \frac{\cos \alpha}{\cos \beta}. \quad (43)$$

Substituting $\gamma = v_m/v$, $\xi = (x_m - x_d)/z_d$, and removing β , we obtain:

$$\tau(\alpha) = \tau_0 \cos \alpha \frac{\xi \gamma \sin \alpha + \sqrt{\xi^2 + 1 - \gamma^2 \sin^2 \alpha}}{1 - \gamma^2 \sin^2 \alpha}. \quad (44)$$

Now, if the migration velocity is correct ($\gamma = 1$), the event present a flat shape. Based on the differences between diffractions and reflections in the dip-angle domain, Klovov and Fomel (2012) presented a method for separating the diffracted events from the wavefield using a hybrid Radon transform (Trad et al., 2001). They use a model for describing reflection events (m_r), and one for describing diffraction events (m_d), connected to data by two operators. The adjoint operators transfer energy from the data space to the model spaces, and the forward operator moves the energy back to the data space. Once the transformation is completed, the energy of reflection events is transferred to the reflection model, while diffraction events are focused in the diffraction model (Klovov and Fomel, 2012). The operator for the adjoint transform from the data domain to the reflection-model domain is denoted by L_r^T :

$$m_r(\gamma, \alpha_0, \tau_0) = \sum_{\alpha} d[\alpha, \tau_{\alpha}(\gamma, \alpha_0, \tau_0)], \quad (45)$$

and forward transform operator L_r from the reflection model domain to the reflection data domain is given by:

$$d_r(\alpha, \tau_{\alpha}) = \sum_{\gamma} \sum_{\alpha_0} m_r[\gamma, \alpha_0, \tau_0(\gamma, \alpha_0, \tau_{\alpha})], \quad (46)$$

where τ_{α} is the imaging time, α_0 is the real reflector dip, and γ is the velocity model accuracy. For diffractions, the operator for the adjoint transform L_d^T from the data domain d to the diffraction domain m_d and the forward operator L_d can be defined as (Klokov and Fomel, 2012):

$$m_d(\gamma, \xi, \tau_0) = \sum_a d[\alpha, \tau_{\alpha}(\gamma, \xi, \tau_0)], \quad (47)$$

$$d_d(\alpha, \tau_{\alpha}) = \sum_{\gamma} \sum_{\xi} m_d[\gamma, \xi, \tau_0(\gamma, \xi, \tau_{\alpha})]. \quad (48)$$

To obtain the inverse Radon operator, we need to minimize a defined cost or objective function (Klokov and Fomel, 2012; Trad et al., 2001):

$$J(m_r, m_d) = \|W_{\text{data}}(L_r m_r + L_d m_d - d)\|_2 + \epsilon_r \|W_r m_r\|_2 + \epsilon_d \|W_d m_d\|_2, \quad (49)$$

where W_{data} consists of a diagonal matrix of weights, W_r and W_d are model space weights, and ϵ_r , ϵ_d are reflection and diffraction measures of sparseness determined heuristically. Klokov and Fomel (2012) minimize the J function by applying a quasi-Newton L-BFGS algorithm (Nocedal, 1980). The authors separate the diffracted wavefield after finding the models m_r and m_d . Thus, the diffraction image is obtained after stacking the diffracted component in the dip-angle direction.

7 Conclusions

This article provides a comprehensive overview of the seismic diffraction imaging methods by conducting a systematic literature review and quantitative analysis. The field of seismic diffractions separation and imaging has been gaining significant attention and recognition within the geoscience community, as evidenced by a notable increase in publications and citations since 2010. This growing interest indicates that the exploration of new methods, applications, and interpretations related to seismic diffractions holds great potential for

generating significant and influential research publications.

The proposed methods exhibit a diverse range of effective applications, highlighting their versatility in addressing various geological structures. Many researchers are actively engaged in characterizing faults, fractures, and small-scale discontinuities, which have significant implications across various fields and render these methods highly valuable for numerous industry applications. Notably, diffraction imaging has gained considerable traction within the oil and gas exploration sector. By leveraging diffractions, it becomes possible to image structures associated with hydrocarbon presence, migration pathways, and other pertinent aspects relevant to oil and gas exploration. The substantial number of publications authored by researchers from industrial institutions indicates a considerable engagement level from these entities.

Recent contributions from Chinese researches shows a growing interest in applying seismic diffractions to coal mining. The information extracted from the diffracted wavefield holds promise for identifying potential coal mining areas, gaining a better understanding of the underlying geological structures that encompass coal deposits, and mitigating the risks of water inrush and coal outbursts. Another emerging application is the utilization of seismic diffractions in near-surface studies, particularly in conjunction with ground-penetrating radar data. The electromagnetic method has the advantage of using high-frequency waves, extending the capability of imaging using diffractions. Further research is required to assess the full potential of seismic diffractions in these new and innovative applications, while also comprehending the associated limitations and challenges.

We noticed that forward modeling plays a crucial role and remains the predominant data type utilized across the analyzed publications. This emphasizes the significance of establishing controlled and reproducible environments to facilitate the testing and validation of novel methods, ensuring consistency and comparability in research outcomes. The field examples are mostly focused on shallower portions of the Earth's crust, not exceeding 20 km or 10 s in depth. We highlight the unexplored possibility of studying deeper portions of the subsurface and exploring new methods' advancements.

In addition, it is worth noting that while there is a wealth of literature showcasing new and improved methods related to seismic diffractions, practical field examples centered around interpretations are relatively scarce compared to synthetic applications. By prioritizing

zing investigations that focus on field applications and case studies, researchers can effectively enhance the applicability and impact of seismic diffraction imaging.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Acknowledgements

The authors would like to thank CNPq for the Research Grant provided.

References

- Ahmed, K., S. Dell, B. Schwarz, D. Gajewski, and N. Ettrich, 2019, 3-D seismic imaging in crystalline rock environments: An approach based on diffraction focusing: *Journal of Applied Geophysics*, **165**, 49–59.
- Alonaizi, F., R. Pevzner, A. Bona, M. Alshamry, E. Caspari, and B. Gurevich, 2014, Application of diffracted wave analysis to time-lapse seismic data for CO₂ leakage detection: *Geophysical Prospecting*, **62**, 197–209.
- Aria, M., and C. Cuccurullo, 2017, bibliometrix: An R-tool for comprehensive science mapping analysis: *Journal of Informetrics*, **11**, 959–975.
- Asgedom, E. G., L. J. Gelius, and M. Tygel, 2013, 2D common-offset traveltimes based diffraction enhancement and imaging: *Geophysical Prospecting*, **61**, 1178–1193.
- Bansal, R., and M. G. Imhof, 2005, Diffraction enhancement in prestack seismic data: *Geophysics*, **70**, V73–V79.
- Barone, P. M., and R. M. D. Maggio, 2019, Forensic geophysics: ground penetrating radar (GPR) techniques and missing persons investigations: *Forensic Sciences Research*, **4**, 337–340.
- Bashir, Y., D. P. Ghosh, H. T. Janjuhah, and C. Weng Sum, 2018a, Diffraction enhancement through pre-image processing: Applications to field data, Sarawak Basin, East Malaysia: *Geosciences*, **8**.
- Bashir, Y., D. P. Ghosh, and C. W. Sum, 2018b, Influence of seismic diffraction for high-resolution imaging: Applications in offshore Malaysia: *Acta Geophysica*, **66**, 305–316.
- Bashir, Y., N. Mohd Muztaza, S. Y. Moussavi Alashloo, S. H. Ali, and D. Prasad Ghosh, 2020, Inspiration for seismic diffraction modelling, separation, and velocity in depth imaging: *Applied Sciences*, **10**.
- Bauer, A., B. Schwarz, and D. Gajewski, 2016, Enhancement of prestack diffraction data and attributes using a traveltimes decomposition approach: *Studia Geophysica et Geodaetica*, **60**, 471–486.
- , 2017, Utilizing diffractions in wavefront tomography: *Geophysics*, **82**, R65–R73.
- Bauer, A., B. Schwarz, T. Werner, and D. Gajewski, 2019, Unsupervised event identification and tagging for diffraction focusing: *Geophysical Journal International*, **217**, no. 3, 2165–2176.
- Belfer, I., I. Bruner, S. Keydar, A. Kravtsov, and E. Landa, 1998, Detection of shallow objects using refracted and diffracted seismic waves: *Journal of Applied Geophysics*, **38**, 155–168.
- Bellezza, C., F. Poletto, B. Farina, G. Pinna, L. Wouters, N. Vandenberghe, and E. Poggiagliolmi, 2022, Processing of seismic diffractions from carbonate nodules in clay formations: *Geophysics*, **87**, D67–D81.
- Berkovitch, A., I. Belfer, Y. Hassin, and E. Landa, 2009, Diffraction imaging by multifocusing: *Geophysics*, **74**, no. 6, WCA75–WCA81.
- Berkovitch, A., I. Belfer, and E. Landa, 2008, Multifocusing as a method of improving subsurface imaging: *The Leading Edge*, **27**, 250–256.
- Berryhill, J. R., 1977, Diffraction response for nonzero separation of source and receiver: *Geophysics*, **42**, 1158–1176.
- Biondi, B., and A. Almomin, 2014, Simultaneous inversion of full data bandwidth by tomographic full-waveform inversion: *Geophysics*, **79**, WA129–WA140.
- Boyack, K. W., and R. Klavans, 2010, Co-citation analysis, bibliographic coupling, and direct citation: Which citation approach represents the research front most accurately?: *Journal of the American Society for Information Science and Technology*, **61**, 2389–2404.
- Burnett, W., A. Klovov, S. Fomel, R. Bansal, E. Liu, and T. Jenkinson, 2015, Seismic diffraction interpretation at Piceance Creek: *Interpretation*, **3**, SF1 – SF14.
- Burnett, W. A., C. Hyslop, J. Park, A. Wawrzynski, A. Srinivasan, S. Lazaratos, G. Ayeni, D. Zhou, D. Carlo, M. Lahey, P. Montoya, W. Espegren, B. M. Phillips, T. T. Lê, T. H. Mai, V. T. Đ, T. Nguyn, T. M. H. Đng, and D. A. Nguyn, 2021, Viscoacoustic diffraction imaging to delineate karst features in a carbonate reservoir: *First Break*, **39**, 39–43.
- Castro, K. C. P. L., and L. S. Cunha, 2021, Forensic investigations with the identification of human remains with ground penetrating radar (GPR): A review: *Estudos Geológicos (UFPE)*, **31**, 64–86.

- Chen, J., 2004, Specular ray parameter extraction and stationary-phase migration: *Geophysics*, **69**, 249–256.
- Chen, W., X. Liu, O. M. Saad, Y. A. S. I. Oboué, L. Yang, H. Wang, and Y. Chen, 2022, 3-D seismic diffraction separation and imaging using the local rank-reduction method: *IEEE Transactions on Geoscience and Remote Sensing*, **60**, 1–10.
- Chen, Z., S. Fomel, and W. Lu, 2013, Accelerated plane-wave destruction: *Geophysics*, **78**, V1–V9.
- Claerbout, J. F., 1985, *Imaging the Earth's interior*: Blackwell scientific publications Oxford, **1**.
- , 1992, *Earth soundings analysis: Processing versus inversion*: Blackwell Scientific Publications London, **6**.
- Coimbra, T. A., J. H. Facciopieri, J. H. Speglich, L.-J. Gelius, and M. Tygel, 2018, Enhancement of diffractions in prestack domain by means of a finite-offset double-square-root traveltime: *Geophysics*, **84**, V81–V96.
- Conyers, L. B., 2013, *Ground-penetrating radar for archaeology*: Altamira Press.
- Conyers, L. B., and J. Leckebusch, 2010, Geophysical archaeology research agendas for the future: Some ground-penetrating radar examples: *Archaeological Prospection*, **17**, 117–123.
- Cortes, C., and V. Vapnik, 1995, Support-vector networks: *Machine Learning*, **20**, 273–297.
- Dafni, R., and W. W. Symes, 2017, Diffraction imaging by prestack reverse-time migration in the dip-angle domain: *Geophysical Prospecting*, **65**, 295–316.
- de Aguiar, G. Z., L. Lins, M. F. de Paulo, S. T. R. Maciel, and A. A. Rocha, 2021, Dielectric permittivity effects in the detection of tree roots using ground-penetrating radar: *Journal of Applied Geophysics*, **193**, 104435.
- De Figueiredo, J., F. Oliveira, E. Esmei, L. Freitas, J. Schleicher, A. Novais, P. Sussner, and S. Green, 2013, Automatic detection and imaging of diffraction points using pattern recognition: *Geophysical Prospecting*, **61**, 368–379.
- Decker, L., and S. Fomel, 2021, A Probabilistic Approach to Seismic Diffraction Imaging: *Lithosphere*, **2021**. (6650633).
- Decker, L., X. Janson, and S. Fomel, 2014, Carbonate reservoir characterization using seismic diffraction imaging: *Interpretation*, **3**, SF21–SF30.
- Decker, L., D. Merzlikin, and S. Fomel, 2017, Diffraction imaging and time-migration velocity analysis using oriented velocity continuation: *Geophysics*, **82**, U25–U35.
- Delf, R., R. G. Bingham, A. Curtis, S. Singh, A. Giannopoulos, B. Schwarz, and C. P. Borstad, 2022, Reanalysis of polythermal glacier thermal structure using radar diffraction focusing: *Journal of Geophysical Research: Earth Surface*, **127**, e2021JF006382.
- Dell, S., I. Abakumov, P. Znak, D. Gajewski, B. Kashan, and A. Ponomarenko, 2019, On the role of diffractions in velocity model building: a full-waveform inversion example: *Studia Geophysica et Geodae-tica*, **63**, 538–553.
- Dell, S., and D. Gajewski, 2011, Common-reflection-surface-based workflow for diffraction imaging: *Geophysics*, **76**, S187–S195.
- Dell, S., A. Hoelker, and D. Gajewski, 2018, Using seismic diffractions for assessment of tectonic overprint and fault interpretation: *Geophysics*, **84**, IM1–IM9.
- Dell, S., J. Walda, A. Hoelker, and D. Gajewski, 2020, Categorizing and correlating diffractivity attributes with seismic-reflection attributes using autoencoder networks: *Geophysics*, **85**, O59–O70.
- Economou, N., A. Vafidis, M. Bano, H. Hamdan, and J. Ortega-Ramirez, 2020, Ground-penetrating radar data diffraction focusing without a velocity model: *Geophysics*, **85**, H13–H24.
- Fagan, J., K. S. Eddens, J. Dolly, N. L. Vanderford, H. Weiss, and J. S. Levens, 2018, Assessing research collaboration through co-authorship network analysis: *The journal of research administration*, **49**, 76–99.
- Ferreira, F. A., 2018, Mapping the field of arts-based management: Bibliographic coupling and co-citation analyses: *Journal of Business Research*, **85**, 348–357.
- Fiedler, S., B. Illich, J. Berger, and M. Graw, 2009, The effectiveness of ground-penetrating radar surveys in the location of unmarked burial sites in modern cemeteries: *Journal of Applied Geophysics*, **68**, 380–385.
- Fomel, S., 2002, Applications of plane-wave destruction filters: *Geophysics*, **67**, 1946–1960.
- Fomel, S., E. Landa, and T. Taner, 2007, Postack velocity analysis by separation and imaging of seismic diffractions: *Geophysics*, **72**, no. 6, U89–U94.
- Fomel, S., P. Sava, I. Vlad, Y. Liu, and V. Bashkardin, 2013, Madagascar: open-source software project for multidimensional data analysis and reproducible computational experiments: *Journal of Open Research Software*.
- Ford, J., R. Urgeles, A. Camerlenghi, and E. Gràcia, 2021, Seismic diffraction imaging to characterize mass-transport complexes: Examples from the Gulf of Cadiz, South West Iberian Margin: *Journal of Geophysical Research: Solid Earth*, **126**,

- e2020JB021474.
- Gelius, L., and E. Asgedom, 2011, Diffraction-limited imaging and beyond – the concept of super resolution: *Geophysical Prospecting*, **59**, 400–421.
- Gelius, L.-J., M. Tygel, A. K. Takahata, E. G. Asgedom, and D. R. Serrano, 2013, High-resolution imaging of diffractions — a window-steered MUSIC approach: *Geophysics*, **78**, S255–S264.
- Grasmueck, M., T. Moser, M. Pelissier, J. Pajchel, and K. Pomar, 2015, Diffraction signatures of fracture intersections: *Interpretation*, **3**, SF55 – SF68.
- Grasmueck, M., M. C. Quintà, K. Pomar, and G. P. Eberli, 2013, Diffraction imaging of sub-vertical fractures and karst with full-resolution 3D Ground-Penetrating Radar: *Geophysical Prospecting*, **61**, 907–918.
- Guo, L., H. Lin, B. Fan, X. Cui, and J. Chen, 2013, Impact of root water content on root biomass estimation using ground penetrating radar: evidence from forward simulations and field controlled experiments: *Plant and soil*, **371**, 503–520.
- Haidich, A.-B., 2010, Meta-analysis in medical research: *Hippokratia*, **14**, 29.
- Harlan, W. S., J. F. Claerbout, and F. Rocca, 1984, Signal/noise separation and velocity estimation: *Geophysics*, **49**, 1869–1880.
- Huang, Y., D. Zhang, and G. T. Schuster, 2015, Tomographic resolution limits for diffraction imaging: **3**, 15–20.
- Hubral, P., 1983, Computing true amplitude reflections in a laterally inhomogeneous earth: *Geophysics*, **48**, 1051–1062.
- Hubral, P., J. Schleicher, and M. Tygel, 1996, A unified approach to 3-D seismic reflection imaging, part I: Basic concepts: *Geophysics*, **61**, 742–758.
- Jiang, J., W. Yang, H. Ren, and M. Chen, 2022, Diffraction separation and imaging using an improved singular value decomposition method: *IEEE Geoscience and Remote Sensing Letters*, **19**, 1–5.
- Jäger, R., J. Mann, G. Höcht, and P. Hubral, 2001, Common-reflection-surface stack: Image and attributes: *Geophysics*, **66**, 97–109.
- Kallweit, R. S., and L. C. Wood, 1982, The limits of resolution of zero-phase wavelets: *Geophysics*, **47**, 1035–1046.
- Kanasewich, E. R., and S. M. Phadke, 1988, Imaging discontinuities on seismic sections: *Geophysics*, **53**, 334–345.
- Kessler, M. M., 1963, Bibliographic coupling between scientific papers: *American documentation*, **14**, 10–25.
- Keydar, S., and E. Landa, 2019, Wave refocusing for linear diffractor using the time-reversal principle: *Geophysical Prospecting*, **67**, 1345–1353.
- Keydar, S., D. Pelman, and M. Ezersky, 2010, Application of seismic diffraction imaging for detecting near-surface inhomogeneities in the Dead Sea area: *Journal of Applied Geophysics*, **71**, 47–52.
- Khaidukov, V., E. Landa, and T. J. Moser, 2004, Diffraction imaging by focusing-defocusing: An outlook on seismic superresolution: *Geophysics*, **69**, no. 6, 1478–1490.
- Khosnavaz, M., A. Bóna, M. Hossain, M. Urosevic, and K. Chambers, 2016, Diffractivity - another attribute for the interpretation of seismic data in hard rock environment, a case study.: *Interpretation*, **4**, no. 4, B23–B32.
- Khosnavaz, M. J., A. Bóna, and M. Urosevic, 2018, Post-stack diffraction imaging in vertical transverse isotropy media using non-hyperbolic moveout approximations: *Geophysical Prospecting*, **66**, 273–281.
- Kim, S., S. Jee Seol, J. Byun, and S. Oh, 2021, Extraction of diffractions from seismic data using convolutional U-net and transfer learning: *Geophysics*, **87**, 1–87.
- Kitchenham, B., O. P. Brereton, D. Budgen, M. Turner, J. Bailey, and S. Linkman, 2009, Systematic literature reviews in software engineering – a systematic literature review: *Information and software technology*, **51**, 7–15.
- Klem-Musatov, K., 1994, Theory of seismic diffractions: *Society of Exploration Geophysicists*.
- Klokov, A., and S. Fomel, 2012, Separation and imaging of seismic diffractions using migrated dip-angle gathers: *Geophysics*, **77**, S131–S143.
- , 2013, Selecting an optimal aperture in Kirchhoff migration using dip-angle images: *Geophysics*, **78**, no. 6, S243–S254.
- Klokov, A., D. Irkabaev, T. Sharova, and N. Munasyrov, 2015, Reef delineation by offset vertical seismic profiling and seismic diffractions: *Journal of Seismic Exploration*, **24**, 135–150.
- Klokov, A., R. H. Treviño, and T. A. Meckel, 2017, Diffraction imaging for seal evaluation using ultra high resolution 3D seismic data: *Marine and Petroleum Geology*, **82**, 85–96.
- Knight, R., 2001, Ground penetrating radar for environmental applications: *Annual Review of Earth and Planetary Sciences*, **29**, 229–255.
- Kong, X., Z. Wang, De and Li, R. Zhang, and Q. Hu, 2017, Diffraction separation by plane-wave prediction filtering: *Applied Geophysics*, **14**, 399–405.
- Krey, T., 1952, The significance of diffraction in the investigation of faults: *Geophysics*, **17**, 843–858.

- Kunz, B. F., 1960, Diffraction problems in fault interpretation: *Geophysical Prospecting*, **8**, 381–388.
- Landa, E., and S. Keydar, 1998, Seismic monitoring of diffraction images for detection of local heterogeneities: *Geophysics*, **63**, 1093–1100.
- Landa, E., V. Shtivelman, and B. Gelchinsky, 1987, A method for detection of diffracted waves on common-offset sections: *Geophysical Prospecting*, **35**, no. 4, 359–374.
- Lellouch, A., and M. Reshef, 2016, Shallow diffraction imaging in an SH-wave crosshole configuration: *Geophysics*, **82**, S9–S18.
- Li, C., J. Gao, Z. Gao, R. Wang, and T. Yang, 2020a, Periodic plane-wave least-squares reverse time migration for diffractions: *Geophysics*, **85**, S185–S198.
- Li, C., J. Gao, R. Wang, and Z. Gao, 2020b, Enhancing subsurface scatters using reflection-damped plane-wave least-squares reverse time migration: *IEEE Geoscience and Remote Sensing Letters*, **17**, 706–710.
- Li, C., S. Peng, X. Cui, and W. Du, 2022a, 3-D pre-stack diffraction separation by extending the PWD method with parametrized local slope: *Geophysical Journal International*, **232**, 750–763.
- Li, C., S. Peng, X. Cui, Q. Liu, and P. Lin, 2022b, Pre-stack diffraction separation by parameterizing the reflection local slope: *Geophysics*, **87**, S35–S44.
- Li, C., S. Peng, P. Lin, X. Cui, and Q. Liu, 2022c, Imaging diffractions using a double-order weight function: *Pure and Applied Geophysics*, **179**, 1053–1067.
- Li, C., S. Peng, J. Zhao, and X. Cui, 2020c, Diffraction imaging using an adaptive phase filter: *Geophysical Prospecting*, **68**, 164–177.
- Li, C., S. Peng, J. Zhao, X. Cui, W. Du, and S. Satibekova, 2018, Polarity-preserved diffraction extracting method using modified apex-shifted Radon transform and double-branch Radon transform: *Journal of Geophysics and Engineering*, **15**, 1991–2000.
- Li, C., J. Zhao, S. Peng, and X. Cui, 2021a, Prestack diffraction separation in the common virtual source gather: *Geophysics*, **86**, S113–S124.
- Li, C., J. Zhao, S. Peng, X. Cui, and P. Lin, 2020d, Separating and imaging diffractions of seismic waves in the full-azimuth dip-angle domain: *Journal of Geophysics and Engineering*, **17**, 339–356.
- Li, C., J. Zhao, S. Peng, and Y. Zhou, 2021b, Diffraction imaging using a mathematical morphological filter with a time-varying structuring element: *Geophysics*, **86**, S185–S196.
- Li, Z., and J. Zhang, 2019, 3D diffraction imaging with Kirchhoff time migration using vertical traveltimes difference gathers: *Geophysics*, **84**, S555–S566.
- Li, Z.-C., and Y.-M. Qu, 2022, Research progress on seismic imaging technology: *Petroleum Science*, **19**, 128–146.
- Lin, P., C. Li, and S. Peng, 2022a, Diffraction extraction using a low-rank matrix approximation method: *IEEE Geoscience and Remote Sensing Letters*, **19**, 1–5.
- Lin, P., S. Peng, X. Cui, W. Du, and C. Li, 2022b, Effective diffraction separation using the improved optimal rank-reduction method: *Geophysics*, **87**, V169–V182.
- Lin, P., S. Peng, X. Huang, Y. Xiang, C. Li, and X. Cui, 2022c, Plane-wave destruction-based workflow for prestack diffraction separation in the shot domain: *Pure and Applied Geophysics*, 1–15.
- Lin, P., S. Peng, J. Zhao, and X. Cui, 2020, Diffraction separation and imaging using multichannel singular-spectrum analysis: *Geophysics*, **85**, V11–V24.
- Lin, P., S. Peng, J. Zhao, X. Cui, and W. Du, 2018a, A new scheme for velocity analysis and imaging of diffractions: *Journal of Geophysics and Engineering*, **15**, 1084–1093.
- , 2018b, Accurate diffraction imaging for detecting small-scale geologic discontinuities: *Geophysics*, **83**, S447–S457.
- Lin, P., S. Peng, J. Zhao, X. Cui, and H. Wang, 2018c, L1-norm regularization and wavelet transform: An improved plane-wave destruction method: *Journal of Applied Geophysics*, **148**, 16–22.
- Lin, P., J. Zhao, and S. Peng, 2021a, Low-rank diffraction separation using an improved MSSA algorithm: *Acta Geophysica*, **69**, 1651–1665.
- Lin, P., J. Zhao, S. Peng, and X. Cui, 2021b, Diffraction separation by variational mode decomposition: *Geophysical Prospecting*, **69**, 1070–1085.
- Lin, P., J. Zhao, S. Peng, X. Cui, C. Li, and D. Xu, 2021c, A robust adaptive rank-reduction method for 3D diffraction separation and imaging: *Pure and Applied Geophysics*, **178**, 2917–2931.
- Liu, J., D. Draganov, R. Ghose, and Q. Bourgeois, 2021, Near-surface diffractor detection at archaeological sites based on an interferometric workflow: *Geophysics*, **86**, WA1–WA11.
- Liu, L., Y. Ma, Y. Zhao, and Y. Luo, 2022, Wave-equation diffraction imaging using pseudo dip-angle gather: *Geophysics*, **87**, S95–S103.
- Liu, L., E. Vincent, X. Ji, F. Qin, and Y. Luo, 2016, Imaging diffractors using wave-equation migration: *Geophysics*, **81**, S459–S468.
- Lowney, B., I. Lokmer, and G. O’Brien, 2021a, Multi-domain diffraction identification: A supervised deep learning technique for seismic diffraction classifica-

- tion: Computers & Geosciences, **155**, 104845.
- Lowney, B., I. Lokmer, G. S. O'Brien, and C. J. Bean, 2021b, Pre-migration diffraction separation using generative adversarial networks: Geophysical Prospecting, **69**, 949–967.
- Lowney, B., I. Lokmer, G. S. O'Brien, L. Amy, C. J. Bean, and M. Igoe, 2020, Enhancing interpretability with diffraction imaging using plane-wave destruction aided by frequency-wavenumber f-k filtering: Interpretation, **8**, T541–T554.
- Maciel, S., and R. Biloti, 2020, A statistics-based descriptor for automatic classification of scatterers in seismic sections: Geophysics, **85**, O83–O96.
- Madiba, G. B., and G. A. McMechan, 2003, Processing, inversion, and interpretation of a 2D seismic data set from the North Viking Graben, North Sea: Geophysics, **68**, 837–848.
- Markovic, M., R. Malehmir, and A. Malehmir, 2022, Diffraction pattern recognition using deep semantic segmentation: Near Surface Geophysics, **20**, 507–518.
- Marques Jr, A., R. K. Horota, E. M. de Souza, L. Kupsinskü, P. Rossa, A. S. Aires, L. Bachi, M. R. Veronez, L. Gonzaga Jr, and C. L. Cazarin, 2020, Virtual and digital outcrops in the petroleum industry: A systematic review: Earth-Science Reviews, **208**, 103260.
- Meng, Z., Z. Sun, and G. Li, 2021, A case study of complex carbonate reservoir connectivity analysis, Tarim Basin, China: Interpretation, **9**, B77–B87.
- Merzlikin, D., and S. Fomel, 2017, Analytical path-summation imaging of seismic diffractions: Geophysics, **82**, S51–S59.
- Merzlikin, D., S. Fomel, and M. K. Sen, 2019, Least-squares path-summation diffraction imaging using sparsity constraints: Geophysics, **84**, S187–S200.
- Merzlikin, D., S. Fomel, and X. Wu, 2020, Least-squares diffraction imaging using shaping regularization by anisotropic smoothing: Geophysics, **85**, S313–S325.
- Merzlikin, D., T. A. Meckel, S. Fomel, and Y. Sripa-nich, 2017, Diffraction imaging of high-resolution 3D P-cable data from the Gulf of Mexico using azimuthal plane-wave destruction: First Break, **35**.
- Moher, D., A. Liberati, J. Tetzlaff, D. G. Altman, and P. Group*, 2009, Preferred reporting items for systematic reviews and meta-analyses: the PRISMA statement: Annals of internal medicine, **151**, 264–269.
- Montazeri, M., L. O. Boldreel, A. Uldall, and L. Nielsen, 2019, Improved seismic interpretation of a salt diapir by utilization of diffractions, exemplified by 2D reflection seismics, Danish sector of the North Sea: Interpretation, **8**, T77–T88.
- Moore, G. F., T. H. Shipley, P. L. Stoffa, D. E. Karig, A. Taira, S. Kuramoto, H. Tokuyama, and K. Suyehiro, 1990, Structure of the Nankai Trough Accretionary Zone from multichannel seismic reflection data: Journal of Geophysical Research: Solid Earth, **95**, 8753–8765.
- Moser, T. J., 2012, Review of ray-Born forward modeling for migration and diffraction analysis: Stud. Geophys. Geod., **56**, 411–432.
- Moser, T. J., and C. B. Howard, 2008, Diffraction imaging in depth: Geophysical Prospecting, **56**, 627–641.
- Mu, X., J. Huang, L. Fu, S. Jian, B. Hu, and Z. Wang, 2021, Diffraction imaging for the fault-karst structure by least-squares reverse time migration: Interpretation, **9**, T385–T394.
- Neidell, N. S., 1997, Perceptions in seismic imaging, part 2: Reflective and diffractive contributions to seismic imaging: The Leading Edge, **16**, 1121–1123.
- Nocedal, J., 1980, Updating quasi-newton matrices with limited storage: Mathematics of Computation, **35**, 773–782.
- Nosjean, N., Y. Khamitov, S. Rodriguez, and R. Yahia-Cherif, 2020, Fracture corridor identification through 3D multifocusing to improve well deliverability, an Algerian tight reservoir case study: Solid Earth Sciences, **5**, 31–49.
- Oesper, L., D. Merico, R. Isserlin, and G. D. Bader, 2011, WordCloud: a Cytoscape plugin to create a visual semantic summary of networks: Source code for biology and medicine, **6**, 7.
- Ogiesoba, O. C., and A. Klovov, 2017, Examples of seismic diffraction imaging from the Austin Chalk and Eagle Ford Shale, Maverick Basin, South Texas: Journal of Petroleum Science and Engineering, **157**, 248–263.
- Ogiesoba, O. C., A. Klovov, and R. Hernandez, 2015, Diffraction imaging of polygonal faults within a submarine volcanic terrain, Maverick Basin, south Texas: Interpretation, **3**, SF81–SF99.
- Orlando, L., and E. Slob, 2009, Using multicomponent GPR to monitor cracks in a historical building: Journal of Applied Geophysics, **67**, 327–334. (Advanced Applications, Systems and Modelling for GPR).
- Papziner, U., and K.-P. Nick, 1998, Automatic detection of hyperbolas in georadargrams by slant-stack processing and migration: First Break, **16**, 219–223.
- Parsekian, A. D., 2018, Inverse Methods To Improve Accuracy of Water Content Estimates from Multi-offset Gpr: Journal of Environmental and Engineering Geophysics, **23**, 349–361.

- Pedersen, , and F. Marcy, 2022, Diffraction imaging: A valuable complement to reflection imaging for sand injectite interpretation in the Norwegian North Sea: *The Leading Edge*, **41**, 247–252.
- Pérez-Gracia, V., and M. Solla, 2015, *in* Inspection Procedures for Effective GPR Surveying of Buildings: Springer International Publishing, 97–123.
- Peterie, S. L., and R. D. Miller, 2014, Near-surface scattering phenomena and implications for tunnel detection: *Interpretation*, **3**, SF43–SF54.
- Peterie, S. L., R. D. Miller, J. Ivanov, and S. D. Sloan, 2020, Shallow tunnel detection using SH-wave diffraction imaging: *Geophysics*, **85**, EN29–EN37.
- Porritt, K., J. Gomersall, and C. Lockwood, 2014, JBI's systematic reviews: study selection and critical appraisal: *AJN The American Journal of Nursing*, **114**, 47–52.
- Prosatov, M., V. Cheverda, and V. Shilikov, 2021, Detection and characterization of fault, cavernous, and fracture zones by constructing seismic diffraction image attributes: *Doklady Earth Sciences*, **499**, 670—674.
- Protasov, M., V. Tcheverda, and V. Shilikov, 2021, 3d diffraction imaging attributes and their application for fault and fracture localization and characterization: *Journal of Applied Geophysics*, **191**, 104346.
- Rad, P. B., and C. J. Hickey, 2021, Seismic diffraction separation in the near surface: Detection of high-contrast voids in unconsolidated soils: *Geophysics*, **86**, WA13–WA23.
- Rad, P. B., B. Schwarz, D. Gajewski, and C. Vanelle, 2018, Common-reflection-surface-based prestack diffraction separation and imaging: *Geophysics*, **83**, S47–S55.
- Rasol, M., J. C. Pais, V. Pérez-Gracia, M. Solla, F. M. Fernandes, S. Fontul, D. Ayala-Cabrera, F. Schmidt, and H. Assadollahi, 2022, GPR monitoring for road transport infrastructure: A systematic review and machine learning insights: *Construction and Building Materials*, **324**, 126686.
- Reshef, M., and E. Landa, 2009, Post-stack velocity analysis in the dip angle domain using diffractions: *Geophysical Prospecting*, **57**, 811–821.
- Reshef, M., and A. Rüger, 2008, Influence of structural dip angles on interval velocity analysis: *Geophysics*, **73**, U13–U18.
- Römer-Stange, N., S. Wenau, V. Bihler, H. Keil, C. A. Ramos Córdova, and V. Spiess, 2022, Boulder detection in the shallow sub-seafloor by diffraction imaging with beamforming on ultra-high resolution seismic data — a feasibility study: *Earth and Space Science*, **9**, e2021EA002156.
- Santos, L. A., W. J. Mansur, and G. A. McMechan, 2012, Tomography of diffraction-based focusing operators: *Geophysics*, **77**, R217–R225.
- Santos, L. A., E. H. P. Neves, A. F. M. Freire, M. A. C. Santos, R. Matsumoto, and C. M. I. Ajus, 2020, Diffraction velocity analysis in a single-channel seismic survey in the Joetsu Basin: *Geophysics*, **85**, U47–U53.
- Santos, L. T., J. Schleicher, M. Tygel, and P. Hubral, 2000, Seismic modeling by demigration: *Geophysics*, **65**, 1281–1289.
- Sava, P., B. Biondi, and J. Etgen, 2005, Wave-equation migration velocity analysis by focusing diffractions and reflections: *Geophysics*, **70**, no. 3, U19–U27.
- Schneider, W. A., 1978, Integral formulation for migration in two and three dimensions: *Geophysics*, **43**, 49–76.
- Schoepp, A., S. Labonté, and E. Landa, 2014, Multifocusing 3D diffraction imaging for detection of fractured zones in mudstone reservoirs: Case history: *Interpretation*, **3**, SF31–SF42.
- , 2015, Multifocusing 3D diffraction imaging for detection of fractured zones in mudstone reservoirs: Case history: *Interpretation*, **3**, SF31–SF42.
- Schwarz, B., 2019a, Chapter one - an introduction to seismic diffraction, *in* Recent Advances in Seismology: Elsevier, volume **60** of *Advances in Geophysics*, 1–64.
- , 2019b, Coherent wavefield subtraction for diffraction separation: *Geophysics*, **84**, no. 3, V157–V168.
- Schwarz, B., and D. Gajewski, 2017, Accessing the diffracted wavefield by coherent subtraction: *Geophysical Journal International*, **211**, 45–49.
- Schwarz, B., and C. M. Krawczyk, 2020, Coherent diffraction imaging for enhanced fault and fracture network characterization: *Solid Earth*, **11**, 1891–1907.
- Schwarz, B., C. Vanelle, D. Gajewski, and B. Kashtan, 2014, Curvatures and inhomogeneities: An improved common-reflection-surface approach: *Geophysics*, **79**, S231–S240.
- Shen, H.-Y., Q. Li, Y.-Y. Yan, X.-X. Li, and J. Zhao, 2020, Separation of diffracted waves via SVD filter: *Petroleum Science*, **17**, 1259–1271.
- Sheng, T., and J. Zhao, 2022, Separation and imaging of diffractions using a dilated convolutional neural network: *Geophysics*, **87**, S117–S127.
- Shustak, M., and E. Landa, 2018, Time reversal for wave refocusing and scatterer detection using machine learning: *Geophysics*, **83**, T257–T263.
- Silvestrov, I., R. Baina, and E. Landa, 2016, Posts-

- tack diffraction imaging using reverse-time migration: *Geophysical Prospecting*, **64**, 129–142.
- Small, H., 1973, Co-citation in the scientific literature: A new measure of the relationship between two documents: *Journal of the American Society for information Science*, **24**, 265–269.
- Sturzu, I., A. Popovici, M. Pelissier, J. Wolak, and T. Moser, 2014, Diffraction imaging of the Eagle Ford shale: *First Break*, **32**, 49–59.
- Sun, J., J. Yang, Z. Li, J. Huang, J. Xu, and S. Zhuang, 2022, Reflection and diffraction separation in the dip-angle common-image gathers using convolutional neural network: *Geophysics*, **88**, WA281–WA291.
- Syed, F. I., A. AlShamsi, A. K. Dahaghi, and S. Neghabhan, 2022, Application of ML & AI to model petrophysical and geomechanical properties of shale reservoirs – a systematic literature review: *Petroleum*, **8**, 158–166. (SI: Computational Petroleum Engineering).
- Taner, T., 2006, Separation and imaging of seismic diffractions using plane-wave decomposition: *SEG Technical Program Expanded Abstracts 2006*, 2401–2405.
- Trad, D., M. D. Sacchi, and T. J. Ulrych, 2001, A hybrid linear-hyperbolic Radon transform: *Journal of Seismic Exploration*, **9**, 303–318.
- Tranfield, D., D. Denyer, and P. Smart, 2003, Towards a methodology for developing evidence-informed management knowledge by means of systematic review: *British journal of management*, **14**, 207–222.
- Trorey, A. W., 1970, A simple theory for seismic diffractions: *Geophysics*, **35**, 762–784.
- Tschannen, V., N. Ettrich, M. Delescluse, and J. Keuper, 2020, Detection of point scatterers using diffraction imaging and deep learning: *Geophysical Prospecting*, **68**, 830–844.
- Tygel, M., J. Schleicher, and P. Hubral, 1996, A unified approach to 3-D seismic reflection imaging, Part II: Theory: *Geophysics*, **61**, 759–775.
- Tyiasning, S., D. Merzlikin, D. Cooke, and S. Fomel, 2016, A comparison of diffraction imaging to incoherence and curvature: *The Leading Edge*, **35**, 86–89.
- Van Eck, N., and L. Waltman, 2010, Software survey: VOSviewer, a computer program for bibliometric mapping: *Scientometrics*, **84**, 523–538.
- Červeny, V., 2001, *Seismic ray theory*: Cambridge University Press.
- Virieux, J., and S. Operto, 2009, An overview of full-waveform inversion in exploration geophysics: *Geophysics*, **74**, WCC1–WCC26.
- Vogel, R., and W. H. Güttel, 2013, The dynamic capability view in strategic management: A bibliometric review: *International Journal of Management Reviews*, **15**, 426–446.
- Vries, D. D., and A. J. Berkhout, 1984, Velocity analysis based on minimum entropy: *Geophysics*, **49**, 2132–2142.
- Wahono, R. S., 2015, A systematic literature review of software defect prediction: *Journal of software engineering*, **1**, 1–16.
- Wang, B., B. Jin, L. Huang, S. Liu, H. Sun, J. Liu, X. Ding, and S. Wang, 2020a, A Hilbert polarization imaging method with breakpoint diffracted wave in front of roadway: *Journal of Applied Geophysics*, **177**, 104032.
- Wang, H., X. Liu, and Y. Chen, 2020b, Separation and imaging of seismic diffractions using a localized rank-reduction method with adaptively selected ranks: *Geophysics*, **85**, V497–V506.
- Wechsler, N., M. Shustak, and E. Ben-Yosef, 2020, Diffraction stack imaging as a potential tool for detecting underground voids – the case of the ancient copper mines of Timna Valley (Israel): *Archaeological Prospection*, **27**, 27–37.
- White, H. D., and B. C. Griffith, 1981, Author cocitation: A literature measure of intellectual structure: *Journal of the American Society for Information Science*, **32**, 163–171.
- Xiang, K., and E. Landa, 2019, Poststack impedance inversion considering the diffractive component of the wavefield: *Geophysics*, **85**, R11–R28.
- Xu, S., H. Chauris, G. Lambaré, and M. Noble, 2001, Common-angle migration: A strategy for imaging complex media: *Geophysics*, **66**, 1877–1894.
- Yin, J., and N. Nakata, 2019, Diffraction imaging using geometric-mean reverse time migration and common-reflection surface: *Geophysics*, **84**, S355–S364.
- Yu, C., J. Zhao, and Y. Wang, 2017, Revealing small-scale diffracting discontinuities by an optimization inversion algorithm: *Journal of Geophysics and Engineering*, **14**, 1–14.
- Yu, C., J. Zhao, Y. Wang, C. Wang, and W. Geng, 2016, Separation and imaging diffractions by a sparsity-promoting model and subspace trust-region algorithm: *Geophysical Journal International*, **208**, 1756–1763.
- Yuan, H., M. Montazeri, M. C. Looms, and L. Nielsen, 2019, Diffraction imaging of ground-penetrating radar data: *Geophysics*, **84**, H1–H12.
- Zhang, D., T. W. Fei, C. Tsingas, and Y. Luo, 2019, Efficient wave-equation-based diffraction imaging: *Geophysics*, **84**, S389–S399.

- , 2020, Diffraction imaging using the two-way imaging condition: *Geophysics*, **85**, H1–H11.
- Zhang, J., 2013, A pre-stack depth diffraction imaging workflow based on angle gathers: *Journal of Seismic Exploration*, **22**, 129–145.
- Zhang, J., and J. Zhang, 2014, Diffraction imaging using shot and opening-angle gathers: A prestack time migration approach: *Geophysics*, **79**, S23–S33.
- Zhao, J., S. Peng, W. Du, and X. Li, 2016, Diffraction imaging method by Mahalanobis-based amplitude damping: *Geophysics*, **81**, no. 6, S399–S408.
- Zhao, J., X. Sun, S. Peng, W. Wei, and T. Liu, 2019a, Separating prestack diffractions with SVMF in the flattened shot domain: *Journal of Geophysics and Engineering*, **16**, 389–398.
- Zhao, J., Y. Wang, and C. Yu, 2015, Diffraction imaging by uniform asymptotic theory and double exponential fitting: *Geophysical Prospecting*, **63**, no. 2, 338–353.
- Zhao, J., C. Yu, S. Peng, and J. Cao, 2021, Least-squares imaging of diffractions by solving a hybrid L1-L2 norm minimization problem: *Geophysics*, **86**, S59–S72.
- Zhao, J., C. Yu, S. Peng, and Z. Chen, 2019b, Online dictionary learning method for extracting GPR diffractions: *Journal of Geophysics and Engineering*, **16**, 1116–1123.
- Zhao, J., C. Yu, S. Peng, and C. Li, 2020, 3D diffraction imaging method using low-rank matrix decomposition: *Geophysics*, **85**, S1–S10.
- Zhou, B., P. Hatherly, and W. Sun, 2017, Enhancing the detection of small coal structures by seismic diffraction imaging: *International Journal of Coal Geology*, **178**, 1–12.
- Zhou, B., M. Pavlova, R. Sliwa, and W. Sun, 2022, Seismic diffraction imaging for improved coal structure detection by a structure-oriented moving average error filter: *IEEE Transactions on Geoscience and Remote Sensing*, **60**, 1–13.
- Zhou, J. W., S. P. Peng, P. Lin, X. Q. Cui, and T. Wang, 2023, Diffraction separation and imaging using ensemble empirical mode decomposition and multichannel singular spectrum analysis: *Geophysical Prospecting*, **71**, 245–262.
- Zhu, X., and R.-S. Wu, 2010, Imaging diffraction points using the local image matrices generated in prestack migration: *Geophysics*, **75**, S1–S9.
- Zhuang, S.-B., J.-P. Huang, J.-D. Yang, and Z.-C. Li, 2022, A computational method for wide-azimuth 3D dip-angle gathers using Gaussian beam migration: *Petroleum Science*, **19**, 2081–2094.

DiffraPy: an open-source Python software for seismic diffractions imaging

GUILHERME ZAKAREWICZ¹, SUSANNE MACIEL², RICARDO BILOTI³

¹Universidade de Brasília (UnB), Instituto de Geociências

²Faculdade UnB Planaltina (FUP)

³Universidade de Campinas

Resumo. As difrações sísmicas, frequentemente negligenciadas como ruído nos métodos convencionais de imageamento sísmico, contêm informações valiosas de alta resolução sobre estruturas e heterogeneidades em subsuperfície. Além disso, o componente difratado do campo de ondas é mascarado pelas reflexões de maiores amplitudes. Existem poucas opções de alternativas de código aberto para o processamento e imageamento de difrações sísmicas. Neste estudo, apresentamos o DiffraPy, um pacote de código aberto em Python especificamente projetado para realizar migrações em que as difrações são preservadas de forma proporcional e as reflexões são suprimidas. Além disso, o código oferece funcionalidades para modelagem sintética de dados sísmicos, o que permite testes e experimentações abrangentes. Demonstramos a aplicação do código usando um modelo simples e avaliamos suas capacidades em um modelo sintético de maior complexidade, em que discutimos suas limitações e potencialidades. Em ambos os casos, o programa suprimiu com sucesso a maior parte da energia refletida na imagem de difrações, o que resulta em uma imagem de maior resolução em comparação com a migração convencional. A imagem final destaca de forma proeminente as respostas de pequenas estruturas espalhadoras, descontinuidades e falhas, fornecendo maiores detalhes sobre a subsuperfície para a interpretação das seções. Esperamos que nosso código ofereça à comunidade das geociências uma alternativa acessível e documentada para testar, reproduzir e explorar exemplos de imageamento de difrações.

Palavras-chaves: Programa Aberto; Python; Imageamento de Difrações; Difrações Sísmicas; Modelagem Sintética; Descontinuidades

Abstract. Seismic diffractions, often overlooked as noise in conventional seismic imaging methods, contain valuable high-resolution information about subsurface structures and heterogeneities. Furthermore, the diffracted component of the wavefield is masked by the stronger reflections. There are limited options of open-source alternatives in seismic diffractions processing and imaging. In this study, we introduce DiffraPy, an open-source Python package specifically designed for performing both conventional and diffraction-oriented migrations. Additionally, the code provides functionality for forward modeling seismic data, allowing for comprehensive testing and experimentation. We demonstrate the workflow of the code using a toy model and evaluate its capabilities on a synthetic salt model, discussing its limitations and potentialities. In both cases, the program successfully suppressed the majority of reflected energy in the diffraction-oriented image. The final image prominently highlights the responses of small lenses, discontinuities, and faults, providing valuable insights into the subsurface for interpretation. We expect our code to offer the geoscience community a freely accessible and well-documented alternative for testing, reproducing, and exploring diffraction imaging examples.

Keywords: Open-software; Python; Diffraction Imaging; Seismic Diffractions; Forward Modeling; Discontinuities

1 Introduction

One of the primary objectives of the exploration geophysics is obtaining realistic images of the subsurface and characterizing the medium through non-invasive approaches. Using reflected seismic waves to study the shallow portions of the Earth has become one of the main exploration methods in the past decades, with great advances in both techniques and applications (Fehler and Huang, 2002). Seismic waves interacting with interfaces or boundaries can produce various phenomena, and their behaviour is typically different (Khaidukov et al., 2004; Trorey, 1970). Specular reflection occurs when the waves encounter a continuous and smooth layer that is larger than the seismic wavelength. Non-specular diffractions, on the other hand, result from small-scale structures like faults, fractures, pinch-outs, channels, wedges, and other discontinuities that do not conform to the assumptions of Snell’s law (Lin et al., 2021). Theoretically, seismic diffraction imaging can achieve “superresolution”, allowing for the retrieval of information of structures that are smaller than the Rayleigh limit, i.e., half of the seismic wavelength (Neidell, 1997; Tschannen et al., 2020; Zhang and Zhang, 2014; Zhao et al., 2019a).

One major issue in obtaining high-resolution images is the fact that the diffracted component of the wavefield is typically considered as noise during the traditional preprocessing workflow (Wang et al., 2020b). Furthermore, the energy of diffractions is significantly lower compared to reflections, which can differ by one or two orders of magnitude (Klem-Musatov, 1994; Reshef and Landa, 2009). As a result, a significant portion of the wavefield is not utilized in subsurface imaging, leading to a loss of valuable information.

Pioneers studies on separating the diffracted energy from the wavefield were conducted by Hagedoorn (1954), Kanasewich and Phadke (1988), Krey (1952), Kunz (1960), and Trorey (1970) for detecting and delineating faults. Recent applications focus on fault, fractures, and small-scale discontinuities characterization (Bansal and Imhof, 2005; Berkovitch et al., 2009; Khaidukov et al., 2004; Moser and Howard, 2008); imaging structures related to the oil and gas industry (Decker et al., 2014; Lin et al., 2018a; Merzlikin and Fomel, 2017; Tyiasning et al., 2016; Zhao et al., 2019a, 2015, 2020), near surface investigations (Keydar et al., 2010; Liu et al., 2021; Maciel and Biloti, 2020; Peterie et al., 2020; Rad and Hickey, 2021), velocity analysis (Decker et al., 2017; Dell and Gajewski, 2011; Fomel et al., 2007; Klovov and Fomel, 2012; Reshef and Landa, 2009; Santos et al., 2020), and imaging small faults and fractures related to coal mining (Li et al.,

2020; Wang et al., 2020a; Zhao et al., 2021; Zhou et al., 2017, 2022). Machine learning techniques have also been applied for automated separation of diffractions from data (De Figueiredo et al., 2013; Kim et al., 2021; Lowney et al., 2021; Maciel and Biloti, 2020; Markovic et al., 2022; Sheng and Zhao, 2022; Shustak and Landa, 2018; Sun et al., 2022; Tschannen et al., 2020).

Recently, the number of well-documented open-source programs for geophysical applications has significantly increased, enabling effective collaboration and development of new methods and techniques for processing and interpreting various data types. Python, a programming language known for its versatility, has become a popular choice for developing non-commercial solutions in this field. Lecocq et al. (2014) introduced MSNoise, a software that monitors seismic velocity variations by using ambient noise data. Krieger and Peacock (2014) developed MTPy, an application that processes and models magnetotellurics (MT) data. Hector and Hinderer (2016) presented pyGrav, a solution for processing relative-gravity data. Rücker et al. (2017) developed pyGIMLI, a framework for modeling and inversion in various geophysical and hydrological methods. Mather and Delhaye (2019) presented PyCurious, a package for computing the Curie depth using magnetic anomaly data. Plattner (2020) developed GPRPy, a software for processing and visualizing ground-penetrating radar (GPR) data. Blanchy et al. (2020) presented ResIPy, a software for electrical data modeling and inversion. Bueno et al. (2020) developed PICOSS, an interface for analyzing and classifying volcano-seismic signals. Guedes et al. (2022) presented Refrapy, a program for analyzing seismic refraction data, including waveform processing, first break picking, and inversion. Tschannen et al. (2022) introduced the *wtie* package for automating the seismic well to tie process. Ajithabh and Patro (2023) developed SigMT, a package for automated processing of MT data.

This study introduces DiffraPy, the first freely available Python-based program designed for performing diffractions imaging. The primary objective of DiffraPy is to provide an open and expandable software that can reproduce geophysical simulations and imaging concerning seismic diffractions. We explain the theory behind the applied methods and outline the general workflow of the code. Finally, we demonstrate the software’s capabilities using synthetic data and discuss its limitations and potentialities.

2 Methods

The theory of the applied methods is described in the section “Seismic Diffraction Imaging Methods”, and

the software structure is explained in the following section “Software Implementation”.

2.1 Seismic Diffractions Imaging Methods

2.1.1 Ray-Tracing

There are two main methods for ray tracing: shooting and bending (Julian and Gubbins, 1977; Sekiguchi, 2021). Shooting is an iterative method that starts at the source position and traces the ray path to the receiver by solving the ray tracing differential equations. It iterates by changing the initial conditions, e.g., the ray angle at the source, until the ray arrives at the receiver. The bending method assumes a single ray connecting the source and receiver positions, and the ray trajectories are modified until it finds the path satisfying the Fermat’s principle (Červeny, 2001). Both methods are simple to be implemented, but present a major drawback as they are computationally intensive.

Here, we apply a different technique for calculating the seismic ray travel-time. Based on the Fermat’s principle (Officer, 1958), the shortest path method calculates the path with the minimum corresponding travel-time between two given points (Zhou and Greenhalgh, 2005). All ray paths are discarded except the one corresponding to a minimum travel-time between two points.

Considering a 2D velocity model represented as a grid with several nodes, the approach is to iteratively calculate several grid intervals that corresponds to a ray path, a process named “sub-ray tracing length”. From the source location, several line paths are traced to the nodes. From these nodes, the program decides which one has the minimum travel-time. Then, it uses the chosen node as a secondary source, following the Huygens’ principle, and repeat the sub-ray tracing process. These steps are repeated until all interested nodes are decided. Moser (1991) states that this method presents no restrictions compared with the classical ray theory as it can correctly find diffracted ray paths and even paths to shadow zones, which is particularly interesting for diffraction imaging methods.

2.1.2 Kirchhoff Modeling and Kirchhoff Migration

Both the Kirchhoff modeling and migration rely on the Huygens’ Principle. Presented by Christiaan Huygens in the study “*Traité de la Lumière*” (Treatise on Light) (Huygens, 1690), the principle states that considering an original wave front, every point acts as a new source of secondary waves. In other words, we can understand the wave front as the envelope of the secondary waves energy; the energy interferes constructively along

the wave front and destructively elsewhere (Enders, 1996). Any reflective smooth surface can be treated as composed by infinitesimals emitting points that generate wave fronts called “elementary diffractions”. The elementary diffractions can not be observed individually in a seismogram, as they are mathematical idealizations (Moser and Howard, 2008). However, real diffractions are generated by the scattered energy of small objects in the subsurface and can be observed individually in a seismogram. The principle also describes the reflection hyperbola as the envelope of the elementary diffractions, named as the “classical elementary Huygens wave” or simply “Huygens wave” (Hubral et al., 1996b).

Kirchhoff Modeling

For a given source-receiver pair (s and r), the Kirchhoff modeling integral calculates the reflected wave fronts as the envelope of all the primary wave fronts along the considered smooth reflective interface. The Kirchhoff modeling integral can be defined as (Santos et al., 2000):

$$U(\xi, t) = \frac{1}{2\pi} \iint W(x, \xi, t) \partial_z M(x, z) d^2x \big|_{z=\zeta(x, \xi, t)}, \quad (1)$$

where the stacking surface z is implicitly defined by $t = \zeta(\xi, x, z)$; ξ is the trace coordinate, t is the time coordinate, (x, z) represents the velocity model coordinate, ζ is the reflective surface, $M(x, z)$ is the model and $W(x, \xi, t)$ consists of an obliquity factor and the reflection coefficient of the incident wave at the reflector. The resulting $U(\xi, t)$ is the numerical data that represents the synthetic seismic section.

Kirchhoff Migration

We consider the Kirchhoff migration integral (Schneider, 1978), also known as the diffraction stack, as the following Equation (Tygel et al., 1996):

$$V(x) = \int d\xi \int dt U(\xi, t) \delta(t - t_d(\xi, x)), \quad (2)$$

where $U(\xi, t)$ is the seismic data parameterized by the vector ξ , δ is the Dirac delta distribution, and t_d is the time-travel curve, i.e., the propagation time of an elementary diffraction at the point x .

There are some corrections that need to be applied to the seismic data before the migration based on the nature of the waveform amplitude and phase: the spherical spreading, the obliquity factor, and the phase shift (Yilmaz, 2008). The spherical spreading factor considers the fact that a farther point a from the wave front’s

emission will present a weaker amplitude than a closer point b . For 2D situations, the energy decays by a factor of $\sqrt{1/cr}$, where c is the velocity and r is the distance between a and the emitting point. The obliquity factor accounts for the angle dependence of the amplitudes. The correction is based on multiplying the amplitude A in a given point of the diffraction hyperbola by the cosine in relation to the vertical axis z . The third factor involves the phase and amplitude restoration of the resulting waveform. The elementary diffractions must respond as a wavelet along the hyperbolas with their characteristic amplitude and phase. Under any different conditions, there would be no amplitude cancellation from close waveform. Yilmaz (2008) defines it as the $\rho(t)$ filter, a 45-degree phase shift and adjustment of the amplitude spectrum by $\sqrt{\omega}$ (ω = frequency). It can also be understood as applying a half-derivative to the wavefield.

Furthermore, there are migration parameters that an analyst needs to consider. For example, as discussed by Klovov and Fomel (2013), the migration operator distributes the seismic energy in a large area, but not all energy contributes constructively. If the migration curve is entirely considered, it may result in artifacts as migration noise, boundary noise and boundary effects (Hertweck et al., 2003). To minimize this problem when imaging the specular energy, it is possible to limit the migration aperture, i.e., the spatial extent of the summation span, measured in terms of traces (Yilmaz, 2008). For non-specular energy, it may be beneficial to use larger apertures (Klovov and Fomel, 2013; Li et al., 2020).

Similarly, the window size determines the extent of the data used for migration. A small window may lead to truncation of useful information and incomplete imaging, while a large window can introduce unnecessary computational burden and include irrelevant data that may degrade the image quality. Selecting an appropriate window size that encompasses the relevant seismic energy and ensures the desired level of imaging accuracy and efficiency is crucial.

2.1.3 Diffraction Imaging

It is possible to implement an isotropic weight factor in Equation 2 (Hubral et al., 1996a), henceforth referred to as stationary phase filter or imaging condition. Considering the parameterization vector ξ as the source s and receiver r position, we can rewrite the Kirchhoff integral (Hubral et al., 1996a; Li et al., 2020; Moser and Howard, 2008):

$$V^D(x) = \int w(s, x, r) U(t, s, r) \delta(t - t_d(s, x, r)) dt ds dr, \quad (3)$$

where $w(s, x, r)$ is the imaging condition. The travel-time of the elementary diffractions $t_d(s, x, r)$ can be considered as

$$t_d(s, x, r) = T(s, x) + T(x, r), \quad (4)$$

where $T(s, x)$ is the travel-time trajectory between the source and the diffractor point x , and $T(r, x)$ is the travel-time trajectory between the diffractor point and the receiver. Considering the vectors from the source and the receiver $p_s = \nabla_x T(s, x)$ and $p_r = \nabla_x T(r, x)$, and a unit vector normal to the reflector n , the imaging condition can be defined as

$$w(s, x, r) = n^T \frac{(p_s + p_r)}{\|p_s + p_r\|}. \quad (5)$$

The image condition defined in Equation 5 can be analyzed as the dot product between the reflector unit normal and the bisectrix of the vectors at location x . This weight reaches its maximum value when the reflector normal and the bisectrix are parallel, indicating a specular reflection. The Fresnel zone, where the migration aperture is limited to the vicinity of the specular reflection point, can be defined as the set of locations where the image condition exceeds a small frequency-dependent value ϵ , given by $w(x, s, r) > 1 - \epsilon$ (Moser and Howard, 2008). This definition emphasizes the contributions of specular reflections in the migration process, which attenuates diffractions coming from outside the Fresnel zone and reduces the aliasing effects (Chen, 2004).

We can modify the Equation 5 to define a new weight that explicitly suppresses specular reflections and emphasizes diffractions (Moser and Howard, 2008):

$$w_d(s, x, r) = 1 - n^T \frac{(p_s + p_r)}{\|p_s + p_r\|}, \quad (6)$$

named ‘‘anti-stationary phase filter’’. The method presented by Moser and Howard (2008) substitutes $w(s, x, r)$ with $w_d(s, x, r)$ in Equation 3 to attenuate the reflections’ contributions to the wavefield and emphasize the diffraction’s energy, resulting in an image with information of small scatterers in subsurface.

2.1.4 Local Slant Stack

The diffraction imaging from the anti-stationary phase filter requires the construction of a reflector dip field for

each grid point. As done by Moser and Howard (2008), we implemented the local slant stack approach, where there is a search for local plane segments for each grid point with maximal semblance S .

From the initial reflectivity model, we define a plane curve $z = p(x - x_0) + z_0$, where (x_0, z_0) represents the coordinates of a given image point, and p is the ray parameter (Claerbout, 1978). We calculate the coherence or semblance S values along the curve (Neidell, 1997), where the value of S is expected to be closer to 1 in the situation where the curve defined by z coincides with the inclination of a reflector. We perform an iterative search for a relevant range of p values. Thus, for each image point x_0, z_0 of the velocity model, a window is defined, i.e., a new grid is defined based on a clipping of the original data; in this window, an iterative search is performed varying values of p , i.e., different dip values, and the code stores the p value that results in the highest S value for each image point. For field data, where a reflectivity model is not available, the local slant stack calculation is done on a migrated image where $w=1$. The objective of implementing the local slant stack is to obtain the horizontal and vertical components of the normal vector n of the Equation 5. Therefore, it calculates the reflector dip field components needed for the anti-stationary phase filter construction.

2.2 Software Implementation

The main algorithm was written in Python (version 3.8.8). Although a graphical user interface (GUI) can facilitate the user experience, Plattner (2020) emphasizes that a software controlled entirely by GUI may provide automating several tasks and reduce efficiency in large-scale projects. The Jupyter IDE is a solution for providing command-line implementations through an interactive interface, which reduces the experience required to effectively operate. For this reason, we implemented the software using the Jupyter IDE (<https://jupyter.org/>). The algorithm is dependent of some well-documented open-source Python libraries. The main libraries and modules can be installed with Anaconda (<https://www.anaconda.com/>): NumPy (Van der Walt et al., 2011) and SciPy (Jones et al., 2007), fundamental packages for scientific computations in Python involving matrix operations and signal analysis; Matplotlib (Hunter, 2007), for plotting graphs and data visualization; and ObsPy (Beyreuther et al., 2010), for seismic data analysis.

Figure 1 shows a general flowchart outlining the main steps of the DiffraPy implementation. The workflow involves pre-defined functions that perform the necessary steps to generate two images: a conventional

migration image and a diffraction image. For a general understanding of the proposed methods, we demonstrate them in a simplified toy model with a constant velocity $c = 250$ m/s. The model consists on eight horizontal reflectors, four small faults, five different small lenses with lateral length of 1 m, and white noise. The parameters used to generate it are listed in Table 1.

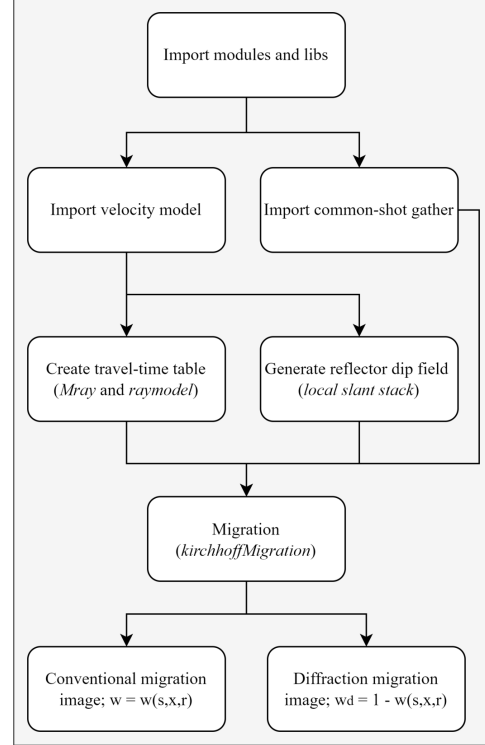


Figure 1: General flow chart diagram showing the main steps for generating the diffraction image.

2.2.1 Diffractions Forward Modeling

Our program handles operations with field and synthetic data. However, when working with synthetic data, there are several important factors to consider. While reflections are generated by smooth and continuous interfaces, diffractions are the result of the interaction of the wave with small structures and do not satisfy the general validation conditions of the ray theory presented by Červený (2001). Thus, not all forward modelers are capable of accurate modeling the non-specular responses of diffractions, e.g., simple ray-tracing modeling. For forward modeling diffractions responses, DiffraPy includes the *kirchhoffModeling* function built-in, but certain parameters must be set first. The func-

Table 1: Parameters applied in the toy model simulation.

Parameter	Value
Constant velocity c	250 m/s
Horizontal grid size nx	240
Vertical grid size nz	120
Horizontal discretization dx	1 m
Vertical discretization dz	1 m
Source frequency fr	40 Hz
Length of the wavelet (points) $npts$	200
Time increment dt	4 ms
Conventional mig. aperture (traces)	20
Diffraction mig. aperture (traces)	80
Migration window (samples)	20

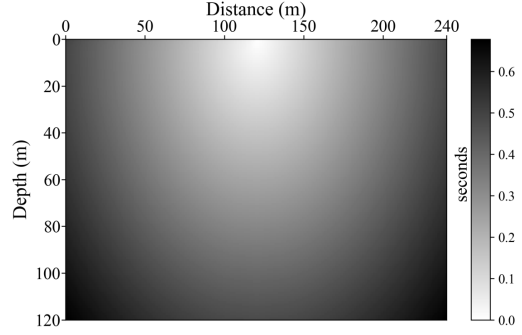
tion requires a smoothly-varying velocity model, i.e., a background model suitable for ray-tracing. Users can either import a smooth velocity model or generate a simple one using DiffraPy, such as a constant velocity or smooth gradient model. Based on the velocity model, travel-times must be calculated. This can be done using various approaches (Moser, 2012), including wavefront construction (Lee and Gibson, 2006; Vinje et al., 1993), eikonal equation solvers (Popovici and Sethian, 2002; Qin et al., 1992), and shortest path techniques (Moser, 1991). The *MRay* function generates a seismic wave travel-time grid for a single source sx using the shortest path method based on the velocity model (Figure 2). The *raymodel* function uses *MRay* to create nx time tables, i.e., one file for each source position sx , resulting in a 3D cube of dimensions $[nx, nz, nx]$. As our migration kernel is dependent on the velocity model, we emphasize that the travel-times calculation with *raymodel* is necessary for both synthetic and field data before generating the final images.

Before modeling the gather shots, it is necessary to create a wavelet that represents the inherent characteristics of the receivers. To create the wavelet, the user inputs the central frequency (fr), number of points or length of the wavelet (nps), and the time increment (dt). The *ricker* function generates a Ricker wavelet $r(t)$, defined as (Yilmaz, 2008)

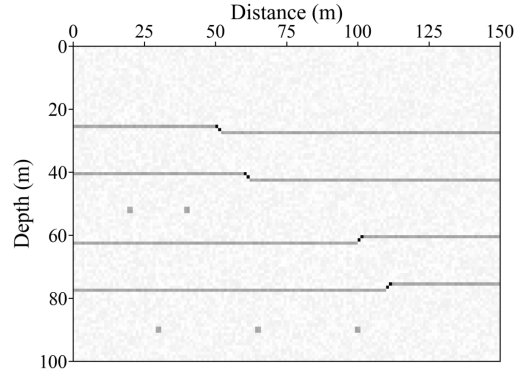
$$r(t) = (1 - 2\pi^2 f_r^2 t^2) e^{-\pi^2 f_r^2 t^2}. \quad (7)$$

2.2.2 Diffractions Imaging

For synthetic data applications, the user can either import or generate the velocity model file. If choosing to generate, the input parameter is the constant velocity c , and the user has to define a simplistic reflectivity model


Figure 2: Travel-time field for shot position $sx = 120$ m resulting from the *MRay* and *raymodel* functions for the toy model.

based on the geometry of the desired reflectors (Figure 3).


Figure 3: Reflectivity of the simple toy model.

Following the procedure presented by Moser and Howard (2008), it is necessary to generate a reflector dip field for each image point of the model grid. This is done with the *slant local* function, which applies the local slant stack approach. For each image point of the model and a relevant range of dips, the function calculates the dip semblances over local plane segments. The dip value with the maximal semblance for each image point is then stored in the resulting file. A separated file is stored with the maximum semblance values for each image point (Figure 4a). The dip field file can be decomposed into the vertical (*dipY*) and horizontal (*dipX*) components of reflector normal (Figure 4b). Differentiating the travel-time table generated by the *raymodel* function results in the ray vectors p_r and p_s (Equation 5). With the components of the reflector normal field (*dipY* and *dipX*), and the ray vectors (p_r and p_s), it is possible to calculate the image condition w .

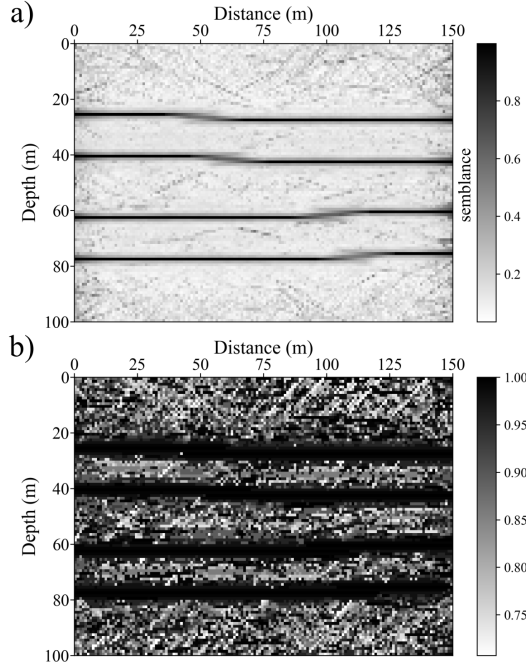


Figure 4: Semblance and dip field obtained by the *slant local* function for the toy model. (a) Maximum semblance values for each image point; (b) vertical component of reflector normal (*dipY*).

The *kirchhoffModeling* function is used to generate synthetic common-shot gathers based on the previously created travel-time table and wavelet. The input parameters are *nz* (vertical grid size), *nx* (horizontal grid size), *dx* (x-axis discretization), *dz* (z-axis discretization), and *dsx* (shots spacing). Thereby, the program can use the necessary files to generate the synthetic common-shot gathers (Figure 5).

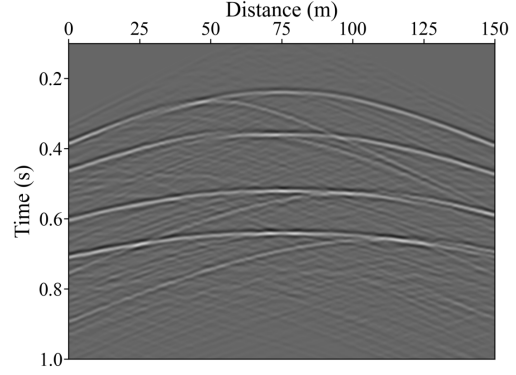


Figure 5: Synthetic common-shot gather resulting from the *kirchhoffModeling* function for the toy model (source position $s_x = 75$ m).

Following, the user inputs the migration parameters: *appref* (aperture size for the conventional migration), *appdif* (aperture size for diffraction migration), and *win* (window length). The *kirchhoffMigration* function generates two migrated images simultaneously: the conventional prestack-depth Kirchhoff migration with the imaging condition, which preserves specular reflections from the Fresnel zone (Figure 6a), and the diffraction prestack-depth Kirchhoff migration with the diffraction-oriented weight ($w_d = 1 - w$), which enhances diffractions and suppresses specular reflections (Figure 6b). Both conventional and diffraction migration images can be exported in high-resolution files.

It is possible to combine both results into a single image (Figure 7) that contains the structures present in the conventional and the diffracted-oriented migrations.

3 Results and Discussion

In order to evaluate the performance of our code, we conducted an analysis of a realistic synthetic example obtained from a portion of the SEG/EAGE salt model (Aminzadeh et al., 1997). The model includes complex reflectors and discontinuities, which pose challenges for seismic imaging techniques. To further test the capability of our code to image small-scale scatterers, we added several point diffractors with a lateral length

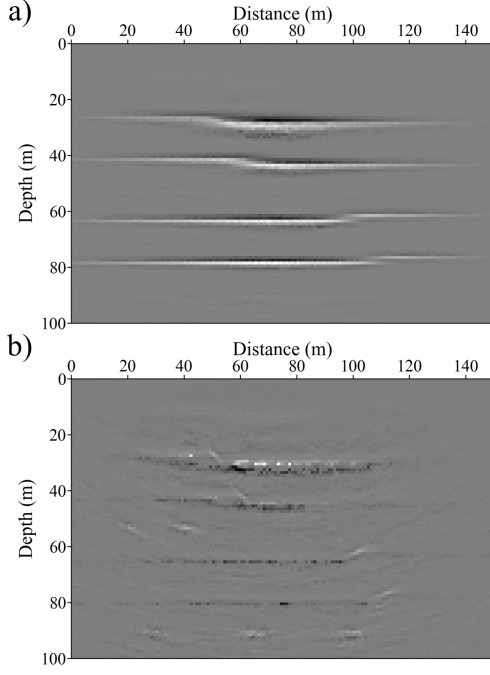


Figure 6: Comparison of conventional and diffraction-oriented prestack-depth Kirchhoff migration images generated by the *kirchhoffMigration* function for the toy model. (a) Migrated image using the conventional weighted Kirchhoff migration. The specular reflections are not suppressed, but the position of the scatterers is not well defined. (b) Migrated image using the diffraction-oriented weight ($w_d = 1 - w$), where diffractions are enhanced and specular reflections are suppressed, resulting in better resolution of scatterer positions.

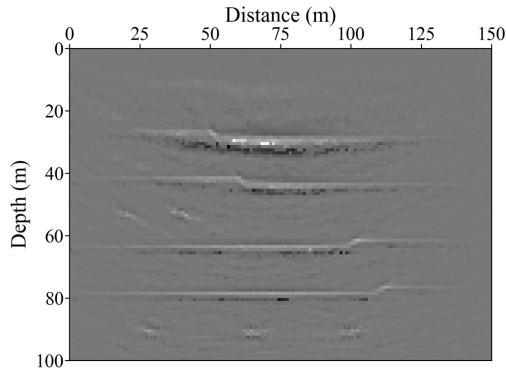


Figure 7: Combined image of the conventional and diffraction-oriented prestack-depth Kirchhoff migrations. The image was generated by summing the conventional migration image with the diffraction migration image. The resulting image highlights both the specular reflections and non-specular diffractions, allowing for a more comprehensive interpretation of the subsurface structures.

of 30 m to the model, simulating the response of point scatterers in the data. As shown in Figure 8a, the velocity model of our synthetic example is heterogeneous, reflecting the geological complexity of the subsurface. This complexity is further highlighted by the reflectivity model in Figure 8b, which shows the presence of five small lenses and white noise in addition to the main reflectors. Table 2 lists the other key parameters used to create the model, including the grid size and discretization parameters.

When generating synthetic data, having the ability to specify and control the velocity model is a key advantage, as it allows for better control over the resulting data and easier comparison with field data. In our software, the user can either import a pre-existing velocity model or generate one by specifying a constant velocity value. This flexibility allows for the creation of custom models that fit specific geological scenarios. One potential limitation of using simplistic models is that they may not accurately capture the complexity and heterogeneity of real-world geological structures. However, by generating synthetic data with controlled models, it is possible to test and optimize processing algorithms, which can later be applied to field data.

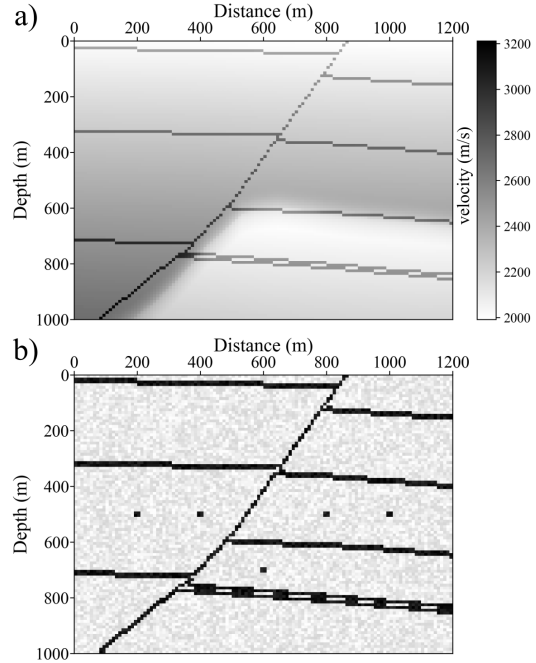


Figure 8: (a) Velocity model of the synthetic example. (b) Reflectivity model with five small lenses and white noise.

Figure 9a shows the maximum semblance values

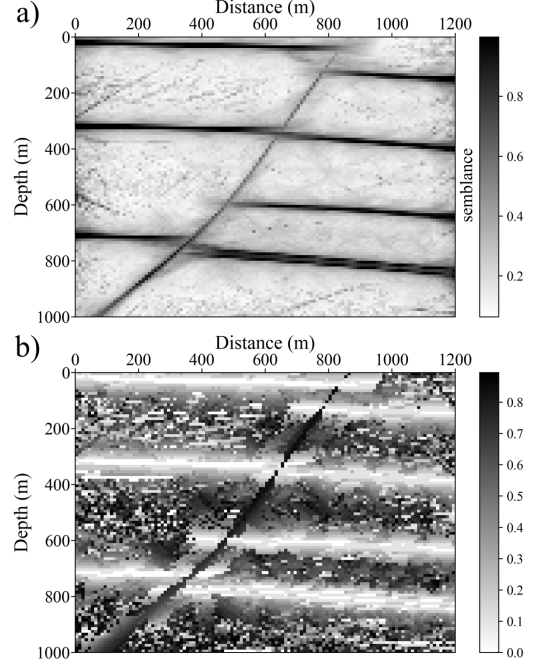
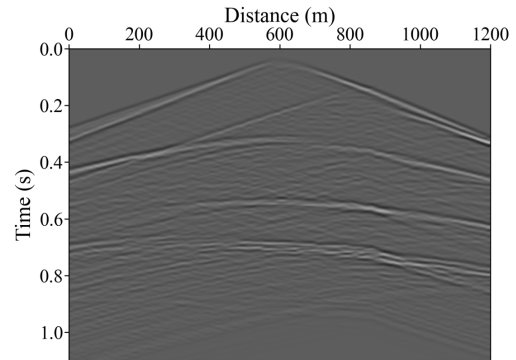
Table 2: Parameters applied in the realistic synthetic example.

Parameter	Value
Horizontal grid size nx	120
Vertical grid size nz	100
Horizontal discretization dx	10 m
Vertical discretization dz	10 m
Source frequency fr	40 Hz
Length of the wavelet (points) $npts$	200
Time increment dt	5 ms
Conventional mig. aperture (traces)	20
Diffraction mig. aperture (traces)	50
Migration window (samples)	10

obtained using the *slant local* function, while Figure 9b shows the horizontal component of the reflector normal field (*dipX*). The horizontal component of the reflector normal is closer to zero near the horizontally oriented reflectors, while it is closer to one near the fault. Given that the normals are more irregular at the faults junctions and reflectors discontinuities, the maximum semblance values are lower at those positions. It is noteworthy that the scattering lenses do not appear to cause any disturbances in either of the images.

The *kirchhoffModeling* function was used to generate synthetic common-shot gathers after creating the time-travel table and wavelet. Figure 10 shows the resulting file for a source at 600 m. Note that the diffraction hyperbolas of the small lenses are not visible individually in the data because their amplitudes were masked by the reflection's energy. Forward modeling of seismic data is a powerful tool that offers several advantages for seismic imaging and interpretation (Moser, 2012). It allows us to generate synthetic seismic data under controlled conditions, which can be used to validate seismic processing workflows and to study the impact of different acquisition and processing parameters on the final image. Forward modeling can also be used to test the sensitivity of the data to changes in the velocity and reflectivity models, allowing us to assess the robustness of our interpretations and identify areas of uncertainty. In addition, it can be used to investigate the illumination of a subsurface target and designing field surveys. Furthermore, synthetic data can provide insights into the seismic wave propagation and scattering mechanisms, allowing us to better understand the subsurface characteristics and to refine our imaging techniques.

The availability of the travel-time grid file is an essential aspect of seismic diffractions imaging using DiffraPy. As discussed, the *raymodel* function generates


Figure 9: Semblance and dip field obtained by the *slant local* function for the synthetic example. (a) Maximum semblance values for each image point; (b) horizontal component of reflector normal (*dipX*).

Figure 10: Synthetic common-shot gather resulting from the *kirchhoffModeling* function for the synthetic example (source position $sx = 600$ m).

this file, which is crucial for performing the migration. For synthetic data modeled with the code, this file is already available. Nonetheless, for field data, the user must run the *raymodel* function before migration.

The final products of the program are shown in Figure 11. The diffraction image (Figure 11b) shows the scattering lenses positions, although residual energy from the reflections is still present contaminating the image. Improving the accuracy of the calculated dip field is crucial for effectively suppressing reflections energy. Therefore, we expect to improve the method applied to generate the reflector normal and the dip field. By refining these aspects, we aim to enhance the suppression of the reflected component of the wavefield. Nevertheless, the code successfully suppressed most of the reflector energy, making the responses from small diffractors, fault junctions, and discontinuities more visible. Additionally, the fault is more prominent in the diffraction image, providing better insight into the subsurface structure for the interpreter.

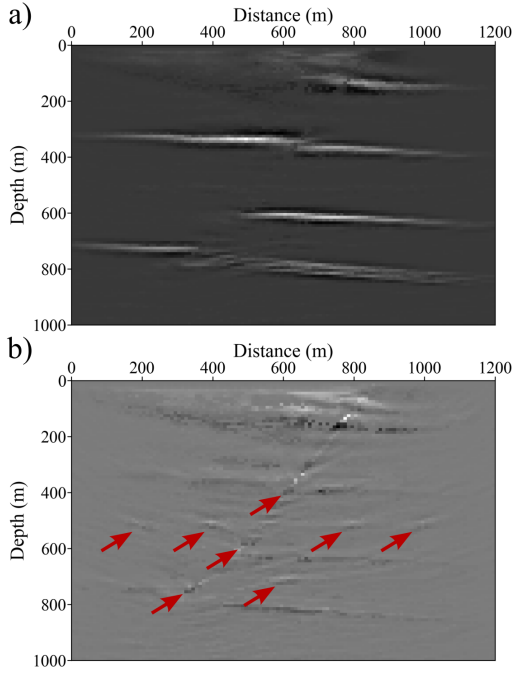


Figure 11: Comparison of conventional and diffraction-oriented prestack-depth Kirchhoff migration images generated by the *kirchhoffMigration* function for the synthetic example. (a) Migrated image using the conventional weighted Kirchhoff migration. (b) Migrated image using the diffraction-oriented weight ($w_d = 1 - w$), where diffractions are enhanced. The arrows indicate the main diffractions positions.

Figure 12 shows the sum of the two migration ima-

ges. By overlaying the two images, it is possible to identify the locations of the diffractions and their relationships with the larger-scale structures. This can help in the interpretation of the geological features of the subsurface, such as identifying areas of potential hydrocarbon presence, structures related to coals seams, or locations of structural complexity. The diffraction migration image may highlight features that are not apparent in the conventional migration image, while the conventional migration image may provide context and a more complete picture of the subsurface. Thus, the combination of both images can lead to a more accurate and comprehensive interpretation of the subsurface structure.

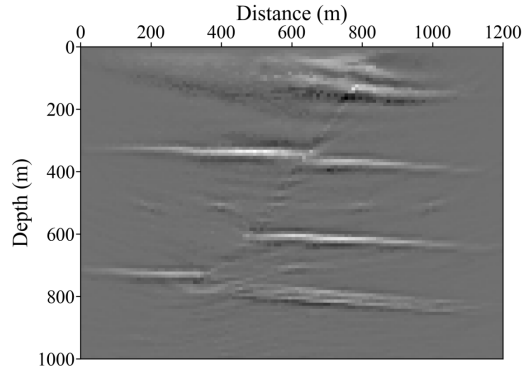


Figure 12: Combined image of the conventional and diffraction-oriented prestack-depth Kirchhoff migrations for the synthetic model.

The proposed method has a potential limitation due to its high dependence on a correct velocity model. As pointed out by Lin et al. (2018b), the stationary phase point determination in the method is highly sensitive to the velocity model and local slopes. Any significant errors in the velocity model used for migrations can result in artifacts and potentially misleading migrated images. To assess the impact of erroneous velocity models on our diffractions-oriented migration, we conducted a comparative analysis using three different velocity models for the toy model: the correct model (Figure 13a), a model with a 5% higher velocity (Figure 13b), and a model with a 5% lower velocity (Figure 13c). Observing the migration results, we found that the inaccurate velocity models leads to the expected effects on the positioning and shape of reflectors and diffractions. In the case of the too high velocity model, the reflectors appear shifted downward, and the diffractions exhibit variations in their shapes. Conversely, with the too low velocity model, the reflectors are shifted upward, and the diffractions also display shapes alterations. Despite

these deviations, the diffractions remain discernible in both erroneous velocity models. A potential avenue for future research could be to investigate the threshold at which errors in the velocity model start to produce migration artifacts. This would help to identify the level of accuracy required for the velocity model to ensure reliable and accurate migrated images.

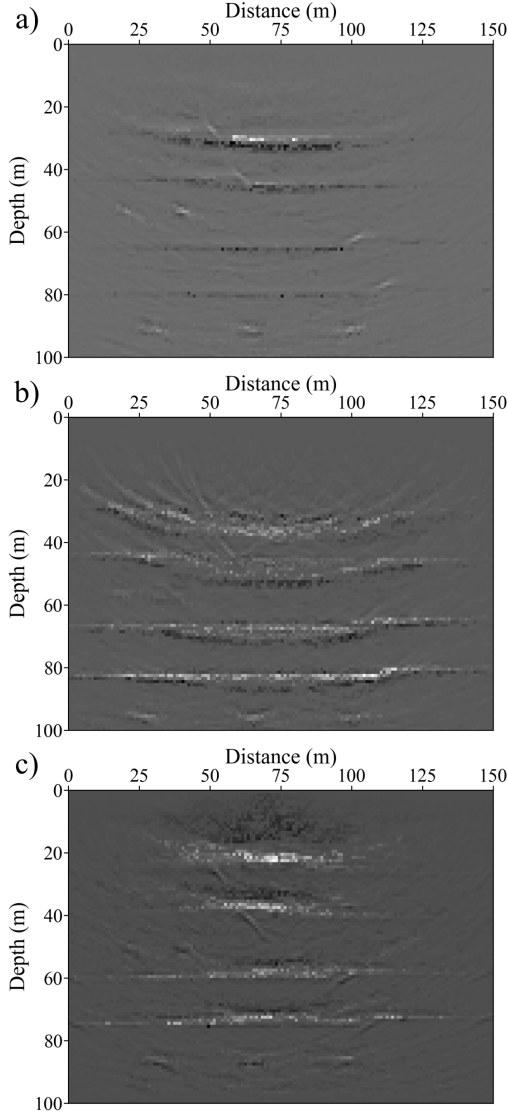


Figure 13: Comparative analysis of diffractions-oriented migration using different velocity models. (a) Migration image with the correct velocity model. (b) Migration image with a velocity model 5% higher than the correct velocity. (c) Migration image with a velocity model 5% lower than the correct velocity.

A critical point of the anti-stationary phase filter mi-

gration is determining the dip field for calculating the components of the reflector normal field (Figures 4b and 9b). For each image point of the reflectivity model, the function *slant local* calculates several dip values and dip semblances over local plane segments. For field data, where the reflectivity model is not available, the *slant local* function is applied on a migrated image where $w=1$. The dip value with the maximum semblance value is stored for each image point. It is important to note that not all dip values correspond to actual reflector dips, as some may be randomly determined from relatively low semblance values. To address this, we incorporate the semblance values (Figures 4a and 9a) as weights during the conventional migration process. This weighting strategy ensures that we prioritize coherent dips, enhancing the reliability and accuracy of the migration results. However, the conventional and the diffraction-oriented migration differ in their treatment of semblance as a weight. For the diffraction-oriented migration, the irregular nature of the normal at fault junctions, reflector discontinuities, and other irregularities results in lower semblance values. Therefore, incorporating the semblance as a weight in the diffraction migration would potentially obscure the precise locations of the diffractions. To ensure the accurate imaging of diffractions, we refrain from using semblance as a weight, allowing us to preserve the visibility and integrity of the diffraction events in the migrated image.

While DiffraPy represents an important contribution as an open-access software dedicated to diffractions imaging, it is important to acknowledge its limitations. One of the critical aspects that require further improvement is the calculation of the dip field and reflector normal. Currently, our implemented solution overlooks artifacts that may arise in the shallower portions of the migrated images, resulting in a perceived blurring effect. To address this limitation, our future work will focus on enhancing and fine-tuning the code responsible for calculating the reflector normal. By refining this component, we aim to minimize artifacts and improve the overall image quality, thereby providing more accurate and detailed representations of subsurface structures in diffractions imaging.

The ground-penetrating radar (GPR) is a non-destructive geophysics method based on the reflection of electromagnetic waves. The method present similar kinematics behaviour to seismic reflection common-offset acquisition, despite the differences in the frequencies magnitude and the reached depths. Diffraction events are frequently observed in GPR profiles as a result of the short wavelength in comparison to the dimensions of the targets and the signal's heightened sus-

ceptibility to rough interfaces (Economou et al., 2020; Markovic et al., 2022). Recent publications focus on separating and imaging diffractions obtained in GPR data for identifying steep fractures, karsts, small-scale objects, near surface utilities, and velocity analysis (De Figueiredo et al., 2013; Delf et al., 2022; Economou et al., 2020; Grasmueck et al., 2015, 2013; Liu et al., 2023; Liu and Shi, 2022; Maciel and Biloti, 2020; Schwarz and Krawczyk, 2020; Yuan et al., 2019; Zhao et al., 2019b). Furthermore, the utilization of GPR diffractions has extended to mapping the internal structure of glaciers (Delf et al., 2022). Given the similarity in the processing workflow, we expect that DiffraPy might be adapted with minor modifications in the input files re-reading step for handling GPR diffractions imaging as well. Currently, DiffraPy assumes a common-shot geometry for data, but by incorporating considerations for common-offset geometry, it can be tailored to accommodate GPR data. Further research and development could expand the software’s capabilities to handle a wider range of geophysical data types, allowing for more efficient and accurate subsurface imaging.

4 Conclusions

DiffraPy is the first open-source freely distributed Python-based software designed for imaging seismic diffractions. The program rely on pre-built functions that perform the necessary steps for generating a diffraction image: calculating the travel-time field, the dip field, and performing the conventional Kirchhoff migration and the diffraction-oriented Kirchhoff migration. Furthermore, DiffraPy offers support for forward modeling of synthetic seismic data, facilitating new testing and experimentation. The diffraction migration is based on the anti-stationary phase filter presented by Moser and Howard (2008), where a modified isotropic weight factor $w_d = 1 - w(s, x, r)$ is incorporated in the migration integral. A key advantage of this approach is that both conventional and diffraction images are generated simultaneously, avoiding additional computational time.

Through the application of DiffraPy on a synthetic example derived from the SEG/EAGE salt model, we have demonstrated its efficacy in locating diffractors and enhancing a fault structure, with the diffraction migration effectively suppressing the majority of reflection energy. However, it is important to acknowledge the current limitations of our implementation, such as artifacts resulting from the reflector normal calculation, which is an area we are actively addressing and refining. Additionally, the accuracy and reliability of the method are contingent upon the availability of an accurate velocity

model, as emphasized in our analysis.

DiffraPy can contribute to the geophysical community with a solution for testing and reproducing diffractions imaging examples. We expect to integrate GPR data applications with minor modifications in our program.

Code availability section

Name of the code: DiffraPy.

Contact: Guilherme Zakarewicz,
guilherme.zakarewicz.97@gmail.com

License: permissive MIT license.

Program language: Python 3.8.8.

Software required: No external software. Python dependencies are listed in the project, as the libraries NumPy, SciPy, Matplotlib, and ObsPy. We tested the software on Ubuntu 20.04 and Windows 10.

More details about DiffraPy, access to the code, and documentation can be found in the project’s repository: <https://github.com/GuilhermeZakarewicz/DiffraPy>.

Author contributions

Zakarewicz, G: Conceptualization, analysis design, main text writing, main code development;

Maciel, S: Conceptualization, original idea conception, main text supervision, review and editing, code development guidance;

Biloti, R: Code development guidance and suggestions, main text supervision, review and editing.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Acknowledgements

The authors would like to thank CNPq for the Research Grant provided.

References

- Ajithabh, K., and P. K. Patro, 2023, SigMT: An open-source Python package for magnetotelluric data processing: *Computers & Geosciences*, **171**, 105270.
- Aminzadeh, F., J. Brac, and T. Kunz, 1997, 3-D Salt and Overthrust models: SEG/EAGE 3-D Modeling Series No.1.
- Bansal, R., and M. G. Imhof, 2005, Diffraction enhancement in prestack seismic data: *Geophysics*, **70**, V73–V79.

- Berkovitch, A., I. Belfer, Y. Hassin, and E. Landa, 2009, Diffraction imaging by multifocusing: *Geophysics*, **74**, no. 6, WCA75–WCA81.
- Beyreuther, M., R. Barsch, L. Krischer, T. Megies, Y. Behr, and J. Wassermann, 2010, ObsPy: A Python Toolbox for Seismology: *Seismological Research Letters*, **81**, 530–533.
- Blanchy, G., S. Saneiyan, J. Boyd, P. McLachlan, and A. Binley, 2020, ResIPy, an intuitive open source software for complex geoelectrical inversion/modeling: *Computers & Geosciences*, **137**, 104423.
- Bueno, A., L. Zuccarello, A. Díaz-Moreno, J. Woollam, M. Titos, C. Benítez, I. Álvarez, J. Prudencio, and S. De Angelis, 2020, PICOSS: Python interface for the classification of seismic signals: *Computers & Geosciences*, **142**, 104531.
- Chen, J., 2004, Specular ray parameter extraction and stationary-phase migration: *Geophysics*, **69**, 249–256.
- Claerbout, J. F., 1978, Snell waves: Stanford Exploration Project Report No. 15, 57–72.
- De Figueiredo, J., F. Oliveira, E. Esmi, L. Freitas, J. Schleicher, A. Novais, P. Sussner, and S. Green, 2013, Automatic detection and imaging of diffraction points using pattern recognition: *Geophysical Prospecting*, **61**, 368–379.
- Decker, L., X. Janson, and S. Fomel, 2014, Carbonate reservoir characterization using seismic diffraction imaging: *Interpretation*, **3**, SF21–SF30.
- Decker, L., D. Merzlikin, and S. Fomel, 2017, Diffraction imaging and time-migration velocity analysis using oriented velocity continuation: *Geophysics*, **82**, U25–U35.
- Delf, R., R. G. Bingham, A. Curtis, S. Singh, A. Giannopoulos, B. Schwarz, and C. P. Borstad, 2022, Reanalysis of polythermal glacier thermal structure using radar diffraction focusing: *Journal of Geophysical Research: Earth Surface*, **127**, e2021JF006382.
- Dell, S., and D. Gajewski, 2011, Common-reflection-surface-based workflow for diffraction imaging: *Geophysics*, **76**, S187–S195.
- Economou, N., A. Vafidis, M. Bano, H. Hamdan, and J. Ortega-Ramirez, 2020, Ground-penetrating radar data diffraction focusing without a velocity model: *Geophysics*, **85**, H13–H24.
- Enders, P., 1996, Huygens principle and the modeling of propagation: *European Journal of Physics*, **17**, 226–235.
- Fehler, M. C., and L. Huang, 2002, Modern imaging using seismic reflection data: *Annual Review of Earth and Planetary Sciences*, **30**, 259–284.
- Fomel, S., E. Landa, and T. Taner, 2007, Poststack velocity analysis by separation and imaging of seismic diffractions: *Geophysics*, **72**, no. 6, U89–U94.
- Grasmueck, M., T. Moser, M. Pelissier, J. Pajchel, and K. Pomar, 2015, Diffraction signatures of fracture intersections: *Interpretation*, **3**, SF55 – SF68.
- Grasmueck, M., M. C. Quintà, K. Pomar, and G. P. Eberli, 2013, Diffraction imaging of sub-vertical fractures and karst with full-resolution 3D Ground-Penetrating Radar: *Geophysical Prospecting*, **61**, 907–918.
- Guedes, V. J. C. B., S. T. R. Maciel, and M. P. Rocha, 2022, Refrapy: A Python program for seismic refraction data analysis: *Computers & Geosciences*, **159**, 105020.
- Hagedoorn, J., 1954, A process of seismic reflection interpretation: *Geophysical Prospecting*, **2**, 85–127.
- Hector, B., and J. Hinderer, 2016, pyGrav, a Python-based program for handling and processing relative gravity data: *Computers & Geosciences*, **91**, 90–97.
- Hertweck, T., C. Jäger, A. Goertz, and J. Schleicher, 2003, Aperture effects in 2.5D Kirchhoff migration: A geometrical explanation: *Geophysics*, **68**, 1673–1684.
- Hubral, P., J. Schleicher, and M. Tygel, 1996a, A unified approach to 3-D seismic reflection imaging, part I: Basic concepts: *Geophysics*, **61**, 742–758.
- Hubral, P., M. Tygel, and J. Schleicher, 1996b, Seismic image waves: *Geophysical Journal International*, **125**, 431–442.
- Hunter, J. D., 2007, Matplotlib: A 2D Graphics Environment: *Computing in Science Engineering*, **9**, 90 – 95.
- Huygens, C., 1690, *Traité de la Lumière*: Pieter van der Aa, Leiden.
- Jones, E., T. Oliphant, and P. Peterson, 2007, SciPy: Open source scientific tools for Python.
- Julian, B., and D. Gubbins, 1977, Three-dimensional seismic ray tracing: *Journal of Geophysics*, **43**, 95–114.
- Kanasewich, E. R., and S. M. Phadke, 1988, Imaging discontinuities on seismic sections: *Geophysics*, **53**, 334–345.
- Keydar, S., D. Pelman, and M. Ezersky, 2010, Application of seismic diffraction imaging for detecting near-surface inhomogeneities in the Dead Sea area: *Journal of Applied Geophysics*, **71**, 47–52.
- Khaidukov, V., E. Landa, and T. J. Moser, 2004, Diffraction imaging by focusing-defocusing: An outlook on seismic superresolution: *Geophysics*, **69**, no. 6, 1478–1490.
- Kim, S., S. Jee Seol, J. Byun, and S. Oh, 2021, Extrac-

- tion of diffractions from seismic data using convolutional U-net and transfer learning: *Geophysics*, **87**, 1–87.
- Klem-Musatov, K., 1994, Theory of seismic diffractions: Society of Exploration Geophysicists.
- Klokov, A., and S. Fomel, 2012, Separation and imaging of seismic diffractions using migrated dip-angle gathers: *Geophysics*, **77**, S131–S143.
- , 2013, Selecting an optimal aperture in Kirchhoff migration using dip-angle images: *Geophysics*, **78**, S243–S254.
- Krey, T., 1952, The significance of diffraction in the investigation of faults: *Geophysics*, **17**, 843–858.
- Krieger, L., and J. R. Peacock, 2014, MTpy: A Python toolbox for magnetotellurics: *Computers & Geosciences*, **72**, 167–175.
- Kunz, B. F., 1960, Diffraction problems in fault interpretation: *Geophysical Prospecting*, **8**, 381–388.
- Lecocq, T., C. Caudron, and F. Brenguier, 2014, MS-Noise, a Python Package for Monitoring Seismic Velocity Changes Using Ambient Seismic Noise: *Seismological Research Letters*, **85**, 715–726.
- Lee, K. J., and J. Gibson, Richard L., 2006, An improved mesh generation scheme for the wavefront construction method: *Geophysics*, **72**, T1–T8.
- Li, C., S. Peng, J. Zhao, and X. Cui, 2020, Diffraction imaging using an adaptive phase filter: *Geophysical Prospecting*, **68**, 164–177.
- Lin, P., S. Peng, J. Zhao, X. Cui, and W. Du, 2018a, Accurate diffraction imaging for detecting small-scale geologic discontinuities: *Geophysics*, **83**, S447–S457.
- Lin, P., S. Peng, J. Zhao, X. Cui, and H. Wang, 2018b, L1-norm regularization and wavelet transform: An improved plane-wave destruction method: *Journal of Applied Geophysics*, **148**, 16–22.
- Lin, P., J. Zhao, S. Peng, and X. Cui, 2021, Diffraction separation by variational mode decomposition: *Geophysical Prospecting*, **69**, 1070–1085.
- Liu, J., D. Draganov, R. Ghose, and Q. Bourgeois, 2021, Near-surface diffractor detection at archaeological sites based on an interferometric workflow: *Geophysics*, **86**, WA1–WA11.
- Liu, Y., J. Irving, and K. Holliger, 2023, Weighted diffraction-based migration velocity analysis of common-offset GPR reflection data: *IEEE Transactions on Geoscience and Remote Sensing*, **61**, 1–9.
- Liu, Y., and Z. Shi, 2022, Mapping water pipeline leakage by ground-penetrating radar diffraction imaging: *Geophysics*, **87**, WB1–WB7.
- Lowney, B., I. Lokmer, and G. O’Brien, 2021, Multi-domain diffraction identification: A supervised deep learning technique for seismic diffraction classification: *Computers & Geosciences*, **155**, 104845.
- Maciel, S., and R. Biloti, 2020, A statistics-based descriptor for automatic classification of scatterers in seismic sections: *Geophysics*, **85**, O83–O96.
- Markovic, M., R. Malehmir, and A. Malehmir, 2022, Diffraction pattern recognition using deep semantic segmentation: *Near Surface Geophysics*, **20**, 507–518.
- Mather, B., and R. Delhay, 2019, PyCurious: A Python module for computing the Curie depth from the magnetic anomaly.: *Journal of Open Source Software*, **4**, 1544.
- Merzlikin, D., and S. Fomel, 2017, Analytical path-summation imaging of seismic diffractions: *Geophysics*, **82**, S51–S59.
- Moser, T. J., 1991, Shortest path calculation of seismic rays: *Geophysics*, **56**, 59–67.
- , 2012, Review of ray-Born forward modeling for migration and diffraction analysis: *Stud. Geophys. Geod.*, **56**, 411–432.
- Moser, T. J., and C. B. Howard, 2008, Diffraction imaging in depth: *Geophysical Prospecting*, **56**, 627–641.
- Neidell, N. S., 1997, Perceptions in seismic imaging, part 2: Reflective and diffractive contributions to seismic imaging: *The Leading Edge*, **16**, 1121–1123.
- Officer, C. B., 1958, Introduction to the theory of sound transmission: McGraw-Hill Book Co.
- Peterie, S. L., R. D. Miller, J. Ivanov, and S. D. Sloan, 2020, Shallow tunnel detection using SH-wave diffraction imaging: *Geophysics*, **85**, EN29–EN37.
- Plattner, A. M., 2020, GPRPy: Open-source ground-penetrating radar processing and visualization software: *The Leading Edge*, **39**, 332–337.
- Popovici, A. M., and J. A. Sethian, 2002, 3-D imaging using higher order fast marching traveltimes: *Geophysics*, **67**, 604–609.
- Qin, F., Y. Luo, K. B. Olsen, W. Cai, and G. T. Schuster, 1992, Finite-difference solution of the eikonal equation along expanding wavefronts: *Geophysics*, **57**, 478–487.
- Rad, P. B., and C. J. Hickey, 2021, Seismic diffraction separation in the near surface: Detection of high-contrast voids in unconsolidated soils: *Geophysics*, **86**, WA13–WA23.
- Reshef, M., and E. Landa, 2009, Post-stack velocity analysis in the dip angle domain using diffractions: *Geophysical Prospecting*, **57**, 811–821.
- Rücker, C., T. Günther, and F. M., 2017, pyGIMLi: An open-source library for modelling and inversion in geophysics: *Computers & Geosciences*, **109**, 106–

- 123.
- Santos, L. A., E. H. P. Neves, A. F. M. Freire, M. A. C. Santos, R. Matsumoto, and C. M. I. Ajus, 2020, Diffraction velocity analysis in a single-channel seismic survey in the Joetsu Basin: *Geophysics*, **85**, U47–U53.
- Santos, L. T., J. Schleicher, M. Tygel, and P. Hubral, 2000, Seismic modeling by demigration: *Geophysics*, **65**, 1281–1289.
- Schneider, W. A., 1978, Integral formulation for migration in two and three dimensions: *Geophysics*, **43**, 49–76.
- Schwarz, B., and C. M. Krawczyk, 2020, Coherent diffraction imaging for enhanced fault and fracture network characterization: *Solid Earth*, **11**, 1891–1907.
- Sekiguchi, S., 2021, Efficient seismic ray tracing based on the shortest path method: *Geophysical Journal International*, **225**, 729–743.
- Sheng, T., and J. Zhao, 2022, Separation and imaging of diffractions using a dilated convolutional neural network: *Geophysics*, **87**, S117–S127.
- Shustak, M., and E. Landa, 2018, Time reversal for wave refocusing and scatterer detection using machine learning: *Geophysics*, **83**, T257–T263.
- Sun, J., J. Yang, Z. Li, J. Huang, J. Xu, and S. Zhuang, 2022, Reflection and diffraction separation in the dip-angle common-image gathers using convolutional neural network: *Geophysics*, **88**, WA281–WA291.
- Trorey, A. W., 1970, A simple theory for seismic diffractions: *Geophysics*, **35**, 762–784.
- Tschannen, V., N. Ettrich, M. Delescluse, and J. Keuper, 2020, Detection of point scatterers using diffraction imaging and deep learning: *Geophysical Prospecting*, **68**, 830–844.
- Tschannen, V., A. Ghanim, and N. Ettrich, 2022, Partial automation of the seismic to well tie with deep learning and bayesian optimization: *Computers & Geosciences*, **164**, 105120.
- Tygel, M., J. Schleicher, and P. Hubral, 1996, A unified approach to 3-D seismic reflection imaging, Part II: Theory: *Geophysics*, **61**, 759–775.
- Tyiasning, S., D. Merzlikin, D. Cooke, and S. Fomel, 2016, A comparison of diffraction imaging to incoherence and curvature: *The Leading Edge*, **35**, 86–89.
- Van der Walt, S., S. C. Colbert, and G. Varoquaux, 2011, The NumPy array: a structure for efficient numerical computation: *Computing in Science Engineering*, **13**, 22–30.
- Červený, V., 2001, *Seismic ray theory*: Cambridge University Press.
- Vinje, V., E. Iversen, and H. Gjoystdal, 1993, Travel-time and amplitude estimation using wavefront construction: *Geophysics*, **58**, 1157–1166.
- Wang, B., B. Jin, L. Huang, S. Liu, H. Sun, J. Liu, X. Ding, and S. Wang, 2020a, A hilbert polarization imaging method with breakpoint diffracted wave in front of roadway: *Journal of Applied Geophysics*, **177**, 104032.
- Wang, H., X. Liu, and Y. Chen, 2020b, Separation and imaging of seismic diffractions using a localized rank-reduction method with adaptively selected ranks: *Geophysics*, **85**, V497–V506.
- Yilmaz, O., 2008, *Seismic data analysis*: Society of Exploration Geophysicists.
- Yuan, H., M. Montazeri, M. C. Looms, and L. Nielsen, 2019, Diffraction imaging of ground-penetrating radar data: *Geophysics*, **84**, H1–H12.
- Zhang, J., and J. Zhang, 2014, Diffraction imaging using shot and opening-angle gathers: A prestack time migration approach: *Geophysics*, **79**, S23–S33.
- Zhao, J., X. Sun, S. Peng, W. Wei, and T. Liu, 2019a, Separating prestack diffractions with SVMF in the flattened shot domain: *Journal of Geophysics and Engineering*, **16**, 389–398.
- Zhao, J., Y. Wang, and C. Yu, 2015, Diffraction imaging by uniform asymptotic theory and double exponential fitting: *Geophysical Prospecting*, **63**, no. 2, 338–353.
- Zhao, J., C. Yu, S. Peng, and J. Cao, 2021, Least-squares imaging of diffractions by solving a hybrid L1-L2 norm minimization problem: *Geophysics*, **86**, S59–S72.
- Zhao, J., C. Yu, S. Peng, and Z. Chen, 2019b, Online dictionary learning method for extracting GPR diffractions: *Journal of Geophysics and Engineering*, **16**, 1116–1123.
- Zhao, J., C. Yu, S. Peng, and C. Li, 2020, 3D diffraction imaging method using low-rank matrix decomposition: *Geophysics*, **85**, S1–S10.
- Zhou, B., and S. A. Greenhalgh, 2005, Shortest path ray tracing for most general 2D/3D anisotropic media: *Journal of Geophysics and Engineering*, **2**, 54.
- Zhou, B., P. Hatherly, and W. Sun, 2017, Enhancing the detection of small coal structures by seismic diffraction imaging: *International Journal of Coal Geology*, **178**, 1–12.
- Zhou, B., M. Pavlova, R. Sliwa, and W. Sun, 2022, Seismic diffraction imaging for improved coal structure detection by a structure-oriented moving average error filter: *IEEE Transactions on Geoscience and Remote Sensing*, **60**, 1–13.

5 Conclusões e Considerações Finais

Imagens que preservam proporcionalmente as difrações sísmicas fornecem informações valiosas sobre estruturas como falhas, quinas, fraturas, cunhas e outras discontinuidades, com implicações significativas em diversas áreas e aplicações industriais. A partir de uma revisão sistemática de literatura e análise quantitativa sobre o imageamento de difrações sísmicas, identificamos um interesse crescente neste tema, com um aumento significativo no número de publicações a partir de 2010. Os métodos propostos apresentam notável diversidade de aplicações, o que permite aplicar o processamento de difrações em diferentes contextos geológicos. A maior parte dos estudos se concentra na caracterização de falhas, fraturas e outras discontinuidades em subsuperfície. As difrações não especulares têm recebido destaque em publicações voltadas para a indústria de óleo e gás, o que possibilita a identificação de estruturas associadas à presença de hidrocarbonetos. Contribuições recentes destacam o uso de difrações para a mineração de carvão, uma vez que é possível imagear estruturas relacionadas com a presença de depósitos e mitigar riscos de inundações e explosões. Além disso, notamos que outra aplicação emergente consiste no processamento de difrações para investigações rasas a partir do uso de dados de GPR. Este método geofísico utiliza ondas eletromagnéticas que superam em várias ordens de magnitude as frequências das investigações sísmicas (1 MHz a 1 GHz) (Annan, 2002), o que aumenta a capacidade de obter imagens de alta resolução.

A maioria das publicações analisadas aborda avanços nos métodos de imageamento de difrações. No entanto, há uma escassez de publicações focadas exclusivamente na interpretação de imagens obtidas a partir de dados de campo. Priorizar estudos que apliquem o processamento de difrações em estudos de caso pode aprimorar significativamente a aplicabilidade e o impacto dos métodos de separação e imageamento de difrações. Apesar do considerável potencial mineral e petrolífero do Brasil, a aplicação de técnicas de processamento de difrações ainda é minimamente explorada no país. Dentre os artigos analisados, notamos somente três estudos que aplicaram o imageamento de difrações para conjuntos de dados obtidos em bacias brasileiras (Asgedom et al., 2013; Coimbra et al., 2018; Gelius et al., 2013). De forma geral, há uma demanda por novos estudos com o objetivo de compreender e avaliar o potencial, os fatores limitantes e os desafios relacionados aos métodos apresentados.

Neste trabalho, apresentamos o primeiro código aberto desenvolvido em Python dedicado especificamente ao processamento e imageamento de difrações sísmicas. O código oferece funções essenciais para gerar imagens de difrações, além de possibilitar a modelagem de seções sintéticas para testes e experimentações. A migração de difrações é baseada na técnica apresentada por Moser e Howard (2008) que considera contribuições de difrações não especulares provenientes de fora da zona de Fresnel. Para isso, incorporamos um peso isotrópico anti-estacionário

$w_d = 1 - w(s, x, r)$ na equação da migração Kirchhoff.

Aplicamos o imageamento de difrações em um modelo sintético e notamos que o programa foi capaz de localizar pequenas estruturas espalhadoras inseridas no modelo, além de suprimir a maior parte da energia refletida. A imagem de difrações resultante também ressaltou a falha presente no modelo, o que facilita a interpretação e compreensão da subsuperfície. No entanto, reconhecemos que nossa implementação inicial apresenta limitações. Entre elas, há a presença de artefatos decorrentes do cálculo da normal do refletor. Além disso, a precisão do método depende da disponibilidade de um modelo de velocidades preciso, embora permita variações de velocidade. No caso da presença de hipérboles de difrações na seção sísmica, é possível empregá-las para uma análise de velocidades por meio do ajuste da curvatura hiperbólica. Adicionamos ruído branco em todas as seções modeladas, fato que não alterou a precisão do código em localizar as estruturas espalhadoras.

Dessa forma, o programa oferece uma solução aberta e documentada para a comunidade geofísica. Esperamos que o código apresentado possa contribuir com as atividades de pesquisadores, estudantes e profissionais da área interessados em testar e reproduzir exemplos de processamento de difrações. Estamos atualmente implementando pequenas modificações para que o código seja capaz de processar dados de GPR. Para estudos futuros, destacamos aplicar o programa em dados coletados em campo, incluindo tanto sísmica como GPR, e a realização de análises mais profundas sobre a capacidade do método em recuperar informações abaixo do limite de Rayleigh. Mais detalhes, acesso e documentação do código podem ser acessados no repositório github.com/GuilhermeZakarewicz/DiffraPy.

Referências Bibliográficas

- Alonaizi, F., R. Pevzner, A. Bona, M. Alshamry, E. Caspari, e B. Gurevich, 2014, Application of diffracted wave analysis to time-lapse seismic data for CO₂ leakage detection: *Geophysical Prospecting*, **62**, 197–209.
- Anderson, F. L., 2021, Huygens’ principle geometric derivation and elimination of the wake and backward wave: *Scientific Reports*, **11**, 1–13.
- Annan, A. P., 2002, GPR — history, trends, and future developments: *Subsurface Sensing Technologies and Applications*, **3**, 253—270.
- Asgedom, E. G., L. J. Gelius, e M. Tygel, 2013, 2D common-offset traveltime based diffraction enhancement and imaging: *Geophysical Prospecting*, **61**, 1178–1193.
- Baker, B. B., e E. T. Copson, 2003, The mathematical theory of Huygens’ principle: *American Mathematical Soc.*, **329**.
- Bansal, R., e M. G. Imhof, 2005, Diffraction enhancement in prestack seismic data: *Geophysics*, **70**, V73–V79.
- Bashir, Y., D. P. Ghosh, e C. W. Sum, 2018, Influence of seismic diffraction for high-resolution imaging: Applications in offshore Malaysia: *Acta Geophysica*, **66**, 305–316.
- Bashir, Y., N. Mohd Muztaza, S. Y. Moussavi Alashloo, S. H. Ali, e D. Prasad Ghosh, 2020, Inspiration for seismic diffraction modelling, separation, and velocity in depth imaging: *Applied Sciences*, **10**.
- Bauer, A., B. Schwarz, e D. Gajewski, 2017, Utilizing diffractions in wavefront tomography: *Geophysics*, **82**, R65–R73.
- Bauer, A., B. Schwarz, T. Werner, e D. Gajewski, 2019, Unsupervised event identification and tagging for diffraction focusing: *Geophysical Journal International*, **217**, no. 3, 2165–2176.
- Bednar, J. B., 2005, A brief history of seismic migration: *Geophysics*, **70**, 3MJ–20MJ.
- Berkovitch, A., I. Belfer, Y. Hassin, e E. Landa, 2009, Diffraction imaging by multifocusing: *Geophysics*, **74**, no. 6, WCA75–WCA81.
- Berryhill, J. R., 1977, Diffraction response for nonzero separation of source and receiver: *Geophysics*, **42**, 1158–1176.
- , 1979, Wave-equation datuming: *Geophysics*, **44**, 1329–1344.
- Castro, K. C. P. L., e L. S. Cunha, 2021, Forensic investigations with the identification of human remains with ground penetrating radar (GPR): A review: *Estudos Geológicos (UFPE)*, **31**, 64–86.
- Coimbra, T. A., J. H. Faccipieri, J. H. Speglich, L.-J. Gelius, e M. Tygel, 2018, Enhancement of diffractions in prestack domain by means of a finite-offset double-square-root traveltime: *Geophysics*, **84**, V81–V96.

- Cortes, C., e V. Vapnik, 1995, Support-vector networks: *Machine Learning*, **20**, 273–297.
- Dafni, R., e W. W. Symes, 2017, Diffraction imaging by prestack reverse-time migration in the dip-angle domain: *Geophysical Prospecting*, **65**, 295–316.
- De Figueiredo, J., F. Oliveira, E. Esmi, L. Freitas, J. Schleicher, A. Novais, P. Sussner, e S. Green, 2013, Automatic detection and imaging of diffraction points using pattern recognition: *Geophysical Prospecting*, **61**, 368–379.
- Dean, D. R., 1991, Robert Mallet and the founding of seismology: *Annals of science*, **48**, 39–67.
- Decker, L., e S. Fomel, 2021, A Probabilistic Approach to Seismic Diffraction Imaging: *Lithosphere*, **2021**. (6650633).
- Decker, L., X. Janson, e S. Fomel, 2014, Carbonate reservoir characterization using seismic diffraction imaging: *Interpretation*, **3**, SF21–SF30.
- Decker, L., D. Merzlikin, e S. Fomel, 2017, Diffraction imaging and time-migration velocity analysis using oriented velocity continuation: *Geophysics*, **82**, U25–U35.
- Delf, R., R. G. Bingham, A. Curtis, S. Singh, A. Giannopoulos, B. Schwarz, e C. P. Borstad, 2022, Reanalysis of polythermal glacier thermal structure using radar diffraction focusing: *Journal of Geophysical Research: Earth Surface*, **127**, e2021JF006382.
- Dell, S., e D. Gajewski, 2011, Common-reflection-surface-based workflow for diffraction imaging: *Geophysics*, **76**, S187–S195.
- Dell, S., A. Hoelker, e D. Gajewski, 2018, Using seismic diffractions for assessment of tectonic overprint and fault interpretation: *Geophysics*, **84**, IM1–IM9.
- Enders, P., 1996, Huygens' principle and the modelling of propagation: *European Journal of Physics*, **17**, 226–235.
- , 2009, Huygens' principle as universal model of propagation: *Latin-American Journal of Physics Education*, **3**, 4.
- Fehler, M. C., e L. Huang, 2002, Modern imaging using seismic reflection data: *Annual Review of Earth and Planetary Sciences*, **30**, 259–284.
- Fomel, S., 2002, Applications of plane-wave destruction filters: *Geophysics*, **67**, 1946–1960.
- Fomel, S., e H. Kaur, 2021, Wave-equation time migration: *Geophysics*, **86**, S103–S111.
- Fomel, S., E. Landa, e T. Taner, 2007, Postack velocity analysis by separation and imaging of seismic diffractions: *Geophysics*, **72**, no. 6, U89–U94.
- Fomel, S., P. Sava, I. Vlad, Y. Liu, e V. Bashkardin, 2013, Madagascar: open-source software project for multidimensional data analysis and reproducible computational experiments: *Journal of Open Research Software*.
- Ford, J., R. Urgeles, A. Camerlenghi, e E. Gràcia, 2021, Seismic diffraction imaging to characterize mass-transport complexes: Examples from the Gulf of Cadiz, South West Iberian Margin: *Journal of Geophysical Research: Solid Earth*, **126**, e2020JB021474.

- Gazdag, J., 1978, Wave equation migration with the phase-shift method: *Geophysics*, **43**, 1342–1351.
- Gelius, L.-J., M. Tygel, A. K. Takahata, E. G. Asgedom, e D. R. Serrano, 2013, High-resolution imaging of diffractions — a window-steered MUSIC approach: *Geophysics*, **78**, S255–S264.
- Hagedoorn, J., 1954, A process of seismic reflection interpretation: *Geophysical Prospecting*, **2**, 85–127.
- Harlan, W. S., J. F. Claerbout, e F. Rocca, 1984, Signal/noise separation and velocity estimation: *Geophysics*, **49**, 1869–1880.
- Hubral, P., J. Schleicher, e M. Tygel, 1996a, A unified approach to 3-D seismic reflection imaging, part I: Basic concepts: *Geophysics*, **61**, 742–758.
- Hubral, P., M. Tygel, e J. Schleicher, 1996b, Seismic image waves: *Geophysical Journal International*, **125**, 431–442.
- Huygens, C., 1690, *Traité de la Lumière*: Pieter van der Aa, Leiden.
- Jiang, J., W. Yang, H. Ren, e M. Chen, 2022, Diffraction separation and imaging using an improved singular value decomposition method: *IEEE Geoscience and Remote Sensing Letters*, **19**, 1–5.
- Kanasewich, E. R., e S. M. Phadke, 1988, Imaging discontinuities on seismic sections: *Geophysics*, **53**, 334–345.
- Keller, J. B., 1962, Geometrical theory of diffractions: *Journal of the Optical Society of America*, **52**, 116–130.
- Khaidukov, V., E. Landa, e T. J. Moser, 2004, Diffraction imaging by focusing-defocusing: An outlook on seismic superresolution: *Geophysics*, **69**, no. 6, 1478–1490.
- Khoshnavaz, M., A. Bóna, M. Hossain, M. Urosevic, e K. Chambers, 2016, Diffractivity - another attribute for the interpretation of seismic data in hard rock environment, a case study.: *Interpretation*, **4**, no. 4, B23–B32.
- Kim, S., S. Jee Seol, J. Byun, e S. Oh, 2021, Extraction of diffractions from seismic data using convolutional U-net and transfer learning: *Geophysics*, **87**, 1–87.
- Kitchenham, B., O. P. Brereton, D. Budgen, M. Turner, J. Bailey, e S. Linkman, 2009, Systematic literature reviews in software engineering—a systematic literature review: *Information and software technology*, **51**, 7–15.
- Klem-Musatov, K., 1994, *Theory of seismic diffractions*: Society of Exploration Geophysicists.
- Klokov, A., e S. Fomel, 2012, Separation and imaging of seismic diffractions using migrated dip-angle gathers: *Geophysics*, **77**, S131–S143.
- , 2013, Selecting an optimal aperture in Kirchhoff migration using dip-angle images: *Geophysics*, **78**, no. 6, S243–S254.
- Krey, T., 1952, The significance of diffraction in the investigation of faults: *Geophysics*, **17**, 843–858.

- Kunz, B. F., 1960, Diffraction problems in fault interpretation: *Geophysical Prospecting*, **8**, 381–388.
- Landa, E., e S. Keydar, 1998, Seismic monitoring of diffraction images for detection of local heterogeneities: *Geophysics*, **63**, 1093–1100.
- Landa, E., V. Shtivelman, e B. Gelchinsky, 1987, A method for detection of diffracted waves on common-offset sections: *Geophysical Prospecting*, **35**, no. 4, 359–374.
- Li, C., J. Gao, R. Wang, e Z. Gao, 2020a, Enhancing subsurface scatters using reflection-damped plane-wave least-squares reverse time migration: *IEEE Geoscience and Remote Sensing Letters*, **17**, 706–710.
- Li, C., S. Peng, X. Cui, e W. Du, 2022a, 3-D pre-stack diffraction separation by extending the PWD method with parametrized local slope: *Geophysical Journal International*, **232**, 750–763.
- Li, C., S. Peng, X. Cui, Q. Liu, e P. Lin, 2022b, Prestack diffraction separation by parameterizing the reflection local slope: *Geophysics*, **87**, S35–S44.
- Li, C., S. Peng, J. Zhao, e X. Cui, 2020b, Diffraction imaging using an adaptive phase filter: *Geophysical Prospecting*, **68**, 164–177.
- Li, C., S. Peng, J. Zhao, X. Cui, W. Du, e S. Satibekova, 2018, Polarity-preserved diffraction extracting method using modified apex-shifted Radon transform and double-branch Radon transform: *Journal of Geophysics and Engineering*, **15**, 1991–2000.
- Li, C., J. Zhao, S. Peng, e X. Cui, 2021, Prestack diffraction separation in the common virtual source gather: *Geophysics*, **86**, S113–S124.
- Li, C., J. Zhao, S. Peng, X. Cui, e P. Lin, 2020c, Separating and imaging diffractions of seismic waves in the full-azimuth dip-angle domain: *Journal of Geophysics and Engineering*, **17**, 339–356.
- Lin, P., C. Li, e S. Peng, 2022a, Diffraction extraction using a low-rank matrix approximation method: *IEEE Geoscience and Remote Sensing Letters*, **19**, 1–5.
- Lin, P., S. Peng, X. Huang, Y. Xiang, C. Li, e X. Cui, 2022b, Plane-wave destruction-based workflow for prestack diffraction separation in the shot domain: *Pure and Applied Geophysics*, 1–15.
- Lin, P., S. Peng, J. Zhao, e X. Cui, 2020, Diffraction separation and imaging using multichannel singular-spectrum analysis: *Geophysics*, **85**, V11–V24.
- Lin, P., S. Peng, J. Zhao, X. Cui, e W. Du, 2018a, A new scheme for velocity analysis and imaging of diffractions: *Journal of Geophysics and Engineering*, **15**, 1084–1093.
- , 2018b, Accurate diffraction imaging for detecting small-scale geologic discontinuities: *Geophysics*, **83**, S447–S457.
- Lin, P., S. Peng, J. Zhao, X. Cui, e H. Wang, 2018c, L1-norm regularization and wavelet transform: An improved plane-wave destruction method: *Journal of Applied Geophysics*, **148**, 16–22.
- Lin, P., J. Zhao, S. Peng, e X. Cui, 2021, Diffraction separation by variational mode decomposition:

- Geophysical Prospecting, **69**, 1070–1085.
- Liu, L., E. Vincent, X. Ji, F. Qin, e Y. Luo, 2016, Imaging diffractors using wave-equation migration: Geophysics, **81**, S459–S468.
- Lockwood, D. J., 2016, *in* Rayleigh and Mie Scattering: Springer New York, 1097–1107.
- Lowney, B., I. Lokmer, e G. O’Brien, 2021a, Multi-domain diffraction identification: A supervised deep learning technique for seismic diffraction classification: Computers & Geosciences, **155**, 104845.
- Lowney, B., I. Lokmer, G. S. O’Brien, e C. J. Bean, 2021b, Pre-migration diffraction separation using generative adversarial networks: Geophysical Prospecting, **69**, 949–967.
- Lowney, B., I. Lokmer, G. S. O’Brien, L. Amy, C. J. Bean, e M. Igoe, 2020, Enhancing interpretability with diffraction imaging using plane-wave destruction aided by frequency-wavenumber f-k filtering: Interpretation, **8**, T541–T554.
- Maciel, S., e R. Biloti, 2020, A statistics-based descriptor for automatic classification of scatterers in seismic sections: Geophysics, **85**, O83–O96.
- Markovic, M., R. Malehmir, e A. Malehmir, 2022, Diffraction pattern recognition using deep semantic segmentation: Near Surface Geophysics, **20**, 507–518.
- Marques Jr, A., R. K. Horota, E. M. de Souza, L. Kupssinskü, P. Rossa, A. S. Aires, L. Bachi, M. R. Veronez, L. Gonzaga Jr, e C. L. Cazarin, 2020, Virtual and digital outcrops in the petroleum industry: A systematic review: Earth-Science Reviews, **208**, 103260.
- Merzlikin, D., e S. Fomel, 2017, Analytical path-summation imaging of seismic diffractions: Geophysics, **82**, S51–S59.
- Merzlikin, D., S. Fomel, e M. K. Sen, 2019, Least-squares path-summation diffraction imaging using sparsity constraints: Geophysics, **84**, S187–S200.
- Merzlikin, D., S. Fomel, e X. Wu, 2020, Least-squares diffraction imaging using shaping regularization by anisotropic smoothing: Geophysics, **85**, S313–S325.
- Moser, T. J., 2012, Review of ray-Born forward modeling for migration and diffraction analysis: Stud. Geophys. Geod., **56**, 411–432.
- Moser, T. J., e C. B. Howard, 2008, Diffraction imaging in depth: Geophysical Prospecting, **56**, 627–641.
- Mulder, W. A., e R.-E. Plessix, 2004, A comparison between one-way and two-way wave-equation migration: Geophysics, **69**, 1491–1504.
- Neidell, N. S., 1997, Perceptions in seismic imaging, part 2: Reflective and diffractive contributions to seismic imaging: The Leading Edge, **16**, 1121–1123.
- Ogiesoba, O. C., e A. Klovov, 2017, Examples of seismic diffraction imaging from the Austin Chalk and Eagle Ford Shale, Maverick Basin, South Texas: Journal of Petroleum Science and Engineering, **157**, 248–263.

- Papziner, U., e K.-P. Nick, 1998, Automatic detection of hyperbolas in georadargrams by slant-stack processing and migration: *First Break*, **16**, 219–223.
- Prosatov, M., V. Cheverda, e V. Shilikov, 2021, Detection and characterization of fault, cavernous, and fracture zones by constructing seismic diffraction image attributes: *Doklady Earth Sciences*, **499**, 670—674.
- Rad, P. B., B. Schwarz, D. Gajewski, e C. Vanelle, 2018, Common-reflection-surface-based pre-stack diffraction separation and imaging: *Geophysics*, **83**, S47–S55.
- Rasol, M., J. C. Pais, V. Pérez-Gracia, M. Solla, F. M. Fernandes, S. Fontul, D. Ayala-Cabrera, F. Schmidt, e H. Assadollahi, 2022, GPR monitoring for road transport infrastructure: A systematic review and machine learning insights: *Construction and Building Materials*, **324**, 126686.
- Reshef, M., e E. Landa, 2009, Post-stack velocity analisys in the dip angle domain using diffracti-
ons: *Geophysical Prospecting*, **57**, 811–821.
- Santos, L. A., W. J. Mansur, e G. A. McMechan, 2012, Tomography of diffraction-based focusing operators: *Geophysics*, **77**, R217–R225.
- Santos, L. A., E. H. P. Neves, A. F. M. Freire, M. A. C. Santos, R. Matsumoto, e C. M. I. Ajus, 2020, Diffraction velocity analysis in a single-channel seismic survey in the Joetsu Basin: *Geophysics*, **85**, U47–U53.
- Sava, P., B. Biondi, e J. Etgen, 2005, Wave-equation migration velocity analisys by focusing dif-
fractions and reflections: *Geophysics*, **70**, no. 3, U19–U27.
- Schneider, W. A., 1978, Integral formulation for migration in two and three dimensions: *Geophy-
sics*, **43**, 49–76.
- Schwarz, B., 2019a, Chapter one - an introduction to seismic diffraction, *in* *Recent Advances in
Seismology*: Elsevier, volume **60** of *Advances in Geophysics*, 1–64.
- , 2019b, Coherent wavefield subtraction for diffraction separation: *Geophysics*, **84**, no. 3,
V157–V168.
- Schwarz, B., e D. Gajewski, 2017, Accessing the diffracted wavefield by coherent subtraction:
Geophysical Journal International, **211**, 45–49.
- Schwarz, B., e C. M. Krawczyk, 2020, Coherent diffraction imaging for enhanced fault and fracture
network characterization: *Solid Earth*, **11**, 1891–1907.
- Schwarz, B., C. Vanelle, D. Gajewski, e B. Kashtan, 2014, Curvatures and inhomogeneities: An
improved common-reflection-surface approach: *Geophysics*, **79**, S231–S240.
- Shen, H.-Y., Q. Li, Y.-Y. Yan, X.-X. Li, e J. Zhao, 2020, Separation of diffracted waves via SVD
filter: *Petroleum Science*, **17**, 1259–1271.
- Sheng, T., e J. Zhao, 2022, Separation and imaging of diffractions using a dilated convolutional
neural network: *Geophysics*, **87**, S117–S127.
- Sheriff, R. E., 1980, Nomogram for Fresnel-zone calculation: *Geophysics*, **45**, 968–972.

- Shustak, M., e E. Landa, 2018, Time reversal for wave refocusing and scatterer detection using machine learning: *Geophysics*, **83**, T257–T263.
- Silvestrov, I., R. Baina, e E. Landa, 2016, Poststack diffraction imaging using reverse-time migration: *Geophysical Prospecting*, **64**, 129–142.
- Stolt, R. H., 1978, Migration by Fourier transform: *Geophysics*, **43**, 23–48.
- Sturzu, I., A. Popovici, M. Pelissier, J. Wolak, e T. Moser, 2014, Diffraction imaging of the eagle ford shale: *First Break*, **32**, 49–59.
- Sun, J., J. Yang, Z. Li, J. Huang, J. Xu, e S. Zhuang, 2022, Reflection and diffraction separation in the dip-angle common-image gathers using convolutional neural network: *Geophysics*, **88**, WA281–WA291.
- Syed, F. I., A. AlShamsi, A. K. Dahaghi, e S. Neghabhan, 2022, Application of ML & AI to model petrophysical and geomechanical properties of shale reservoirs – a systematic literature review: *Petroleum*, **8**, 158–166. (SI: Computational Petroleum Engineering).
- Taner, T., 2006, Separation and imaging of seismic diffractions using plane-wave decomposition: SEG Technical Program Expanded Abstracts 2006, 2401–2405.
- Tranfield, D., D. Denyer, e P. Smart, 2003, Towards a methodology for developing evidence-informed management knowledge by means of systematic review: *British journal of management*, **14**, 207–222.
- Trorey, A. W., 1970, A simple theory for seismic diffractions: *Geophysics*, **35**, 762–784.
- Tschannen, V., N. Ettrich, M. Delescluse, e J. Keuper, 2020, Detection of point scatterers using diffraction imaging and deep learning: *Geophysical Prospecting*, **68**, 830–844.
- Tygel, M., J. Schleicher, e P. Hubral, 1994, Kirchhoff-Helmholtz theory in modelling and migration: *Journal of Seismic Exploration*, **3**, 203–214.
- , 1996, A unified approach to 3-D seismic reflection imaging, Part II: Theory: *Geophysics*, **61**, 759–775.
- Tyiasning, S., D. Merzlikin, D. Cooke, e S. Fomel, 2016, A comparison of diffraction imaging to incoherence and curvature: *The Leading Edge*, **35**, 86–89.
- Červený, V., 2001, *Seismic ray theory*: Cambridge University Press.
- Vries, D. D., e A. J. Berkhout, 1984, Velocity analysis based on minimum entropy: *Geophysics*, **49**, 2132–2142.
- Wang, H., X. Liu, e Y. Chen, 2020, Separation and imaging of seismic diffractions using a localized rank-reduction method with adaptively selected ranks: *Geophysics*, **85**, V497–V506.
- Weill, A., 2014, *in* *Acoustic Waves, Scattering*: Springer New York, 13–16.
- Xu, S., e A. Stovas, 2017, Fresnel zone in VTI and orthorhombic media: *Geophysical Journal International*, **213**, 181–193.
- Yilmaz, O., 2008, *Seismic data analysis*: Society of Exploration Geophysicists.

- Yuan, H., M. Montazeri, M. C. Looms, e L. Nielsen, 2019, Diffraction imaging of ground-penetrating radar data: *Geophysics*, **84**, H1–H12.
- Zhang, D., T. W. Fei, C. Tsingas, e Y. Luo, 2020, Diffraction imaging using the two-way imaging condition: *Geophysics*, **85**, H1–H11.
- Zhang, J., e J. Zhang, 2014, Diffraction imaging using shot and opening-angle gathers: A prestack time migration approach: *Geophysics*, **79**, S23–S33.
- Zhao, J., S. Peng, W. Du, e X. Li, 2016, Diffraction imaging method by Mahalanobis-based amplitude damping: *Geophysics*, **81**, no. 6, S399–S408.
- Zhao, J., X. Sun, S. Peng, W. Wei, e T. Liu, 2019, Separating prestack diffractions with SVMF in the flattened shot domain: *Journal of Geophysics and Engineering*, **16**, 389–398.
- Zhao, J., Y. Wang, e C. Yu, 2015, Diffraction imaging by uniform asymptotic theory and double exponential fitting: *Geophysical Prospecting*, **63**, no. 2, 338–353.
- Zhao, J., C. Yu, S. Peng, e C. Li, 2020, 3D diffraction imaging method using low-rank matrix decomposition: *Geophysics*, **85**, S1–S10.
- Zhou, B., P. Hatherly, e W. Sun, 2017, Enhancing the detection of small coal structures by seismic diffraction imaging: *International Journal of Coal Geology*, **178**, 1–12.
- Zhou, J. W., S. P. Peng, P. Lin, X. Q. Cui, e T. Wang, 2023, Diffraction separation and imaging using ensemble empirical mode decomposition and multichannel singular spectrum analysis: *Geophysical Prospecting*, **71**, 245–262.
- Zhu, J., e L. R. Lines, 1998, Comparison of Kirchhoff and reverse-time migration methods with applications to prestack depth imaging of complex structures: *Geophysics*, **63**, 1166–1176.
- Zhu, X., e R.-S. Wu, 2010, Imaging diffraction points using the local image matrices generated in prestack migration: *Geophysics*, **75**, S1–S9.

A Principais Funções do Código DiffraPy

```
def ricker(nps, fr, dt):
    """
    Generate a Ricker wavelet signal (Mexican Hat).

    Parameters:
    -----
    nps : int
        The number of samples in the output signal.
    fr : float
        The central frequency of the wavelet in Hz.
    dt : float
        The time step of the output signal in seconds.

    Returns:
    -----
    numpy.ndarray
        The Ricker wavelet signal as a 1-D numpy array.

    """
    npt = nps * dt
    t = np.arange(-npt/2, npt/2, dt)
    rick1 = (1 - t**2 * fr**2 * np.pi**2) * np.exp(- t**2 * np.pi**2 * fr**2)
    rick = rick1[(int(np.round(nps/2)) - (int(np.round(1/fr/dt)) + 1): nps)]
    l = len(rick)
    if l < nps:
        rick2 = np.append(rick, np.zeros([1, nps - l]))
    l = nps
    rick = rick2
    return np.array(rick)
```



```
def kirchhoffModeling(nsx, ngx, dsx, nx, nt, dt, TTh, R, W, filename):
    """
    Computes synthetic seismic common-shot gathers by Kirchhoff modeling.

    The method consists of computing the travel times of seismic waves from
    a source point to all the grid points in the model using a ray tracing
    algorithm, and then calculating the seismic amplitudes at each receiver
    location by summing the contributions of all the rays that reach that point.

    The amplitudes of the seismic waves are calculated using the Kirchhoff
    integral, which is an integral equation that relates the incident wave
    field to the scattered wave field at a given receiver location.

    Parameters:
    -----
    nsx : int
        Number of sources (shots).
    ngx : int
        Number of receivers (traces).
    dsx : int
        Step size for sources along the x-axis.
    nx : int
        Number of grid points along the x-axis.
    nt : int
        Number of time samples.
    dt : float
        Time step.
    TTh : numpy.ndarray
        A 3D array of shape [nsx, nz, nx] containing the travel times of
        all the seismic waves from all the source points to all the grid points
        in the model. It is calculated before by the 'raymodel3' function.
    R : numpy.ndarray
        A 2D array of shape [nz, nx] representing the reflectivity coefficients
        of the subsurface model.
    W : numpy.ndarray
        A 1D array of length nt representing the temporal source waveform.
        It is a Ricker wavelet calculated by the 'ricker' function.
```

```

filename : str
    Base name of the files to save the computed common-shot gathers. One
    file will be saved for each source point, with a suffix indicating
    the source index.

Returns:
-----
files : list of numpy.ndarray
    A list of length nsx containing the common-shot gathers computed for
    each source point. Each element in the list is a 2D array of shape
    [nt-2,ngx], representing the seismic trace data for a given source
    (sx).

"""

nsx=nx
ngx=nx

#Loop over shots
for isx in tqdm(range(0,nsx,dx)):
    D = np.zeros([nt,ngx])
    TSX = (TTh[isx, :, :]/ dt+1).astype(int) # Traveltime (indexes)

    # Loop Over Traces
    for gx in range(0,ngx):
        TXG=(TTh[gx, :, :]/ dt+1).astype(int) # Traveltime in heterogeneous medium (indexes)

        #Loop over time sample in a trace
        for t in range(0,nt):
            M=W[t-(TSX+TXG)+nt+1]*R
            #D[t,gx]=np.sum(M.flatten('F'));
            D[t,gx]=np.sum(M.flatten())
            gather1=np.diff(D[:, :, :], n=2, axis=0)

        file = str(filename)+"_"+str(isx)
        with open(file, "wb") as f:
            np.save(f, gather1)

files = []

for i in range(0,nsx,dx):
    file = str(filename)+"_"+str(isx)
    with open(file, 'rb') as file:
        (gather) = np.load(file)
        files.append(gather)

return files

def peso(TTh,dt,X,Y,igx,isx):
    """
    Computes the Kirchhoff weighting function w(s,r,x) for a given source-
    receiver pair in a 2D subsurface model, also called imaging condition
    or anti-stationary phase filter.

    Parameters:
    -----
    TTh : numpy.ndarray
        A 3D array of shape [nsx,nz,nx] containing the travel times of
        all the seismic waves from all the source points to all the grid points
        in the model. It is calculated by the 'raymodel3' function.
    dt : float
        The time sampling interval in seconds.
    X : numpy.ndarray
        A 2D array of shape [nz,ntr] representing the x component of the dip
        flocal_windowfield obtained by a local slant-stack approach with the function
        ''.
    Y : numpy.ndarray
        A 2D array of shape [nz,ntr] representing the y component of the dip
        field obtained by a local slant-stack approach with the function
        'local_window'.
    igx : int

```

```

        The index of the receiver point where the Kirchhoff integral is being
        computed.
    isx : int
        The index of the source point where the Kirchhoff integral is being
        computed.

    Returns:
    -----
    w : numpy.ndarray
        A 2D array of shape [nz,ntr] representing the Kirchhoff weighting
        function for the given source-receiver pair.

    References:
    -----
    [1] Moser, T.j. and Howard, C.B. (2008). Diffraction imaging in depth.
    Geophysical Prospecting.

    """

    gH = np.gradient(TTh, axis=2) #gradiente horizontal #diferen a entre colunas (do modelo de velocidade)
    gV = np.gradient(TTh, axis=1) #gradiente vertical #diferen a entre linhas

    prV = gV[igx,:,:]
    prH = gH[igx,:,:]

    psV = gV[isx,:,:]
    psH = gH[isx,:,:]

    pH = psH + prH
    pV = psV + prV

    norma = np.sqrt(pH**2 + pV**2)

    for idx, x in np.ndenumerate(norma):
        if x==0:
            norma[idx]=1e-16 #avoid nan's

    w = (pH/norma * X) + (pV/norma * Y)

    return w

def kirchhoffMigration(gather, isx, dx, dz, dt, win, dwin, app_ref, app_diff, TTh, X, Y, sm):
    """
    Perform Kirchhoff Migration on seismic data to create conventional
    reflection and diffraction migration images. It includes the
    obliquity factor, phase shift, aperture, window, and weighting
    function w(s,x,r).

    Parameters:
    -----
    gather : array-like
        A single gather or an array of gathers to be migrated.
    isx : int
        The index of the source trace in the gather, considered for
        the single gather case.
    dx : float
        The spacing between traces in the x direction.
    dz : float
        The spacing between samples in the z direction.
    dt : float
        The sampling interval in seconds.
    win : int
        The half-length of the migration window in samples.
    dwin : float
        The step size between migration windows in seconds.
    app_ref : int
        The aperture size in number of traces for the conventional migration.
    app_diff : int
        The aperture size in number of traces for the diffraction-oriented migration.
    TTh : array-like
        The travelttime table calculated by the raymodel3 function.
    X : array-like
        The X component of the dip field. Should be calculated with np.sin(m_theta).

```

```

        It is obtained with a local slant-stack approach by the function
        'local_window'.
Y : array-like
    The Y component of the dip field. Should be calculated with np.cos(m_theta).
    It is obtained with a local slant-stack approach by the function
    'local_window'.
sm : array-like
    Maximum correlation coefficients computed from local windowed slant stacks calculated from the local_window function.

Returns:
-----
refl_mig : array-like
    The conventional reflection migration image where  $w = w(s, x, r)$ .
diff_mig : array-like
    The diffraction migration image where  $w = (1 - w(s, x, r))$ .

Notes:
-----
- The function first checks whether the input data is for a single shot or multiple
shots, then processes the seismic data by phase shifting and applying the obliquity
factor. It then applies a window to the data, tapers the traces, and calculates the
migration for each window position. Finally, if the input data is composed by multiple
shots, it stacks the migration images for each window position to produce the final migration
image. Both conventional and diffraction migration images are generated simultaneously.

- An important step of the migration process is finding the optimal parameters of window
and aperture. A small window may overlook the event's energy, while a large one may include
noise and unrelated energy. Similarly, a small aperture may disregard diffraction energy,
while a large one may consider unrelated noise and energy. Generating an migrated image without
considering the effects of these parameters may lead to spurious results.
"""

gather = np.array(gather)

#####
#single shot#
#####
if gather.ndim == 2:

    gather = phase_shift(gather) #Phase Shift

    timer=np.round(TTh/dt)+1

    window = np.arange(-win,win,dwin)
    [nt,ntr]=gather.shape
    [ntr2,nz,nx]=timer.shape
    if ntr!=ntr2:
        print('Gather and travelttime table have different trace numbers')

    refl_mig = np.zeros([nz,nx])
    diff_mig = np.zeros([nz,nx])

    IX = np.arange(0,nx*dx,dx)
    IZ = np.arange(0,nz*dz,dz)
    [IIX,IIZ] = np.meshgrid(IX,IZ)

    # Loop over each trace of the shot gather at src isx
    for igx in tqdm(range(0,ntr)):

        w = peso(TTh,dt,X,Y,igx,isx)
        w = np.abs(w)

        ##### Single Shot
        #####
        #####
        w_reff = (w**4)*(np.sqrt(sm))
        w_diff = (1-w**4)
        #####
        #####
        #####

        trace_reflwin = np.zeros([nz,nx])
        trace_diffwin = np.zeros([nz,nx])
        R = np.sqrt(IIZ**2 + (IIX-(igx+isx)/2*dx)**2)

```

```

r_mask = (R==0)
R[r_mask]= dx/1000
obli = IIZ/R #Obliquity Factor
trace_appref = taper(ntr,nz,app_ref,ixx,igx) #Tapering
trace_appdif = taper(ntr,nz,app_dif,ixx,igx) #Tapering
sf = ssf(nz,nx,dx,R,SW) #Spherical Spreading Factor

for j in range(len(window)): #somar amplitudes da curva de difra o com uma janela
    t = timer[ixx,0:nz,0:nx] + timer[igx,0:nz,0:nx] #t_{d}
    twin = t + window[j]
    t2 = (twin<nt)*twin
    trace1=gather.T[np.ix_([igx],t2.flatten().astype(np.int32))]
    trace_refl1 = trace1.reshape([nz,nx])*(w_reff)
    trace_diff1 = trace1.reshape([nz,nx])*(w_diff)
    trace_refl = trace_refl1*trace_appref
    trace_diff = trace_diff1*trace_appdif
    trace_reflwin = trace_reflwin + trace_refl
    trace_diffwin = trace_diffwin + trace_diff

    refl_mig[0:nz,0:nx] = refl_mig[0:nz,0:nx] + trace_reflwin*obli*sf
    diff_mig[0:nz,0:nx] = diff_mig[0:nz,0:nx] + trace_diffwin*obli*sf

#####
#multiple shots , stack
#####
elif gather.ndim == 3:

    gathers_shifted = []

    for i in gather:
        gather_shifted = phase_shift(i)
        gathers_shifted.append(gather_shifted)

    files = gathers_shifted
    timer=np.round(TTh/dt)+1

    n_isx = np.shape(files[0])[1] #ntr
    d_isx = len(files) #number of shots
    isxs = np.linspace(0,n_isx,d_isx,endpoint=False) #array with the isx indexes

    refl_migs = []
    diff_migs = []

    for count,gather in tqdm(enumerate(files)):
        isx = int(np.round(isxs[count])) #accessing the correct index for source position isx
        window = np.arange(-win,win,dwin)
        [nt,ntr] = gather.shape
        [ntr2,nz,nx] = timer.shape
        if ntr!=ntr2:
            print('Gather and travelttime table have different trace numbers')

        refl_mig_isx = np.zeros([nz,nx])
        diff_mig_isx = np.zeros([nz,nx])

        IX = np.arange(0,nx*dx,dx)
        IZ = np.arange(0,nz*dz,dz)
        [IIX,IIZ] = np.meshgrid(IX,IZ)

        for igx in range(0,ntr):
            w = peso(TTh,dt,X,Y,igx,ixx)

            ##### #Stack
            #####
            #####
            w_reff = (np.abs(w)**4)*(np.sqrt(sm))
            w_diff = (1-np.abs(w)**4)
            #####
            #####

            trace_reflwin = np.zeros([nz,nx])
            trace_diffwin = np.zeros([nz,nx])
            R = np.sqrt(IIZ**2 + (IIX-(igx+isx)/2*dx)**2)

```

```

r_mask = (R==0)
R[r_mask]= dx/1000
obli = 11Z/R
trace_appref = taper(ntr,nz,app_ref, isx, igx)
trace_appdif = taper(ntr,nz,app_dif, isx, igx)
sf = ssf(nz,nx,dx,R,SW) #Spherical Spreading Factor

for j in range(len(window)):
    t = timer[isx,0:nz,0:nx] + timer[igx,0:nz,0:nx]
    twin = t + window[j]
    t2 = (twin<nt)*twin
    trace1 = gather.T[np.ix_([igx],t2.flatten()).astype(np.int32)]
    trace_refl1 = trace1.reshape([nz,nx])*(w_reff)
    trace_diff1 = trace1.reshape([nz,nx])*(w_diff)
    trace_refl = trace_refl1*trace_appref
    trace_diff = trace_diff1*trace_appdif
    trace_reflwin = trace_reflwin + trace_refl
    trace_diffwin = trace_diffwin + trace_diff

    refl_mig_isx[0:nz,0:nx] = refl_mig_isx[0:nz,0:nx] + trace_reflwin*obli*sf
    diff_mig_isx[0:nz,0:nx] = diff_mig_isx[0:nz,0:nx] + trace_diffwin*obli*sf

    refl_migs.append(refl_mig_isx)
    diff_migs.append(diff_mig_isx)

    refl_mig = np.add.reduce(refl_migs)
    diff_mig = np.add.reduce(diff_migs)

return refl_mig, diff_mig

def plot(grid, filename, nx, dx, nz, dz, xlabel="", ylabel="", cmp="viridis", it=None, color_bar=False):
    """
    Plots a 2D grid using matplotlib.

    Parameters
    -----
    grid : numpy.ndarray
        2D array of data to plot.
    filename : str
        Name of the output file to save the plot.
    nx : int
        Number of grid points in the x-direction.
    dx : int
        Grid spacing in the x-direction.
    nz : int
        Number of grid points in the z-direction.
    dz : int
        Grid spacing in the z-direction.
    xlabel : str, optional
        Label for the x-axis, by default "".
    ylabel : str, optional
        Label for the y-axis, by default "".
    cmp : str, optional
        Matplotlib colormap to use, by default "viridis". Try other colormaps: "Greys",
        "hot", "seismic", "plasma"...
        https://matplotlib.org/stable/tutorials/colors/colormaps.html
    it : str, optional
        Interpolation method to use, by default None. Try others: "bicubic", "gaussian",
        "nearest", "sinc"...
        https://matplotlib.org/stable/gallery/images_contours_and_fields/interpolation_methods.html
    color_bar : boolean, optional
        Set color_bar = True for visualizing the colorbar.

    Returns
    -----
    None

    Notes
    -----
    Saves the plot as a PNG image in the "Data" directory (300 dpi).
    """

```

```

csfont = {'fontname': 'Times New Roman'}

fig, (ax1) = plt.subplots(1,1,figsize=(8,6))

im1 = ax1.imshow(grid, extent=[0, nx*dx, nz*dz, 0], aspect='auto', cmap=cmp, interpolation=it)

ax1.set_xlabel(xlabel, **csfont, fontsize=17)
ax1.set_ylabel(ylabel, **csfont, fontsize=17)

ax1.xaxis.tick_top()
ax1.xaxis.set_label_position('top')
ax1.tick_params(axis='x', which='major', pad=0, labelsize=15)
ax1.tick_params(axis='y', which='major', pad=1, labelsize=15)

for tick in ax1.get_xticklabels():
    tick.set_fontname("Times New Roman")
for tick in ax1.get_yticklabels():
    tick.set_fontname("Times New Roman")

if color_bar:
    divider1 = make_axes_locatable(ax1)

    cax1 = divider1.append_axes("right", size="5%", pad=0.3)
    cbar1 = plt.colorbar(im1, cax=cax1)
    cbar1.ax.tick_params(labelsize=12)
    for l in cbar1.ax.yaxis.get_ticklabels():
        l.set_family("Times New Roman")

plt.savefig("Data/{ }.png".format(str(filename)), dpi=300)
plt.show()

```