Universidade de Brasília

Instituto de Psicologia

Departamento de Processos Psicológicos Básicos

Programa de Pós-Graduação em Ciências do Comportamento

Master’s Dissertation

PARENTS THINK AND PARENTS DO, BUT NOT AS EXPECTED: PATTERNS IN HOME LEARNING STRUCTURE AND CHILD OUTCOMES

Fábio de Barros Correia Gomes Filho

Brasília, March 2026.

Universidade de Brasília

Instituto de Psicologia

Departamento de Processos Psicológicos Básicos

Programa de Pós-Graduação em Ciências do Comportamento

PARENTS THINK AND PARENTS DO, BUT NOT AS EXPECTED: PATTERNS IN HOME LEARNING STRUCTURE AND CHILD OUTCOMES

Fábio de Barros Correia Gomes Filho

Dissertação de Mestrado apresentada ao Programa de Pós-Graduação em Ciências do Comportamento, do Instituto de Psicologia da Universidade de Brasília, como parte dos requisitos para obtenção do título de Mestre em Ciências do Comportamento (Área de concentração: Cognição e Neurociências do Comportamento).

Orientador: Prof. Dr. Ricardo José de Moura

Brasília, março de 2026.

Comissão Examinadora

______________(*Assinado em ata*) ______________

Prof. Dr. Ricardo José de Moura (Presidente)

Programa de Pós-Graduação em Ciências do Comportamento

Universidade de Brasília – UnB

______________(*Assinado em ata*) ______________

Dra. Fernanda de Oliveira Ferreira (Membro externo)

Programa de Pós-Graduação em Psicologia

Universidade Federal de Juiz de Fora – UFJF

______________(*Assinado em ata*) ______________

Dra. Gabriela Sousa de Melo Mietto (Membro interno)

Programa de Pós-Graduação em Psicologia Clínica e Cultura

Universidade de Brasília – UnB

Brasília, 26 de março de 2026.

To Leonardo, Catarina, and every child and family in Brazil.

A Leonardo, Catarina, e todas as crianças e famílias do Brasil.

Acknowledgments

I am grateful to my advisor, Professor Dr. Ricardo José de Moura, for his guidance and mentorship. He consistently pushed me to refine this work's analytical quality while providing intellectual freedom. This allowed me to explore my own ideas while benefiting from his experience.

This project was a collaborative effort, and it would not have been possible to collect child data without my colleague, Bruna Mariz Silva Egreja. I extend my sincere thanks to her for her work in leading the demanding task of child data collection. I also thank the team of research interns who assisted with data collection, and especially Maria Isabel Fernandes Maia and Isadora Peixoto Alves for their assistance with data entry.

Finally, I thank my family for their support, and above all, my wife, Alexandra, who bore the long days and nights this work consumed.

Parenthood is as vast as life itself. To ask what makes a good father or a good mother is to ask what it means to be a good person, how to lead a good life. These are the first questions philosophy sought to answer. While I do not intend to encompass all of life in this study, we can learn something meaningful by observing the small fragment of reality within our reach.

A parentalidade é do tamanho da vida. Perguntar o que é um bom pai ou mãe é o mesmo que perguntar o que é uma boa pessoa, como viver bem. Essas são as primeiras perguntas que a filosofia buscou responder. Ainda que eu não tenha a pretensão de abarcar a vida toda neste estudo, podemos aprender algo significativo ao observar o pequeno fragmento da realidade ao nosso alcance.

Table of Contents

List of Tables

List of Figures

Resumo

Pais Pensam e Pais Agem, mas Não Como Esperado: Padrões na Estrutura do Aprendizado Doméstico e Desempenho Infantil

Este trabalho aborda o ambiente de aprendizagem doméstico a partir de duas perspectivas. O delineamento correlacional do Artigo 1 busca compreender como variáveis parentais se associam ao desempenho infantil. A abordagem descritiva do Artigo 2 busca revelar como os pais organizam essas mesmas variáveis nos domínios de literacia e numeracia. O Artigo 1 apresenta dois modelos de trajetórias com variáveis observadas, estimados para testar a mediação de atitudes e expectativas parentais sobre o desempenho infantil em literacia e numeracia. O primeiro modelo utilizou escalas conforme definidas no questionário, enquanto o segundo utilizou fatores derivados de análises fatoriais exploratórias. Efeitos diretos das crenças parentais também foram avaliados. A mediação não foi confirmada em nenhum dos modelos. A frequência de engajamento apresentou capacidade preditiva limitada sobre o desempenho infantil. Atitudes e expectativas foram preditivas de engajamento, mas apenas expectativas predisseram o desempenho infantil. Análises de correlação bivariada ofereceram evidências exploratórias adicionais ao comparar diferentes tipos de variáveis de expectativa e engajamento parental. Nossos resultados sugerem que variáveis de crenças influenciam as habilidades infantis por vias independentes do engajamento. O Artigo 2 é um estudo descritivo sobre como os pais organizam cognitiva e comportamentalmente variáveis de crenças e engajamento relacionadas ao ambiente de aprendizagem doméstico. Isso foi feito por meio de análises fatoriais exploratórias em escalas de engajamento, expectativas e atitudes combinando domínios de literacia e numeracia, associadas a um novo procedimento desenvolvido para este estudo denominado projeção fatorial em eixos teóricos. Este trabalho mostra que os pais diferenciam entre literacia e numeracia quando as atividades são formais ou as expectativas são avançadas, mas essa diferenciação se dissolve sob engajamento informal e expectativas básicas. Este é um achado cognitivo e comportamental inédito sobre a organização parental da aprendizagem doméstica, com implicações diretas para como educadores orientam famílias.

Palavras-chave: ambiente de aprendizagem doméstico, expectativas parentais, engajamento parental, diferenciação de domínio, análise fatorial.

Abstract

Parents Think and Parents Do, but Not as Expected: Patterns in Home Learning Structure and Child Outcomes

This work addresses the home learning environment from two perspectives. Article 1's correlational design aims to understand how parental variables associate with child outcomes. Article 2's descriptive approach seeks to unveil how parents organize those same variables across literacy and numeracy domains. Article 1 presents two observed-variable path models that were estimated to test the mediation of parental attitudes and expectations on child literacy and numeracy outcomes. The first model used composites as defined in the questionnaire, while the second used factors derived from exploratory factor analyses. Direct paths from parental beliefs were also assessed. The mediation was not supported in either model. Engagement frequency showed limited prediction of child outcomes. Attitudes and expectations predicted engagement, but only expectations predicted child outcomes. Bivariate correlation analyses offered additional exploratory insights by comparing different types of parental expectation and engagement variables. Our results suggest that belief variables influence child abilities through pathways independent from engagement. Article 2 is a descriptive study on how parents cognitively and behaviorally organize beliefs and engagement variables related to the home learning environment. This was done using exploratory factor analyses on engagement, expectation and attitudes cross-domain scales combined with a new procedure developed for this study called factor projection in theoretical axes. This work shows that parents differentiate between literacy and numeracy when activities are formal or expectations are advanced, but this differentiation dissolves under informal engagement and basic expectations. This is a novel cognitive and behavioral finding about parental organization of home learning, with direct implications for how educators advise families.

Keywords: home learning environment, parental expectations, parental engagement, domain differentiation, factor analysis.

Presentation

This dissertation is organized in manuscript format. It is composed of two empirical articles based on a dataset of 124 parent–child dyads from four public preschools in Brasília, Brazil.

Article 1, *Parental Expectations Outperform Home Engagement Frequency as Predictors of Preschoolers' Numeracy Skills*, presents two observed-variable path models that were estimated to test the mediation of parental attitudes and expectations on child literacy and numeracy outcomes through home learning engagement. The first model used composites as defined in the questionnaire, while the second used factors derived from exploratory factor analyses. Direct paths from parental beliefs were also assessed. The mediation was not supported in either model. Engagement frequency showed limited prediction of child outcomes. Attitudes and expectations predicted engagement, but only expectations predicted child outcomes. Bivariate correlation analyses offered additional exploratory insights by comparing different types of parental expectation and engagement variables. Our results suggest that belief variables influence child abilities through pathways independent from engagement.

Article 2, *Domain Differentiation Dissolves Under Informal Engagement and Basic Expectations: How Parents Structure Home Learning Across Literacy and Numeracy*, is a descriptive study on how parents cognitively and behaviorally organize beliefs and engagement variables related to the home learning environment. This was done using exploratory factor analyses on engagement, expectation and attitudes cross-domain scales combined with a new procedure developed for this study called factor projection in theoretical axes. This work shows that parents differentiate between literacy and numeracy when activities are formal or expectations are advanced, but this differentiation dissolves under informal engagement and basic expectations. This is a novel cognitive and behavioral finding about parental organization of home learning, with direct implications for how educators advise families.

Together, the two articles address complementary questions about the home learning environment. Article 1’s correlational design strives to understand how parental variables associate with child outcomes. Article 2’s descriptive approach seeks to unveil how parents organize those same variables across literacy and numeracy domains. A concluding chapter integrates findings across articles. Supplementary materials appear in the appendices.

Article 1: Parental Expectations Outperform Home Engagement Frequency as Predictors of Preschoolers' Numeracy Skills

Manuscript submitted for publication

Abstract

This study investigated whether parental home learning engagement mediates the relationship between parental beliefs (expectations and attitudes) and preschool children's cognitive outcomes in literacy and numeracy. Participants were 124 parent–child dyads from four public preschools in Brasília, Brazil. Children (aged 5 to 6) completed five numeracy tasks, a rhyme detection task and Raven's CPM. Parents reported home engagement frequency, academic expectations, and attitudes toward literacy and mathematics. Two observed-variable path models tested the mediation hypothesis: Model 1 used domain-level composites, while Model 2 used empirically derived factors from exploratory factor analyses. Child age, sex, fluid intelligence, and socioeconomic status were controlled as covariates. The hypothesized mediation was not supported in either model. In Model 1, numeracy engagement predicted only number identification, but neither numeracy expectations nor attitudes predicted engagement. Parents' attitudes toward literacy, in turn, predicted literacy engagement. Concrete numeracy expectations directly predicted all five numeracy outcomes. In Model 2, no engagement factor predicted child outcomes. Attitudes and expectations in both literacy and numeracy domains were predictive of engagement, though lower attitudes toward math predicted more playful engagement. Only expectations about basic numeracy skills were directly predictive of numeracy outcomes. These findings suggest that what parents believe their children should learn may be more important than the mere frequency of home learning activities. Direct paths from expectations to outcomes may capture unobserved channels through which beliefs shape child skills, such as quality and appropriateness of interaction. Limitations of frequency measures of home engagement and implications for policy are discussed.

Keywords: home learning environment, parental expectations, numeracy, preschool, path analysis

Introduction

Fostering children's cognitive development at home is no trivial task. Parents can achieve better results when provided with concrete guidance (Berkowitz et al., 2015; Sénéchal & Young, 2008; Starkey & Klein, 2000). Without it, parental engagement tends to be poorly calibrated, even though well-intentioned, as when parents resort to providing answers rather than encouraging reasoning (Vandermaas-Peeler et al., 2019). In this sense, understanding the *home learning environment* (HLE), that is, the resources and parent–child interactions that support cognitive development outside of school (LeFevre et al., 2009; Lehrl et al., 2020; Sénéchal & LeFevre, 2002; Skwarchuk et al., 2014), is crucial to how policymakers effectively advise families.

Early cognitive skills are strong predictors of later academic achievement (Aunio & Niemivirta, 2010; Claessens et al., 2009; Jordan et al., 2007). This extends to the home environment: early numeracy experiences at home predict mathematical achievement years later, even after controlling for school-based instruction (Hwang, 2020). Despite widespread advice to "read to your child," knowledge about which specific parental behaviors are predictive of better outcomes remains limited (Hornburg et al., 2021). The stakes extend beyond individual families, since numerical skills are linked to higher productivity on the individual level and economic growth at the national level (National Mathematics Advisory Panel, 2008), and literacy is crucial to civic participation and workforce readiness (National Research Council, 1998). Additionally, early skill development represents the highest rate of return for human capital investment (Heckman, 2008).

It has been suggested that different aspects of the home learning environment predict distinct child outcomes through separate pathways. In literacy, Sénéchal and LeFevre (2002) found that two uncorrelated measures showed distinct longitudinal associations with child outcomes: storybook exposure predicted receptive language and later reading comprehension, while direct teaching predicted emergent literacy and early word reading. In numeracy, LeFevre et al. (2009) also found differential effects: game-related engagement was associated with both mathematical knowledge and fluency, whereas direct number instruction correlated with fluency but not with knowledge.

LeFevre et al. (2009) also proposed a questionnaire that has been extensively adapted across countries for assessing the home learning environment. The original questionnaire included items assessing parents' academic expectations and their own attitudes toward mathematics and literacy, which showed no relevant associations with child outcomes, and these results were not reported in detail (LeFevre et al., 2009). However, LeFevre, Polyzoi, et al. (2010) found that, for Greek families, engagement mediated the effect of parental attitudes and expectations on child numerical outcomes. In Canadian families, these parental beliefs showed direct associations with child performance, and the mediation hypothesis was not supported.

Skwarchuk et al. (2014), working with similar constructs, found that expectations predicted formal engagement, and numeracy attitudes directly predicted child outcomes. However, mediation effects were not formally estimated. Martini and Sénéchal (2012) provided convergent evidence from home literacy research, finding that expectations and child interest predicted literacy outcomes beyond what teaching frequency alone explained.

Research on the HLE has focused predominantly on parental engagement, which refers to activities parents and children share, measured through self-report questionnaires or direct observation of parent–child interactions (Mutaf-Yıldız et al., 2020). Yet, a recent international consensus called for broadening measurements to other variables, such as beliefs, attitudes and expectations, while identifying the need for a testable theoretical framework in which alternative plausible models of the home mathematics environment are explicitly contrasted (Hornburg et al., 2021). The same need applies to the broader HLE. The present study responds to this call by testing two path models that integrate parental beliefs and engagement as predictors of child outcomes across both numeracy and literacy domains.

The evidence for parental engagement's association with child outcomes is positive on balance. A *p*-curve analysis confirmed that the effect of home numeracy activities on children's mathematical skills has evidential value, though with modest effect sizes and considerable heterogeneity (Mutaf-Yıldız et al., 2020). Some studies bear contradictory findings, with no statistically significant relations between parental engagement and child outcomes (Missall et al., 2015), or even negative associations (Ciping et al., 2015), though the latter was flagged as children's poor performance activating more parental engagement.

One difficulty is that engagement has been predominantly operationalized as self-reported frequency, which obscures quality of interaction. Responsiveness, scaffolding, feedback, purpose of the interaction, and language specificity are all lost on the simple measure of engagement frequency (Purpura et al., 2020). The contradiction is that counting mechanically with a child is considered equivalent to counting responsively while reflecting on quantities and scaffolding, if both activities occur at the same monthly rate. Another challenge is that several classification schemes for home engagement activities coexist, such as formal and informal

(Sénéchal & LeFevre, 2002), direct and indirect (LeFevre et al., 2009), and operational versus mapping (Susperreguy et al., 2020). Each carries legitimate differential meaning, but with definitions that may be subtle and inconsistently applied across studies. Studies nominally measuring the same construct may in practice be capturing different aspects of parental engagement, since clear and universal definitions are not yet established (Cahoon et al., 2017; Hornburg et al., 2021).

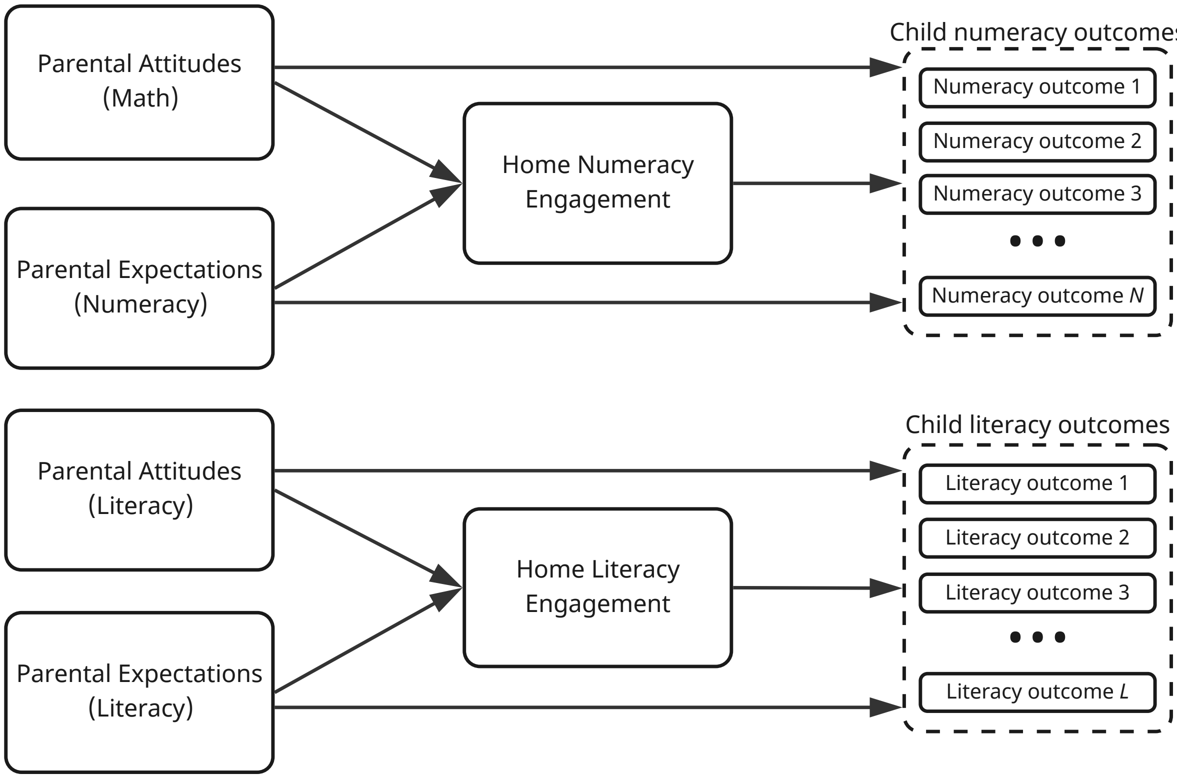
Parental expectations are usually understood as drivers of home engagement (Lefevre et al., 2002; LeFevre, Polyzoi, et al., 2010; Martini & Sénéchal, 2012; Skwarchuk et al., 2014). In the original parental questionnaire (LeFevre et al., 2009), this construct was operationalized as the importance parents attributed to specific academic benchmarks (such as counting, identifying numbers and letters, and calculating simple sums) prior to school entry. Subsequent studies referred to these ratings as parental expectations (Skwarchuk et al., 2014). This construct reflects parental normative beliefs about children's academic readiness. Regarding attitudes, LeFevre et al. (2009) measured them through items that assessed self-perceptions of competence, enjoyment, and perceived importance of mathematics and language arts. Skwarchuk et al. (2014) simplified the measure to two items per domain (enjoyment and avoidance) and found that numeracy attitudes directly predicted child outcomes. Some attitude items (e.g., avoidance of mathematical situations) may overlap with math anxiety, a construct that has received growing attention (Elliott et al., 2020; Hornburg et al., 2021).

Much of the existing HLE research has focused on school-age children, for whom relationships between home environment variables and academic outcomes are more consistently documented (Mutaf-Yıldız et al., 2020; Thompson et al., 2017). For preschoolers, findings tend to be more variable, possibly because the relationships between home environment and child cognition are still consolidating (Thompson et al., 2017). Furthermore, the literature has developed mostly with samples from Western, Educated, Industrialized, Rich, and Democratic (WEIRD) populations (Hornburg et al., 2021), which can limit generalizability of findings, as cultural variation in how parents approach early teaching has been documented even among populations that share similar economic profiles (Huntsinger et al., 1997). This study addresses both gaps by examining Brazilian preschoolers and their parents.

We propose a theoretical framework in which parental beliefs, here represented by expectations and attitudes, and engagement frequency predict child outcomes across literacy and numeracy domains (Figure 1). Critically, the model also includes direct paths from each belief variable to child outcomes, which may capture other unobserved channels through which beliefs influence child skills. The same theoretical structure is specified symmetrically for both domains. Finally, the model makes a general claim that the proposed mechanism applies to every skill within each domain, without enumerating exhaustively each child outcome. The *N* and *L* outcome boxes in Figure 1 represent, then, the full theoretical set of child skills within numeracy and literacy domains, respectively. In practice, any given study instantiates this framework with the particular outcomes it assesses, and the empirical question becomes which paths are supported for which specific skills.

Figure 1

Conceptual Model of Home Learning Activities



Note. Dashed borders group variables. Each labeled arrow represents one or more individual paths.

This study advances prior work in several ways. First, we estimate all predictors and outcomes simultaneously within observed-variable path models. This approach allows direct and indirect effects to compete for shared variance in a way that separate regressions cannot account for. Second, the models explicitly test mediation effects alongside direct paths from parental beliefs to child outcomes. Third, we test the theoretical framework with two models: one that uses composites as defined in the questionnaire and the other that uses EFA-derived factors. Fourth, we examine relationships between alternative expectations (school-subject and long-term educational aspirations) and engagement (school involvement) through bivariate comparisons with our model composites for additional exploratory insights. Finally, the sample extends this literature to Brazilian preschoolers and their families.

Three questions guided our study: i) Does engagement mediate the relationship between belief variables and child outcomes? ii) Do expectations and attitudes play distinct roles in predicting engagement and outcomes? iii) Does disaggregating parental constructs into empirically derived factors alter these associations?

Method

This study examined parental variables linked to the HLE and their associations to children's cognitive outcomes through a cross-sectional and correlational design.

Participants

Recruitment, Sample and Data Screening. Parent–child dyads were recruited from four public preschools in Brasilia, Brazil, through convenience sampling. For children, eligibility involved being enrolled in the final preschool year, which corresponds to ages 5 or 6, and without neurodevelopmental diagnoses or significant uncorrected visual or auditory deficits, as reported by parents. For parents, the respondent had to be the caregiver who spent most time with the child outside school hours. While respondents were primarily parents, hence the parent–child dyad terminology used throughout, two grandparents (1.5%) were enrolled as primary caregivers.

Out of 164 recruited dyads, three were excluded due to assessment error (n = 1), post-enrollment autism diagnosis (n = 1), and parental withdrawal (n = 1). Thus, 161 children completed cognitive assessments. Of these, 29 caregivers did not return the parental questionnaires, yielding 132 dyads. Attention check performance was used to determine the exclusion of 8 additional parents (scoring ≤ 5 of 8 on attention checks). This exclusion was associated with lower SES ($W = 719.5$, $p = .033$) and maternal education ($W = 733.5$, $p = .020$). Intraclass correlations across the four schools were negligible, except for a moderate age effect (ICC = .28). Multilevel modeling, therefore, was not pursued. Inspection of distributions revealed no severe skewness, kurtosis, or outliers for the planned analyses. Data quality procedures were fully reported in Supplementary Materials (Appendix A). The final analytic sample was composed of 124 parent–child dyads.

Sample Characteristics. Children ($N = 124$) were 5 to 6 years old ($M = 68.8$ months, $SD = 3.9$, range = 60–78 months), with 68 boys and 56 girls. Adults ($N = 124$) were 79.8% mothers, 18.5% fathers, and 1.6% grandparents. Family socioeconomic status distribution was 11.3% at level A (highest), 24.2% at B1, 41.1% at B2, 17.7% at C1, and 5.6% at C2, with no families classified in the lowest categories (D-E). Among responding parents, 70.2% held higher education degrees (undergraduate or above), 28.2% had completed high school or technical education, and 1.6% had not completed high school.

Ethical Considerations. This study was approved by the Ethics Committee for Research (CEP-UnB) of the University of Brasília (Protocol 02345418.6.0000.5540). Parents provided written informed consent. Child assent was obtained verbally before each assessment session.

Procedure

Child data collection occurred in 2024 and was conducted by trained research assistants during school visits. The standardized task order was: Ordinality, Counting Range, Number Reading, Rhyme Detection, Raven's CPM, Number Identification, and Number Writing. The full assessment battery included additional tasks not analyzed in this study. Children were assessed individually for about 60 minutes in a quiet school room during school time, and the assessment could be split across days if the child showed signs of fatigue.

Parental data collection occurred from August 2024 to April 2025 and was done exclusively through Google Forms questionnaires sent by email and phone.

Measures

Reliability measures are presented for each scale mentioned below. Cronbach's alpha is reported for comparability with prior research, presented alongside McDonald's omega (Dunn et al., 2014).

Child Cognitive Assessments. Given that this study is part of a broader research project focused on preschoolers' mathematical abilities, child assessments emphasized numeracy, with five dedicated tasks. Rhyme detection was included as a single literacy outcome for exploratory cross-domain analyses. These tasks were developed in Moura (2026).

In Ordinality ($\alpha = .96$, $\omega = .96$), the examiner spoke 11 numbers aloud, and the child identified the numbers immediately before and after each. In Counting Range, children counted aloud as high as possible, with a maximum of 100. In Number Reading ($\alpha = .90$, $\omega = .91$), 18 Arabic numerals were presented successively for the child to read aloud. In Number Identification ($\alpha = .79$, $\omega = .80$), four Arabic numerals were presented, and the child pointed to the one spoken by the examiner, across 16 trials. In Number Writing ($\alpha = .84$, $\omega = .85$), children wrote 18 dictated numbers.

Rhyme detection ($\alpha = .79$, $\omega = .79$) assessed phonological awareness. The examiner presented 15 target words aloud, and for each word the child selected which of two options rhymed with the target.

Fluid intelligence was assessed through Raven's Coloured Progressive Matrices, a measure of nonverbal reasoning (Angelini et al., 1999).

Parent Questionnaire. The questionnaire comprised sections on family demographics, socioeconomic classification, home educational practices, and parental opinions about education and technology. It assessed both literacy and numeracy domains symmetrically and was based on the Brazilian Portuguese adaptation by Silva-Chelles (2023), which drew from Sénéchal & LeFevre (2002) for literacy items and LeFevre et al. (2009) for most other scales. For the present study, the scales described below were retained identically or with minimal modifications.

The following scales used 5-point response formats. Numeracy engagement (14 items; α = .87, ω = .87) and literacy engagement (15 items; α = .90, ω = .91) assessed the frequency of home learning activities (1 = *rarely or never* to 5 = *almost every day*), covering formal and informal activities. Numeracy expectations (9 items; α = .89, ω = .89) and literacy expectations (10 items; α = .90, ω = .90) assessed the importance parents attributed to children acquiring various school-readiness skills before first grade (1 = *not important* to 5 = *extremely important*). An additional expectations scale assessed the importance of performance in school subjects such as math, arts, and sciences (8 items; α = .90, ω = .90). Math attitudes (4 items; α = .78, ω = .80) and literacy attitudes (5 items; α = .73, ω = .76) assessed the respondent's enjoyment, avoidance, and self-perceptions about abilities and school performance (1 = *strongly disagree* to 5 = *strongly agree*).

Additionally, parents identified titles they recognized from a list that included real and fictitious children's games (25 items, 5 foils) and books (60 items, 17 foils). Scores were computed as the number of correctly identified real titles.

Two scales were created for this study. Long-term educational aspirations (5 items; α = .73, ω = .76) assessed the importance parents attributed to the child's future educational milestones, such as attending university (1 = *strongly disagree* to 5 = *strongly agree*). School involvement (9 items; α = .82, ω = .82) assessed parental participation in school life, including attending meetings, communicating with teachers, and talking with the child about the school day (1 = *never* to 5 = *always*).

Socioeconomic status was measured through *Critério Brasil* (Associação Brasileira de Empresas de Pesquisa, 2024), a standardized instrument that covers household assets, domestic workers, housing infrastructure (water source, street paving), and head of household education. Responses yielded a composite score categorized into socioeconomic strata (A through E).

Data Processing. Child assessments were collected with pen and paper, being later manually entered into a spreadsheet with built-in verification formulas. Parent responses were captured via Google Forms. Both datasets were imported and processed exclusively through an automated R pipeline. The script handled exclusions, deduplications (five parents who answered twice, keeping the first response), and computed all scale scores. Database generation was fully reproducible from raw inputs.

Analytic Strategy

Exploratory factor analyses were conducted on each of the six parental scales that composed the path analysis model (numeracy expectations, literacy expectations, math attitudes, literacy attitudes, numeracy engagement, and literacy engagement). Factorability was assessed through Bartlett's sphericity and Kaiser-Meyer-Olkin tests (Bartlett's $p < .05$, KMO > .60; Kaiser, 1974). EFA used maximum likelihood extraction and oblimin rotation, as correlations between factors were expected (Costello & Osborne, 2005; Fabrigar et al., 1999). The number of factors was determined by parallel analysis (Horn, 1965). Items with cross-loadings above .30 were assigned to the factor with the highest loading. Loadings below .30 were suppressed.

Internal consistency for each factor was assessed using McDonald's omega. Factor-derived composites were computed as the mean of items assigned to each factor.

Bivariate associations among child outcomes, parental variables, and demographic covariates were examined through Pearson correlations. This included composites as defined in the questionnaire and EFA-derived factors. Even though the path models were the primary analytical tools, these correlations provided exploratory insights on variables outside the models and served as sensitivity checks.

Two observed-variable path models were estimated to test, for literacy and numeracy domains, direct effects of parental attitudes and expectations on child outcomes, as well as indirect effects mediated through home learning engagement (Kline, 2016). Models were estimated using maximum likelihood with robust standard errors (MLR) and full information maximum likelihood (FIML) for missing data, though all 124 cases had complete data on all model variables. The first path model used composites as defined in the questionnaire, while the second replaced these with finer-grained, EFA-derived factors to test differential associations that the original composites may mask. The examination of both models tests the robustness of theoretical conclusions to construct granularity. All six child outcomes (five numeracy measures and Rhyme Detection) were estimated simultaneously, with residual covariances freely estimated among outcomes, mediators, and exogenous predictors. All outcomes were controlled for child age, sex, SES, and fluid intelligence. Indirect effects were computed as the product of the constituent paths (MacKinnon et al., 2002).

All analyses were conducted in R version 4.4.2 (R Core Team, 2024). Exploratory factor analyses used the psych package (Revelle, 2025). Path analyses were estimated using lavaan (Rosseel, 2012).

Results

Factor Structure of Parental Measures

Exploratory factor analyses were conducted separately on each of the six parental scales used in path analysis. All scales showed factorability (all Bartlett's *p*s < .001) and middling to meritorious sampling adequacy (KMO = .70–.88) (Kaiser, 1974). Each factor was labelled by post hoc interpretation of similarities between items that loaded on each factor.

Tables 1 and 2 present the factor loadings for math attitudes (4 items) and literacy attitudes (5 items). Both scales were unidimensional. The math attitudes factor explained 51.5% of variance (ω = .80), and the literacy attitudes factor explained 40.9% of variance (ω = .76). For literacy attitudes, one item loaded below .30 in the single-factor solution.

Table 1

EFA: Attitudes Toward Mathematics (1 factor)

Item	Factor 1
I was good at math in school	0.80
I like math	0.95
Math is part of my job	0.52
I avoid situations that involve math	0.49
Eigenvalue	2.07
% Variance	51.5
Cronbach's α	0.77
McDonald's ω	0.80

Note. N = 124. Loadings < .30 suppressed. ML extraction, oblimin rotation.

Table 2

EFA: Attitudes Toward Literacy (1 factor)

Item	Factor 1
I was good at language and reading in school	0.62
Writing and preparing reports are part of my job	
I like to read	0.65
I like to write	0.92
I avoid situations that involve writing	0.60
Eigenvalue	2.06
% Variance	40.9
Cronbach's α	0.69
McDonald's ω	0.76

Note. $N = 124$. Loadings < .30 suppressed. ML extraction, oblimin rotation.

Table 3 presents the factor loadings for numeracy expectations (9 items). The two factors explained 61.1% of variance. The first factor, Basic Expectations ($\omega = .87$), captured expectations for age-appropriate skills. The second factor, Advanced Expectations ($\omega = .88$), captured skills usually considered beyond the preschool curriculum.

Table 3

EFA: Numeracy Expectations (2 factors)

Item	Factor 1	Factor 2
Counting to 10	0.77	
Counting to 100		0.76
Counting to 1000		0.85
Reading numbers up to 10	0.78	
Reading numbers up to 100		0.88

Item	Factor 1	Factor 2
Knowing simple sums	0.46	
Calculating simple multiplications, like 2 x 6		0.66
Identifying which group has more or fewer objects	0.75	
Identifying the position of a number (e.g., 4 comes after 3, 5 comes before 6, etc.)	0.81	
Eigenvalue	4.42	1.10
% Variance	31.1	30.0
Cronbach's α	0.86	0.87
McDonald's ω	0.87	0.88

Note. $N = 124$. Loadings < .30 suppressed. ML extraction, oblimin rotation.

Table 4 presents the factor loadings for literacy expectations (10 items). The two factors explained 60.9% of variance. The first factor, Alphabet Knowledge Expectations (ω = .88), captured more basic expectations, related to letter recognition. The second factor, Reading Expectations (ω = .85), captured more advanced expectations, related to word and text reading (which presupposes alphabet knowledge). Four items showed cross-loadings above .30, and each was assigned to the factor with the highest loading.

Table 4

EFA: Literacy Expectations (2 factors)

Item	Factor 1	Factor 2
Knowing some letters of the alphabet	0.77	
Writing their own name	0.77	
Reading a few words	0.34	0.58
Knowing all the letters of the alphabet	0.65	0.31
Knowing how to write all the letters of the alphabet	0.47	0.45
Identifying brand names on packages/billboards		0.54
Associating letters with the names of people/objects (e.g., A for Amanda)	0.55	0.32

Item	Factor 1	Factor 2
Knowing the difference between letters and numbers	0.84	
Reading simple books with pictures and words		0.70
Reading books with words only		0.95
Eigenvalue	4.97	1.12
% Variance	33.4	27.5
Cronbach's α	0.88	0.84
McDonald's ω	0.88	0.85

Note. N = 124. Loadings < .30 suppressed. ML extraction, oblimin rotation.

Table 5 presents the factor loadings for numeracy engagement (13 items). Parallel analysis suggested four factors. Only three were retained, since the fourth factor comprised a single item and was excluded from subsequent analyses. The three retained factors explained 43.1% of variance. The first factor, Number Sense Engagement (ω = .83), captured basic number activities. The second factor, Applied Numeracy Engagement (ω = .75), captured applied activities involving measures and calculations. The third factor, Playful Engagement (ω = .70), captured informal and play-based activities.

Table 5

EFA: Numeracy Engagement (4 factors, with 3 retained)

Item	Factor 1	Factor 2	Factor 3	Factor 4
I help the child learn simple sums (2+2; 5+5..)		0.66		
I encourage the child to do mental calculations		0.65		
We talk about time using clocks and calendars		0.64		
I help the child to weigh, measure, and compare quantities		0.43		0.41
We play games that involve counting, addition, and subtraction				0.96
I teach the child to recognize written numbers	0.54			

Item	Factor 1	Factor 2	Factor 3	Factor 4
We sort objects by color, shape, and size	0.71			
I ask the child questions about quantity (e.g., how many spoons are on the table?)	0.82			
I encourage them to collect objects (stickers, comics, action figures..)			0.33	
I help the child say numbers in order	0.70			
We sing songs that involve numbers (e.g., "Five Little Ducks")			0.72	
I encourage the use of fingers to indicate quantities			0.39	
We play hopscotch			0.56	
Eigenvalue	4.52	1.01	0.80	0.63
% Variance	17.8	14.7	10.6	10.2
Cronbach's α	0.83	0.75	0.69	—
McDonald's ω	0.83	0.75	0.70	—

Note. N = 124. Loadings < .30 suppressed. ML extraction, oblimin rotation.

Table 6 presents the factor loadings for literacy engagement (15 items). The two factors explained 47.2% of variance. The first factor, At-Home Literacy Engagement (ω = .92), captured home-based interactions. The second factor, Outside-Home Literacy Engagement (ω = .66), captured literacy exposure beyond the home. Two items showed no loading above .30 on either factor.

Table 6

EFA: Literacy Engagement (2 factors)

Item	Factor 1	Factor 2
I help the child read words	0.81	
I ask the child to point to words/letters when we read	0.69	

Item	Factor 1	Factor 2
I teach the child to recognize written words (e.g., traffic signs, store names)		0.70
I help the child write words	0.82	
We identify words on signs (e.g., stop, exit)		0.94
I teach the child the sounds of the letters	0.90	
I teach the child words and their meanings	0.69	
I help the child say/sing the alphabet	0.74	
We make up rhymes for songs	0.34	
I ask questions when we read together	0.43	
We visit bookstores or libraries with children's books		0.32
I teach my child to recognize printed letters	0.68	
I teach my child to write their own name	0.71	
I help the child send text messages to friends, classmates, and/or family on mobile apps (e.g., WhatsApp)		
We play Scattergories/Categories		
Eigenvalue	6.30	0.85
% Variance	34.5	12.7
Cronbach's α	0.91	0.61
McDonald's ω	0.92	0.66

Note. $N = 124$. Loadings < .30 suppressed. ML extraction, oblimin rotation.

Bivariate Associations Among Study Variables

Figure 2 presents bivariate correlations among child outcomes, parental domain composites, and covariates. Figure 3 presents correlations between the EFA-derived parental factors and the same child outcomes and covariates. Only significant correlations ($p < .05$) are displayed in both figures.

Figure 2

Bivariate Correlations Among Domain-Level Composites, Covariates, and Child Outcomes

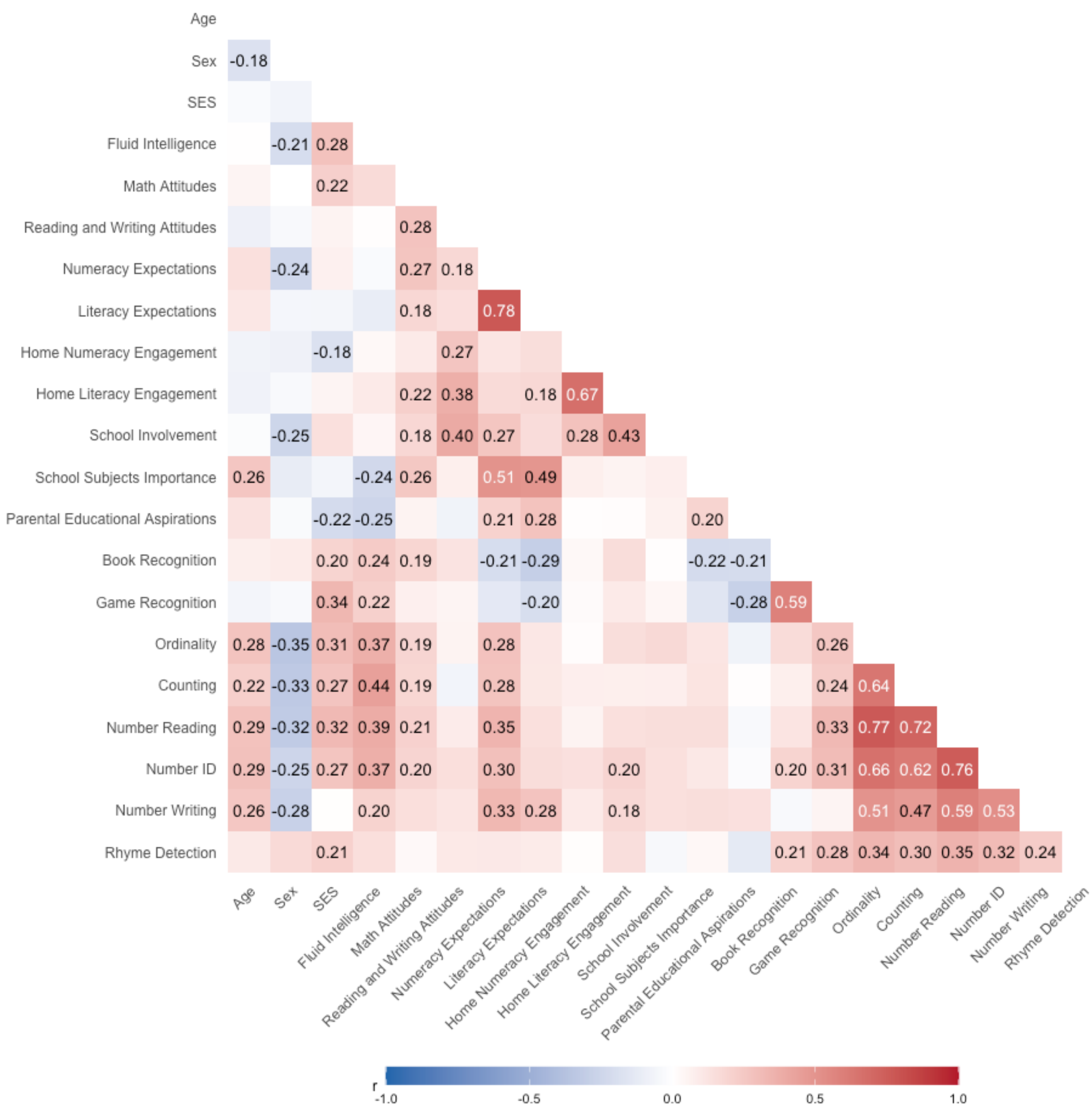
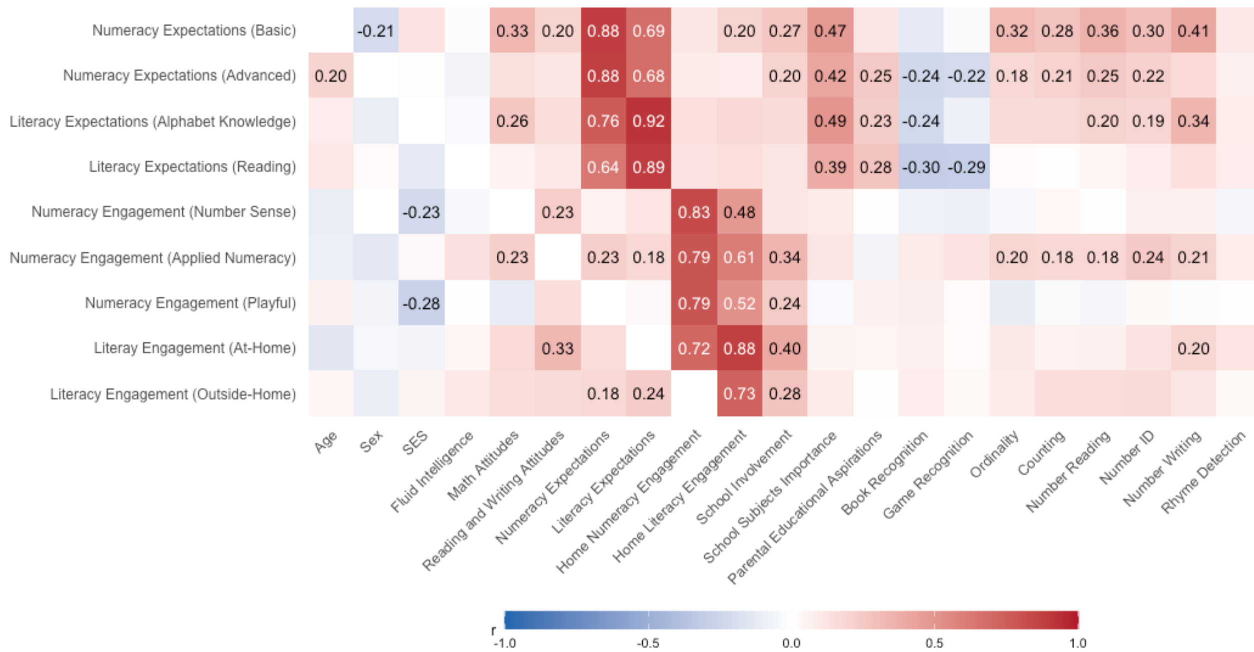


Figure 3

Bivariate Correlations Between EFA-Derived Factors and Domain-Level Variables



Path Analysis of Parental Variables and Child Outcomes

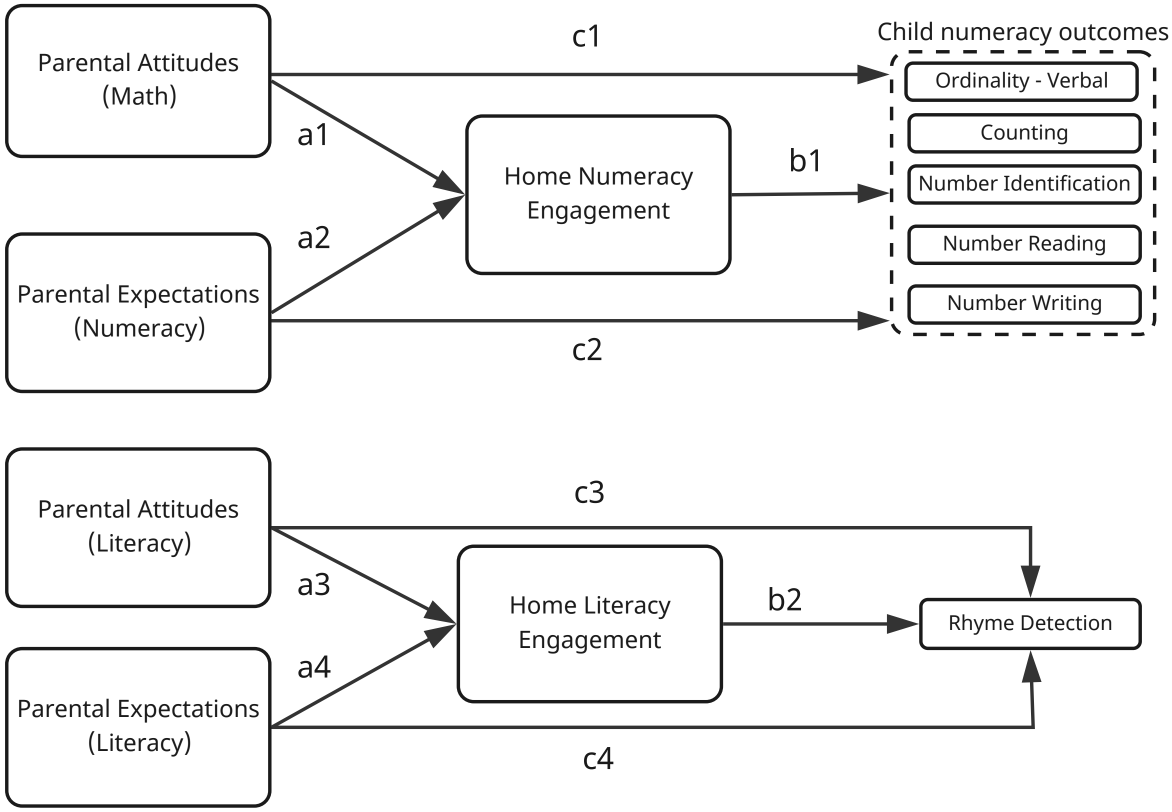
The two observed-variable path models were specified according to the description in Analytic Strategy. Model 1 used composites as defined in the questionnaire (see Parent Questionnaire in Method). Model 2 maintained the general mediational logic but used EFA-derived factors to test differential associations with finer-grained constructs (see Factor Structure of Parental Measures in Results). Attitudes remained unidimensional in both models.

In both path diagrams, each labeled path represents a single path if connecting only two variables or a set of individual paths: a paths connected attitudes and expectations to home engagement; b paths connected home engagement to child outcomes; and c paths were direct paths from expectations and attitudes to outcomes. For example, b1 in Model 1 comprises five paths, one from numeracy engagement to each numeracy outcome, while b1 in Model 2 represents 15 paths, one from each of the three numeracy engagement factors to each of the five numeracy outcomes.

Model 1 specified questionnaire-defined composites. In the numeracy domain, paths were specified from math attitudes and numeracy expectations to numeracy engagement (2 a paths), from numeracy engagement to the five numeracy outcomes (5 b paths), and directly from attitudes and expectations to outcomes (10 c paths). In the literacy domain, paths were specified from literacy attitudes and literacy expectations to literacy engagement (2 a paths), from literacy engagement to Rhyme Detection (1 b path), and directly from attitudes and expectations to Rhyme Detection (2 c paths). The path diagram is presented in Figure 4.

Figure 4

Model 1: Path Analysis for Home Learning and Cognitive Outcomes



Note. Dashed borders group variables. Each labeled arrow represents one or more individual paths.

Model 1 showed acceptable fit ($\chi^2(46) = 83.12$, p = 0.001; CFI = 0.948; TLI = 0.870; RMSEA = 0.081; SRMR = 0.063). It explained, after accounting for child age, sex, SES, and fluid intelligence, 20–37% of variance in numeracy outcomes ($R^2 = .20$ for Number Writing to .37 for Number Reading) and 16% for Rhyme Detection. Belief variables explained less than 1% of variance in numeracy engagement ($R^2 = .008$) and 6% in literacy engagement.

Table 7 presents significant path coefficients for Model 1. The only engagement-to-outcome path was numeracy engagement to number identification. Numeracy expectations directly predicted all five numeracy outcomes (βs = .17–.25), bypassing engagement. Literacy engagement did not predict Rhyme Detection. On the predictor-to-engagement side, only literacy attitudes predicted literacy engagement. No indirect effect was significant.

Table 7

Significant Path Coefficients for Model 1 (Domain Composites)

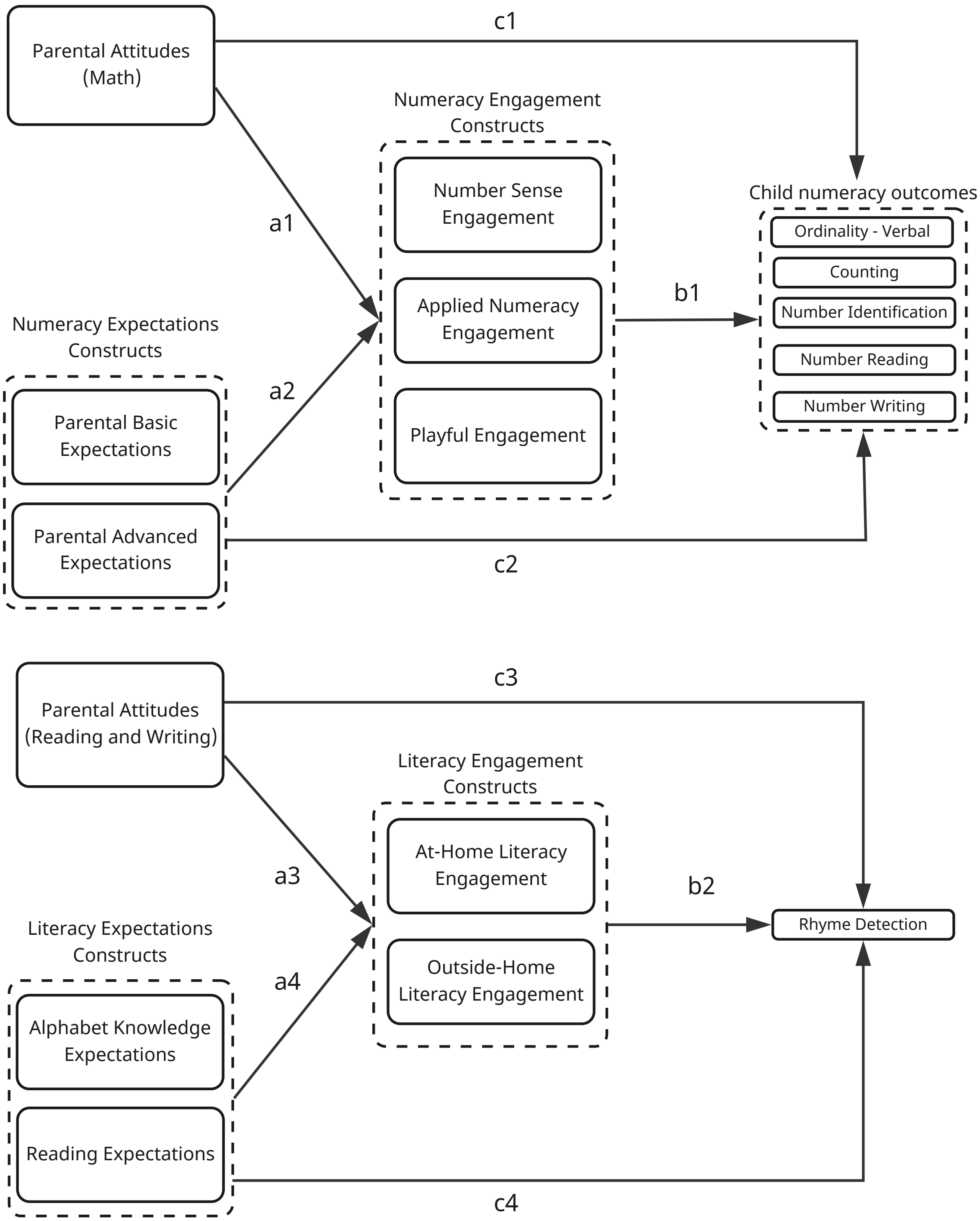
Code	Path	β	p
a3	Reading and Writing Attitudes → Literacy Engagement	0.21	0.002
b1	Numeracy Engagement → Number ID	0.16	0.034
c2	Numeracy Expectations → Counting	0.18	0.043
c2	Numeracy Expectations → Number ID	0.20	0.022
c2	Numeracy Expectations → Number Reading	0.25	< .001
c2	Numeracy Expectations → Number Writing	0.25	0.002
c2	Numeracy Expectations → Ordinality (Verbal)	0.17	0.047

Note. $N = 124$. Only paths with $p < .05$ shown. Full model structure depicted in Figure 4. Model fit: $\chi^2(46) = 83.12$, $p = 0.001$; CFI = 0.948; TLI = 0.870; RMSEA = 0.081; SRMR = 0.063.

Model 2 replaced composites with EFA-derived factors. In the numeracy domain, paths were specified from math attitudes and two expectation factors (basic and advanced) to three engagement factors (basic, formal, and playful; 9 a paths), from each engagement factor to the five numeracy outcomes (15 b paths), and directly from the three belief variables to outcomes (15 c paths). In the literacy domain, paths were specified from literacy attitudes and two expectation factors (foundational and advanced) to two engagement factors (direct and environmental; 6 a paths), from each engagement factor to Rhyme Detection (2 b paths), and directly from the three belief variables to Rhyme Detection (3 c paths). The path diagram is presented in Figure 5.

Figure 5

Model 2: Path Analysis for Home Learning and Cognitive Outcomes



Note. Dashed borders group variables. Each labeled arrow represents one or more individual paths.

Model 2 showed improved fit across all indices as compared to Model 1 ($\chi^2(90) = 125.27$, $p = 0.008$; CFI = 0.966; TLI = 0.924; RMSEA = 0.056; SRMR = 0.066), even though more paths were specified. This model explained, after accounting for child age, sex, SES, and fluid intelligence, 33–39% of variance in numeracy outcomes ($R^2 = .33$ for Number Writing to .39 for Number Reading) and 19% for Rhyme Detection. Belief variables explained 1–8% of variance in numeracy engagement factors ($R^2 = .01$ for Number Sense to .08 for Applied Numeracy) and 5–7% for literacy engagement factors.

Table 8 presents significant path coefficients for Model 2. Only basic numeracy expectations predicted three of the five numeracy outcomes (βs = .17–.41). Advanced numeracy expectations showed no significant association with any outcome. Parallel to Model 1, literacy attitudes predicted at-home literacy engagement. Disaggregation revealed new paths: basic numeracy expectations predicted applied numeracy engagement, alphabet knowledge expectations predicted at-home literacy engagement, and math attitudes showed a small negative association with playful numeracy engagement. As in Model 1, no indirect effect reached significance, but for a different reason: the additional paths revealed upstream relations between beliefs and engagement, but, in Model 2, engagement factors did not predict any child outcomes.

Table 8

Significant Path Coefficients for Model 2 (EFA-Derived Factors)

Code	Path	β	p
a1	Math Attitudes → Playful Numeracy Engagement	-0.17	0.034
a2	Basic Numeracy Expectations → Applied Numeracy Engagement	0.25	0.013
a3	Reading and Writing Attitudes → At-Home Literacy Engagement	0.14	0.022
a4	Alphabet Knowledge Expectations → At-Home Literacy Engagement	0.22	0.013
c2	Basic Numeracy Expectations → Number Reading	0.20	0.018

Code	Path	β	p
c2	Basic Numeracy Expectations → Number Writing	0.41	< .001
c2	Basic Numeracy Expectations → Ordinality	0.17	0.048

Note. $N = 124$. Only paths with $p < .05$ shown. Full model structure depicted in Figure 5. Model fit: $\chi^2(90) = 125.27$, $p = 0.008$; CFI = 0.966; TLI = 0.924; RMSEA = 0.056; SRMR = 0.066.

Discussion

Parental Expectations Outperform Home Engagement Frequency in Predicting Child Outcomes

Summary of Findings. Using two path models differing in granularity of constructs, we studied the relationships between parental variables related to the HLE and child outcomes. Both models tested two hypotheses. The first, a mediational relation between beliefs (expectations and attitudes) and child outcomes through home engagement frequency, was not confirmed. Engagement predicted only number identification in Model 1 and no child outcomes in Model 2, and significant paths from beliefs to engagement were disconnected from significant paths from engagement to child outcomes in both instances. The second, a direct association between belief variables and child outcomes, was confirmed: expectations predicted all child outcomes directly in Model 1, and three in Model 2. Attitudes did not predict any child outcomes, but predicted engagement frequency.

The underlying patterns reveal additional complexity: applied numeracy engagement was the only engagement factor consistently associated with child outcomes at the bivariate level; game and book recognition showed stronger associations with outcomes than engagement frequency; basic expectations drove predictions while advanced expectations did not; and concrete skill expectations were associated with outcomes while abstract educational aspirations were not.

Limited Predictions from Home Engagement Frequency to Child Outcomes. Parental Engagement Frequency showed limited predictions of preschooler performance in this sample. In Model 1, the only path from engagement to child outcomes that was significant was numeracy engagement to number identification ($\beta = .16$, $p = .03$). In Model 2, no engagement constructs predicted any outcomes. Applied numeracy engagement (helping the child learn simple sums, encouraging mental calculations, talking about time with clocks and calendars, and helping measure and compare quantities) deserves closer inspection. This factor resembles the "Applications" factor identified by LeFevre et al. (2009), which captured real-world quantitative activities and, in that study, predicted math fluency. In our study, it was the only engagement scale to correlate with numeracy outcomes at the bivariate level, and it correlated with all five numeracy measures ($r = .18$-$.24$). It was also the only engagement factor predicted by a belief variable (basic expectations, $\beta = .25$, $p = .013$). Two options emerge: either these connections to beliefs and outcomes make applied engagement the closest approximation to the hypothetical mediator between beliefs and outcomes, or it is a behavioral expression of concrete parental expectations. Either way, applied numeracy engagement warrants attention in future research.

School involvement was assessed as an alternative direct engagement measure through bivariate correlations but was not included in the path models. It showed no significant associations with any child outcome.

The lack of significant engagement-to-outcome paths in literacy side reveals an asymmetry that should be interpreted cautiously, since five numeracy outcomes were tested while only a single literacy measure (Rhyme Detection) was included. It is possible that a set of tasks more representative of broader literacy knowledge could yield different results.

Despite numeracy engagement predicting number identification in Model 1 and the notable properties of applied engagement, the broader pattern of limited associations between home engagement and child outcomes, even though not the predominant finding in the literature, is well documented (De Keyser et al., 2020; Mutaf-Yıldız et al., 2020). Many possible explanations for this result have been proposed. First, even in the literacy domain, where effects are more established, a critical review estimated that home reading explains no more than 8% of variance in children's literacy outcomes (Scarborough & Dobrich, 1994), so null findings are not surprising. Second, it may be due to different approaches to measuring home learning environment (Hornburg et al., 2021). Third, age plays an important role, with results for younger children being more equivocal (Thompson et al., 2017). Fourth, the same activity description (e.g., "I help the child to weigh, measure and compare") can refer to different procedures, with potentially varying difficulty levels for the child. Fifth, our measurement focused on frequency of activities by the parents, which can mask quality of interaction issues (Skwarchuk et al., 2014). In the literacy domain, a meta-analysis of family intervention studies found that having parents teach specific literacy skills was twice as effective as having parents listen to their children read and six times more effective than simply encouraging parents to read to their children (Sénéchal, 2006), underscoring that the type of engagement may matter more than its frequency. Quality of interaction includes the appropriateness (or lack thereof) of the activity to the child's prior knowledge of literacy and numeracy (Ausubel, 1968), as well as to the child's developmental readiness at the time of the interaction (Vygotsky, 1978).

To illustrate this last point (and, as a consequence, the inherent fragility of frequency measures for home engagement), imagine three children of the same age doing a certain home learning activity that is developmentally appropriate (for example, a math game). For one child, the activity may be too hard, since they have not mastered previous and more basic knowledge on which the activity depends (say, cardinality). For another, the activity is too easy, and represents no real challenge, as it is a mere repetition of what the child knows. For these two children, a higher frequency of playing that game would not translate into better outcomes, and would be arguably counterproductive if it felt forced or against their preferences (Lukie et al., 2014; Scarborough & Dobrich, 1994). Finally, suppose that, for the last child, the activity fits perfectly their knowledge prerequisites and their learning needs. A higher frequency of this activity for this child would, expectedly, translate into better outcomes. This example illustrates the complex dynamics of causation in home learning.

The interpretation regarding the importance of quality measures for the HLE is supported by two variables from our study: parental game and book recognition were assessed as alternative home engagement measures for bivariate comparison and were not included in models. Using title recognition as a proxy for home literacy exposure is consistent with prior work (Sénéchal & LeFevre, 2002). The number of children's games parents could identify from a list correlated with four of five numeracy outcomes and with Rhyme Detection (*r*s = .24–.33). Book recognition correlated with number identification and Rhyme Detection (*r*s = .20-.21). These associations were somewhat stronger than the correlations between applied engagement frequency and child outcomes. These findings suggest that familiarity with children's games and books may partly capture the quality dimension of the HLE, though recognition scores also correlated with SES (*r*s = .34 and .20, respectively), indicating that the associations may partly reflect socioeconomic access to materials. Future research should examine whether recognition measures better capture engagement quality than frequency reports.

Beliefs Predict Engagement and Outcomes, but Differentially. Beliefs are crucial to understanding the home learning environment (Hornburg et al., 2021). In Model 1, only attitudes predicted engagement; in Model 2, disaggregation revealed that expectations also drove engagement through specific pathways. Across both models, however, only expectations directly predicted child outcomes, and no indirect path from beliefs through engagement to outcomes reached significance. This is consistent with the limited amount of variance explained: belief variables accounted for less than 1% of variance in numeracy engagement in Model 1, and only 1–8% across engagement factors in Model 2. Prior studies typically tested attitudes and expectations in separate regressions, an approach that cannot partition their shared variance. Our simultaneous estimation reveals that they play distinguishable roles.

Attitudes and Expectations Predict Engagement. Findings suggest that domain-level composites in Model 1 masked belief-to-engagement associations that only became visible with finer-grained measurement. In Model 1, literacy attitudes was the only significant belief-to-engagement path, with no corresponding significant path on the numeracy side. Model 2 revealed a richer and somewhat symmetrical picture, with exactly one path from attitudes and one from expectations to engagement in each domain. Literacy attitudes predicted at-home literacy engagement, which replicates the finding from the first model. Basic numeracy expectations predicted applied numeracy engagement, and alphabet knowledge expectations (which included the more foundational items within the literacy expectations scale) predicted at-home literacy engagement. Notably, in both domains it was the basic, proximal expectations that predicted engagement, not the advanced ones.

Interestingly, parents with lower math attitudes engaged more frequently in playful numeracy activities such as songs about numbers, finger counting, hopscotch, and collectibles. This may indicate that parents with lower math self-perceptions tend to gravitate toward informal activities, possibly reflecting the role of math anxiety (which was not directly assessed in this study) in shaping parental behavior (Elliott et al., 2020; Hornburg et al., 2021).

Expectations Predict Child Outcomes, Attitudes Do Not. The first model, using composites as defined in the questionnaire, outperformed the disaggregated factors in predictive reach. In Model 1, numeracy expectations directly predicted all five numeracy outcomes. In Model 2, basic numeracy expectations predicted three outcomes: Ordinality, Number Reading, and Number Writing. The second model, while showing fewer associations between expectations and child outcomes, showed a surprisingly strong prediction between basic numeracy expectations and number writing ($\beta = .41$, $p = <.001$). Indeed, basic and advanced expectations correlated .88 with the original composite, indicating that much of the composite's predictive power resided in their shared variance. Also, advanced numeracy expectations correlated with four out of five child numeracy outcomes (*r*s = .18-.25) at the bivariate level. The richer insight is that our findings suggest that both basic and advanced expectations matter, but may contribute to child outcomes through different mechanisms.

At the bivariate level, math attitudes showed associations (*r*s = .19-.21) with four numeracy outcomes, but these were not significant in path analysis. The fact that basic numeracy expectations predicted child outcomes instead of advanced numeracy expectations or math attitudes may reflect a pattern in which predictors that describe proximal skills more concretely tend to absorb the shared variance in path models.

It is worth noting that our expectations measure is normative ("how important is it that the child…") rather than predictive ("I expect my child will…"). This is a distinction that may influence how expectations relate to other HLE variables.

Finally, neither literacy expectations nor attitudes predicted Rhyme Detection in either model.

Concrete Expectations Predict Child Outcomes, Abstract Aspirations Do Not. The path models included only concrete skill expectations, that is, items referring to specific competencies children should acquire before first grade. Two additional expectation measures, school subject importance and long-term educational aspirations (e.g., attending university), were assessed through bivariate correlations but not included in the path models. Neither showed significant associations with any child outcome. This suggests that expectations are predictive, at least in the short run, only when they are concrete and proximal, referring to specific skills the child is currently developing. Abstract aspirations about the child's educational future, however well-intentioned, may be too removed from daily interactions to shape current outcomes.

This finding, however, does not imply that distal expectations are inconsequential in the long run. Froiland et al. (2013) showed that parental expectations for post-secondary attainment measured in kindergarten predicted achievement in 8th grade, but this effect was entirely indirect, operating through the maintenance of those expectations over time and their eventual transmission to children's own aspirations. In a cross-sectional preschool sample such as the present one, this longitudinal cascade has not yet had the opportunity to unfold, which may explain why only proximal and skill-specific expectations showed concurrent associations with child outcomes. Other longitudinal studies have supported the indirect pathway, with parent beliefs operating through involvement behaviors to reach achievement (Sy & Schulenberg, 2005). Whether direct or mediated effects predominate may depend on the developmental period, cultural context, or how involvement is operationalized.

Interestingly, parental expectations correlated negatively with game and book recognition (*r*s = −.20 to −.30), while both expectations and game and book knowledge correlated positively with child outcomes. This suggests they represent distinct channels through which parents influence child development. Parents who hold strong concrete expectations may orient more toward formal instruction, while parents more familiar with children's games and books may create enriched environments through less structured channels. Recognition may capture dimensions of the HLE, such as quality of interaction and household resources, that are distinct from parental expectations.

Our results reveal that concrete expectations and attitudes, belief variables more directly related to day-to-day learning, were more consistently associated with child outcomes than behavioral ones, whether engagement in activities or school involvement. Beyond the inherent limitations in measurement of home engagement exclusively through frequency of activities, this may suggest that causal pathways traditionally studied in the field (Skwarchuk et al., 2014; Susperreguy et al., 2020), may not capture other important channels through which parental cognition shapes child development. Indeed, what parents believe their child should learn appears to matter more than the mere frequency of home learning activities.

Construct Coherence and Demographic Associations

Beyond the main findings reported above, bivariate correlations among study variables were examined to assess construct coherence and demographic patterns. Given the large number of comparisons, individual correlations should be interpreted as exploratory.

Child outcomes patterns. Child numeracy measures showed moderate to strong intercorrelations, as theoretically expected (Jordan et al., 2007). Rhyme Detection showed more modest correlations with numeracy outcomes. Fluid intelligence correlated with all child numeracy outcomes (*r*s = .20–.44). The absence of correlation between fluid intelligence and Rhyme Detection was expected, as, by definition, Raven's CPM measures non-verbal reasoning.

Child age and socioeconomic status correlated positively with most child outcomes, as anticipated (DeFlorio & Beliakoff, 2015; Hooper et al., 2010). Regarding sex, evidence indicates that mean differences in mathematics are negligible (Hyde et al., 2008). Still, in this sample, boys scored higher than girls on most numeracy tasks. Girls showed better performance in Rhyme Detection ($r = .19$, $p = .03$), a pattern consistent with literature (National Research Council, 1998). Girls were slightly younger than boys ($r = -.18$ between sex and age), but partial correlations controlling for age confirmed that sex differences remained significant.

Parental variable patterns. Parental home engagement frequencies for literacy and numeracy were strongly correlated ($r = .67$). School involvement correlated with both home literacy ($r = .43$) and home numeracy engagement ($r = .28$), suggesting parents who are more involved at school also do more at home. Expectations were also highly intercorrelated: concrete, proximal expectations for literacy and numeracy were highly correlated ($r = .78$), with moderate associations to school subject expectations ($r \sim .50$) and weaker associations to long-term educational aspirations ($r = .20$–$.28$), suggesting a gradient from concrete skill expectations to abstract aspirations.

Lower-SES parents reported more frequent home numeracy engagement ($r = -.18$), a pattern documented elsewhere, though not universal (LeFevre, Fast, et al., 2010; Niklas & Schneider, 2014). By looking at EFA-derived scales, however, SES negatively related only to number sense engagement (*r* = -.23) and to playful engagement (*r* = -.28). Applied numeracy engagement, which correlated with child outcomes, was unrelated to SES. Additionally, lower-SES parents reported higher long-term educational aspirations (*r* = −.22), which may reflect perceived importance of education for social mobility. Yet aspirations did not predict outcomes while, in contrast, concrete skill expectations predicted outcomes and were unrelated to SES.

Limitations and Future Directions

The main limitation that should be considered is that our cross-sectional design precludes causal inference. In this work, terms such as "predictor" and "predicted" are used exclusively in their statistical sense. The central finding that parental expectations predicted child numeracy outcomes could reflect reverse causation or bidirectionality. The most natural extension of this work is longitudinal or experimental designs that can assess causality.

Assertions comparing literacy and numeracy domains are limited by the asymmetrical number of child outcome variables: five numeracy measures versus a single literacy task (Rhyme Detection). The upstream paths from beliefs to engagement were symmetrically specified across domains and revealed real differences. The downstream paths from beliefs and engagement to child outcomes, however, had more opportunities to reach significance on the numeracy side. Furthermore, rhyme detection is not the strongest predictor of later literacy outcomes (Fricke et al., 2016).

Regarding child assessment, future work should expand in several directions. Spatial skills, increasingly recognized as a component of early mathematical cognition (Purpura et al., 2020), were not assessed. Literacy outcomes need substantial broadening beyond rhyme detection to include at minimum phonemic awareness, emergent alphabetic knowledge, vocabulary, and narrative comprehension (National Research Council, 1998). A broader literacy assessment might have revealed different patterns. Additionally, general cognitive predictors of reading success such as rapid automatized naming (RAN) of letters, digits, objects, and colors, and phonological memory (National Early Literacy Panel, 2008) would enable a more complete picture of the cognitive architecture underlying early academic development. A particularly promising direction would be the inclusion of executive function measures (Diamond, 2013), either as covariates that may account for variance in child outcomes not captured by Raven's CPM, or as outcomes in their own right, given growing evidence that executive functions are trainable, even though we are not certain to what extent (Diamond, 2013). If home learning activities can influence executive function development, this would open an additional domain through which the HLE can be understood.

Our sample size ($N = 124$) was sufficient for path analyses with observed variables, which assumes error-free measurement and typically attenuates estimated relationships. This concern, though, is mitigated by generally high scale reliability ($\omega = .76$–$.96$). A larger sample would enable structural equation modeling with latent variables, which could reveal additional effects. Additionally, the sequential assessment of schools meant that child age and school affiliation were confounded (ICC = .28 for age), though main predictors did not differ across schools. Also, parental and child variables were not measured simultaneously. Children were assessed during the 2024 school year, while parental questionnaires were from August 2024 to April 2025. During this interval, both child abilities and parental perceptions could have changed.

Participants were recruited through convenience sampling. All families were categorized as A, B, or C in socioeconomic status; the D-E categories, representing 21.82% of the Brazilian population (Neri, 2026), were unrepresented. Although no families were classified in the lower socioeconomic categories as defined by Critério Brasil (which is based on household assets and education), the sample included families reporting total household income below the minimum wage, indicating variability within the lower economic strata. Still, the restricted SES range may have attenuated the detection of effects that might emerge in more heterogeneous samples. However, the relatively high education and SES levels may make findings more comparable to the predominantly middle-class Western samples that dominate the HLE literature. Future work should strive for representative sampling.

Home learning engagement was measured exclusively through self-reported frequency of activities. As discussed above, frequency measures cannot capture quality of interaction, developmental appropriateness, or child responsiveness, all of which may determine whether an activity translates into cognitive gains (Hornburg et al., 2021; Purpura et al., 2020), with the measurement of informal numeracy experiences identified as a standalone open challenge (Alcock et al., 2016). Even basic features such as duration and intensity are obscured by frequency reports. A parent who engages two times a week for one hour differs meaningfully from one who engages daily for fifteen minutes, and which approach yields better outcomes, if any, is an open empirical question. The limited findings for engagement should therefore be interpreted as a limitation of frequency-based measurement, not as evidence that home learning activities are unimportant. Survey-based approaches should coexist with observational methods, even though research suggests they may measure different constructs (Mutaf-Yıldız et al., 2020). Additionally, parental beliefs and engagement were reported by the same respondent, introducing common-source bias, though this concern is mitigated by the fact that expectations predicted independently assessed child outcomes while engagement did not.

Given the strong role of parental expectations, future research should also assess children's own attitudes toward learning. These may moderate the relationship between parental expectations or engagement and outcomes, or mediate it, with parental expectations shaping the child's disposition toward learning. Similarly, parental attitudinal constructs that are more closely related to self-efficacy in teaching or supporting the child may prove more informative than attitudes towards a discipline. The investigation of math anxiety is particularly important.

Future research should also broaden scope beyond the primary caregiver. Our study, like most in the field, measured only one respondent per family, yet children's learning ecology includes other caregivers, siblings, and school environments. Important steps have been taken in this direction by studies that include both mothers and fathers (Del Río et al., 2017; Susperreguy et al., 2020), with evidence suggesting that mothers' and fathers' engagement may have differential associations with child outcomes.

Finally, another direction concerns how the complexity of causal relationships between the HLE and child outcomes demands new measurements. Research may benefit from low-aggregation approaches, including small-*N* designs that can generate very fine-grained data. This can be the difference between knowing that a parent engages in a specific type of activity every week, versus understanding patterns such as times of the day, duration, number of words spoken, child reactions, whether the interaction was more formal or more informal, how learning resources impacted the interaction, and many others. Only quasi-ethnographic methods could produce such micro-level data, which could generate new hypotheses now invisible to current designs.

Policy Implications

The following implications assume that the associations identified reflect, at least in part, causal processes. The cross-sectional design cannot establish directionality, but if confirmed by longitudinal or experimental work, the implications would be substantial.

What we say to parents matters. Simply encouraging parents to increase the frequency of home learning activities may not be the most effective approach, even though engagement frequency yielded some associations with child outcomes. Game and book recognition showed stronger associations with child outcomes than frequency measures, suggesting that quality and intentionality matter more than quantity. Yet parental expectations were consistently the strongest predictor across both models. The emphasis should shift from “do more” to “understand what your child should learn and do it with intention”: what parents believe their child should learn matters more than how often they practice.

Concrete expectations predicted child outcomes and were unrelated to socioeconomic status, making them a promising and equitable policy target. However, holding concrete expectations presupposes that parents understand what their child currently knows and what should come next developmentally. This informed awareness cannot be captured by raw frequency of engagement measures and is a learnable skill. Teachers, schools, and public health professionals may be the best communicators for such knowledge. Also, what might be called a “parental curriculum” (that is, a basic and easily actionable prompt for daily interactions aligned with the child’s developmental level) could translate these findings into practical guidance regardless of parents’ own educational background. Additionally, communicating to parents that their own negative attitudes toward math need not constrain their expectations about their children’s learning may be a simple but fruitful message.

What we say to educators matters. Educators occupy a privileged position: they observe the child daily and can advise families on which skills to work on next. A parental assessment questionnaire mapping family engagement and beliefs could make this guidance more tailored to each family's need. Research suggests that the quality of parent-educator communication is itself positively associated with the home learning environment (Lin et al., 2019). It is known that, in the Brazilian context, educators face substantial administrative burdens (Organisation for Economic Co-operation and Development, 2025). Such tools should be designed to facilitate rather than add to their workload.

What policy makers can do. Policy makers can support these efforts through institutional adoption of parental assessment tools and continuing education programs for parents and teachers relating to the HLE and to making the interface between schools and families as informative as possible. Another priority is data production. Since most HLE research has been conducted with WEIRD samples (Hornburg et al., 2021), large-scale longitudinal studies conducted through public school systems, ideally in partnership with universities, would advance knowledge and enable schools to receive research-informed insights.

Improving the home learning environment through parental engagement and beliefs is a real opportunity for our children. We hope the present study provides locally grounded evidence that Brazilian policy makers can use to inform early childhood educational policy and strengthen the interface between schools and families.

Article 2: Domain Differentiation Dissolves Under Informal Engagement and Basic Expectations: How Parents Structure Home Learning Across Literacy and Numeracy

Manuscript in preparation

Abstract

While numeracy and literacy as categories are foundational to research in the home learning environment and make theoretical sense, they may not necessarily reflect how parents understand learning activities or children's skills. Research on the home learning environment has focused predominantly on whether parental variables predict child outcomes, with less attention to how parents cognitively and behaviorally organize home learning itself. This study uses a cross-sectional descriptive design to evaluate the internal structure of parental engagement frequency, expectations and attitudes across literacy and numeracy domains. This was measured through a parental questionnaire completed by 124 parents. Scale data were then analyzed with exploratory factor analysis (EFA), yielding item groupings according to covariance patterns. A supplementary analytic procedure developed for this work, factor projection in theoretical axes, consisting of weighted-average procedure with linear regressions, was employed to evaluate how factor structure aligns with theoretical dimensions such as domain, formality and complexity. Our results indicate that domain differentiation dissolves under low formality for parental engagement frequency and under low complexity for parental expectations. These findings indicate that parents distinguish between literacy and numeracy when teaching explicitly or holding advanced expectations, but not when engaging informally or considering foundational skills. This is a cognitive and behavioral finding with implications for how educators guide families in supporting learning across domains.

Keywords: home learning environment, factor analysis, domain differentiation, parental engagement, parental expectations

Introduction

Much research effort has been directed at understanding the home learning environment (HLE), particularly parental engagement in learning activities. Mutaf-Yıldız et al. (2020) reviewed the literature specifically on engagement frequency, which has become the de facto canonical measure of parental engagement in both literacy and numeracy domains (LeFevre et al., 2009). Belief variables, including expectations and attitudes, have also received sustained attention (Sénéchal & LeFevre, 2002; Skwarchuk et al., 2014). This body of work is predominantly oriented toward prediction, since the central question has been whether and how parental variables predict child outcomes (Jordan et al., 2007; LeFevre et al., 2009; Skwarchuk et al., 2014).

A less explored question is how parents structure home learning itself, as the internal structure of parental engagement remains underexamined (Purpura et al., 2020). This shifts the focus from the relationship between the HLE and child outcomes to understanding how parents cognitively and behaviorally organize the HLE. Researchers, educators, and policy makers can benefit from this knowledge in two senses. First, it permits verifying if usual categorizations used in the literature (literacy, numeracy, engagement, beliefs) correspond to actual parental practice, which is a current gap (Cahoon et al., 2017). Second, it helps to understand if specific guidance reaches families as intended.

The empirical question is, then, whether parents actually organize the HLE according to those theoretical classifications.

Indeed, understanding how parents organize home learning activities is both a scientific and a policy question. The international research agenda on the home mathematics environment has called explicitly for more factor analytic work to understand the structure of home learning measures (Hornburg et al., 2021; Purpura et al., 2020), and for clearer operational definitions of the constructs these measures capture. The present study responds to this call.

The most common approach to measuring home learning engagement is through self-reported frequency of activities. Parents indicate how often they engage in each activity on a scale ranging from daily to almost never. Many foundational studies follow this pattern in literacy (e.g., Sénéchal & LeFevre, 2002) and numeracy (e.g., LeFevre et al., 2009). This design was tested within an integrative model by Skwarchuk et al. (2014), and was adapted to the Brazilian context by Silva-Chelles (2023).

Frequency data, even though having serious limitations to the broader understanding of the HLE (see Article 1), carries valuable cognitive and behavioral data from parents that descriptive statistics or path coefficients alone cannot consider. Theoretically, activities vary at least regarding domain and formality level. Domain refers to the type of skill that is being developed in the activity, and, for our purposes, can be classified as literacy (reading, writing, letter recognition) or numeracy (counting, arithmetic, quantity comparison). Formality level refers to the type of engagement, and can be classified as formal (closer to direct instruction) or informal (closer to playful engagement). Even though those classifications are intuitive, many activities do not fit nicely in these classifications, suggesting the need to consider a continuous (instead of categorical) perspective. Drawing, for instance, can be formally associated with fine motor skills training, but also serves both literacy and numeracy dimensions as being relevant to writing letters and numbers.

Parental frequency of engagement in certain activities cannot reveal what substantively those activities are. This is a theoretical matter. But it can reveal how they understand activities in relation to one another. When parents report engagement on frequency scales, the covariance patterns reveal how parents cognitively group activities together. If the instrument assesses both literacy and numeracy activities, or formal and informal, the examination of these covariances through exploratory factor analysis (EFA) unveils whether parents understand these activities as researchers categorize them. The resulting factor structure is not a property of the activities themselves, but a finding about parental cognition and behavior regarding these activities.

Up to this point, we used parental frequency of engagement as the main example, but this reasoning can be applied to any cross-domain scale, such as parental expectations regarding skills the child should acquire before first grade or parental attitudes toward math or literacy. This study focuses on understanding how the HLE is structured by examining three parental variables: engagement frequency, expectations and attitudes. All scales assess literacy and numeracy content in parallel, which makes them suitable for examining whether parents draw the same boundaries between domains that researchers do. In Article 1, we found a direct link from expectations to child cognitive outcomes, bypassing the proposed engagement mediation, which underscores the importance of understanding beliefs as variables in their own right.

This leads to our research questions: i) Does the factor structure of parental measures (engagement and expectations) empirically align with theoretical distinctions assumed in the literature, such as domain specificity or the formality and complexity divides? ii) Does the factor structure differentiate clearly between the relevant dimensions, such as domain, formality or complexity? If so, does differentiation along one dimension depend on position on the other?

To address these questions, we combine exploratory factor analysis with a simple loading-weighted procedure developed by the first author for this study, that projects empirical factors onto theoretical axes and tests for dimensional interactions through linear regression.

Method

This study uses a cross-sectional descriptive design to evaluate the internal structure of parental engagement frequency, expectations and attitudes across literacy and numeracy domains. This was measured in a parental questionnaire through the three cross-domain scales from Article 1. Scale data are then analyzed with exploratory factor analysis (EFA), yielding item groupings according to covariance patterns. Then, we employ a novel supplementary weighted-average procedure with linear regressions to evaluate how factor structure aligns with theoretical dimensions.

Participants, Procedure, and Measures

The same sample, instruments, and data described in Article 1 apply to this study. Full methodological details are described there. However, even though Article 1 used the same parental questionnaire, engagement, expectation, and attitudes scales were reported as separate domain-specific instruments, that is, numeracy engagement, literacy engagement, numeracy expectations, literacy expectations, attitudes towards math, and attitudes toward literacy. By design, we now want to assess how parents understand these variables in a cross-domain fashion, so the present study combines these scales.

The engagement frequency scale ($\alpha = .93$, $\omega = .93$) was composed of 29 items (13 numeracy + 15 literacy + 1 mixed), assessed the frequency of home learning activities, and covered formal and informal activities. One item (“We play drawing games”), excluded from the domain-specific scales in Article 1 for lacking domain specificity, was reintroduced here. The expectations scale ($\alpha = .94$, $\omega = .94$) was composed of 19 items (9 numeracy + 10 literacy), assessed the importance parents attributed to children acquiring various school-readiness skills before first grade, and covered basic and advanced skills. The attitudes scale ($\alpha = .78$, $\omega = .79$) was composed of 10 items (4 math + 5 literacy + 1 mixed) and assessed enjoyment, avoidance, and self-perceptions about abilities and school performance, with domain as the only one theoretical dimension.

Cross-domain reliability was higher than domain-specific scales (see Article 1), consistent with the broader item pool and the strong cross-domain correlations observed between literacy and numeracy engagement ($r = .67$) and expectations ($r = .78$). Attitudes showed improved reliability measures, even with somewhat weak correlations between the domain-specific constructs ($r = .28$).

All analyses were conducted in R (R Core Team, 2024) using the psych package for EFA (Revelle, 2025).

Analytic Strategy

Exploratory Factor Analyses. Both parental scales, engagement frequency (29 items) and expectations (19 items), were examined with exploratory factor analysis. Factorability was confirmed through Bartlett's sphericity and Kaiser-Meyer-Olkin tests (Bartlett's $p < .05$, KMO > .60; Kaiser, 1974). EFA used maximum likelihood extraction with oblimin rotation, as factor correlations were expected (Costello & Osborne, 2005; Fabrigar et al., 1999). The number of factors was determined through parallel analysis (Horn, 1965).

Projecting Factors onto Theoretical Axes. Each item received a priori theoretical classifications on two dimensions using trichotomous scales. This was done independently of EFA results. For engagement, items were classified on Domain (+1 Literacy, 0 Mixed, -1

Numeracy) and Formality (+1 Formal, 0 Mixed, -1 Informal). For expectations, items were classified on Domain (+1 Literacy, 0 Mixed, -1 Numeracy) and Complexity (+1 Advanced, 0 Mixed, -1 Basic).

Consider that EFA produces a loading matrix $\mathbf{\Lambda}$ where element λ_{ik} represents the relationship between item i and factor k. Each factor's position is computed on the theoretical axes by weighting item classifications by their loadings. For factor k, the position on theoretical dimension j is:

$$F_{kj} = \frac{\sum_{i=1}^{p} \lambda_{ik} \cdot t_{ij}}{\sum_{i=1}^{p} |\lambda_{ik}|}$$

where λ_{ik} is the loading of item i on factor k, t_{ij} is the theoretical classification of item i on dimension j and p is the total number of items. The resulting coordinate represents the factor's "theoretical center of gravity", that is, a loading-weighted average of where its items fall on that dimension.

These coordinates permit an objective comparison between a factor's position in theory space and the researcher's post hoc labeling. Since we are working with two dimensions, this procedure yields coordinates that can be plotted in a Cartesian plane: domain versus formality for engagement, domain versus complexity for expectations.

Beyond assessing factor labeling, we can test dimensional differentiation along the counterpart theoretical dimension. We first compute the absolute value of each factor's coordinate on the first axis. The absolute value represents the degree of differentiation, regardless of direction. Then, we regress it on the factor's coordinate on the second axis:

$$|F_{k,1}| = \beta_0 + \beta_1 \cdot F_{k,2} + \epsilon_k$$

where $F_{k,1}$ is the coordinate of factor k on the domain axis, $F_{k,2}$ is its coordinate on the second theoretical axis (formality for engagement, complexity for expectations), β_0 is the intercept representing expected domain differentiation when the second axis is zero, and ϵ_k is the residual. A significant positive β_1 indicates that domain differentiation increases with position on the second axis.

A significant positive slope indicates that differentiation along the first axis increases with position on the second. With two theoretical dimensions, this can yield two complementary regressions: absolute domain differentiation regressed on the second axis, and absolute second-axis differentiation regressed on domain.

For visualization purposes, we plot all factor coordinates in theory space, and project the regression line. This regression line is mirrored across the domain midpoint (Domain = 0), producing a wedge shape. The vertex of this wedge is the point on the second axis where predicted domain differentiation fully collapses (that is, is equal to zero) and is calculated as $F = -\beta_0/\beta_1$. This vertex is only visible in the plot when the regression slope is steep enough for the predicted zero-crossing to fall within between -1 and +1, the theoretical range of the second axis.

Results

Attitudes, expectations and engagement scales showed factorability (Bartlett's *p*s < .001) and middling to meritorious (Kaiser, 1974) sampling adequacy (KMO = .71, .87 and .89, respectively). Factors were labelled by post hoc interpretation of similarities between items that loaded on each factor.

Parental Engagement Scale

The parental engagement scale had 29 items categorized a priori in domain and formality dimensions. EFA yielded four factors, explaining in total 47.8% of variance. Table 9 shows theoretical classifications and EFA results.

Table 9

Parental Engagement Scale: Theoretical Classifications and EFA Factor Loadings (4 factors)

Item	Domain	Formality	Factor 1	Factor 2	Factor 3	Factor 4
I help the child learn simple sums (2+2; 5+5..)	Numeracy	Formal	0.30	0.33		
I encourage the child to do mental calculations	Numeracy	Informal				0.41
We talk about time using clocks and calendars	Numeracy	Informal	0.38			0.35
I help the child to weigh, measure, and compare quantities	Numeracy	Mixed				0.51
We play games that involve counting, addition, and subtraction	Numeracy	Informal			0.49	
I teach the child to recognize written numbers	Numeracy	Formal		0.69		
We sort objects by color, shape, and size	Numeracy	Formal		0.69		
I ask the child questions about quantity (e.g., how many spoons are on the table?)	Numeracy	Formal		0.83		
I encourage them to collect objects (stickers, comics, action figures..)	Mixed	Informal			0.39	
I help the child say numbers in order	Numeracy	Formal		0.75		
We sing songs that involve numbers (e.g., "Five Little Ducks")	Numeracy	Informal		0.31	0.52	
I encourage the use of fingers to indicate quantities	Numeracy	Informal		0.32		
We play hopscotch	Mixed	Informal			0.66	
We play drawing games	Mixed	Informal	0.31	0.45		
I help the child read words	Literacy	Formal	0.94			
I ask the child to point to words/letters when we read	Literacy	Formal	0.60			

Item	Domain	Formality	Factor 1	Factor 2	Factor 3	Factor 4
I teach the child to recognize written words (e.g., traffic signs, store names)	Literacy	Formal	0.45		0.32	
I help the child write words	Literacy	Formal	0.82			
We identify words on signs (e.g., stop, exit)	Literacy	Informal	0.34		0.39	
I teach the child the sounds of the letters	Literacy	Formal	0.79			
I teach the child words and their meanings	Literacy	Formal	0.75			
I help the child say/sing the alphabet	Literacy	Formal	0.48			-0.34
We make up rhymes for songs	Literacy	Informal			0.38	
I ask questions when we read together	Literacy	Formal	0.36			
We visit bookstores or libraries with children's books	Literacy	Informal			0.48	
I teach my child to recognize printed letters	Literacy	Formal	0.54			
I teach my child to write their own name	Literacy	Formal	0.56			
I help the child send text messages to friends, classmates, and/or family on mobile apps (e.g., WhatsApp)	Literacy	Informal			0.39	
We play Scattergories/Categories	Literacy	Informal				0.34
Eigenvalue			10.10	1.51	1.25	1.04
% Variance			20.2	13.1	10.1	4.4
Cronbach's α			0.92	0.85	0.77	0.56
McDonald's ω			0.92	0.85	0.77	0.64

Note. N = 124. Loadings < .30 suppressed. ML extraction, oblimin rotation. Domain and Formality columns are theoretically attributed.

The first factor, Formal Literacy Engagement, was labelled as such for its predominance in formal literacy items. The second factor, Formal Numeracy Engagement, loaded all five items classified a priori as formal numeracy, along with a few informal numeracy items. The third factor, Informal Playful Engagement, captured informal activities of a more playful or routine nature, also spanning both domains. The fourth factor, Informal Applied Engagement, captured a small set of informal activities that were more applied and cognitively demanding in nature.

Six items showed cross-loadings above .30. Each of these items was assigned to the factor with the highest loading. One item ("I help the child say/sing the alphabet") had a small negative cross-loading (-.34) on Factor 4, while loading primarily on Factor 1 (.48). This likely reflects instability in partial coefficients, as Factor 4 accounted for only 4.4% of total variance with modest reliability ($\omega = .64$).

Beyond cross-loadings, one item ("We talk about time using clocks and calendars," a numeracy item) loaded highest on Factor 1 (.38), the Formal Literacy factor. This factor grouped ten formal literacy items and this numeracy item. A label cannot accommodate this. The projection procedure, on the other hand, handles it naturally, since the presence of a numeracy item pulls Factor 1's domain coordinate away from the literacy pole ($D = 0.62$ instead of a hypothetical D closer to 1.0), quantifying the impurity that a label must ignore.

We computed loading-weighted theoretical coordinates for each factor, as shown in Table 10.

Table 10

Parental Engagement Scale: Loading-Weighted Factor Coordinates

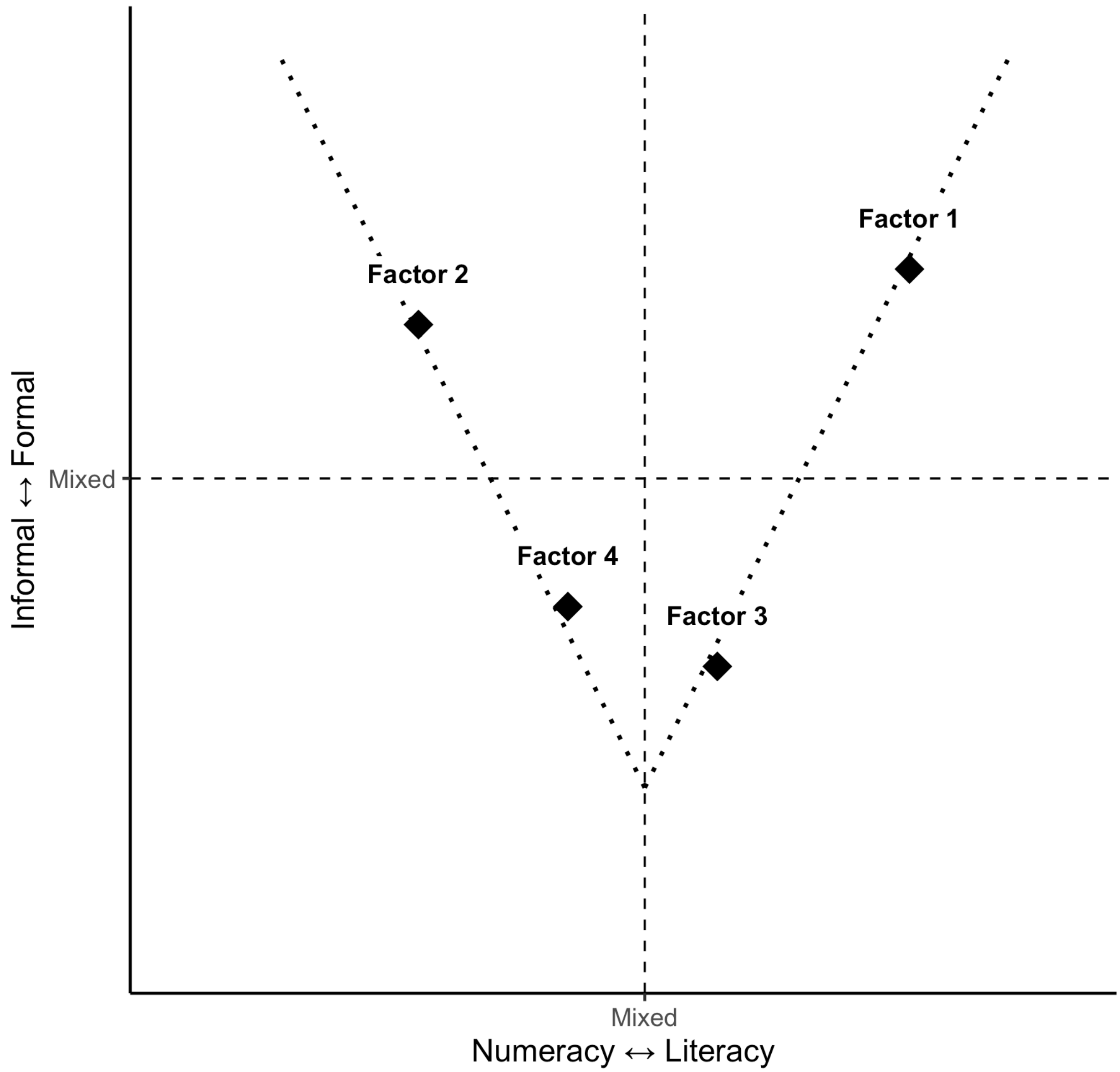
Factor	Domain	Formality
Factor 1	0.62	0.49
Factor 2	-0.53	0.36
Factor 3	0.17	-0.44
Factor 4	-0.18	-0.30

Factors 1 and 2 show strong domain differentiation ($D = 0.62$ and $D = -0.53$, respectively) and positive formality coordinates ($F = 0.49$ and $F = 0.36$). Factors 3 and 4 sit closer to the domain midpoint (D = 0.17 and D = -0.18) with negative formality coordinates (F = -0.44 and F = -0.30), confirming that informal factors mix domains while formal factors separate them.

Absolute domain differentiation was regressed on formality at the factor level ($r = .99$, $p = .006$, slope = 0.50). The vertex fell at -0.72 on the formality axis, beyond the most informally positioned factor centroid (Factor 3 at F = -0.44), but within the theoretical range of -1. Figure 6 shows the plot of factor centroids with the regression slope.

Figure 6

Factor Centroids Projected onto Domain and Formality Axes for Parental Engagement



Note. Diamonds represent factor centroids positioned on theoretical axes. Dotted lines represent the regression of absolute domain differentiation on formality (r = .99, p = .006, slope = 0.50).

The complementary regression, absolute formality differentiation on domain, yielded no significant result ($r = .81$, $p = .19$, slope = 0.14): parents distinguish formal from informal activities regardless of whether those activities are literacy or numeracy.

Parental Expectations Scale

The parental expectations scale had 19 items categorized a priori in domain and complexity dimensions. EFA yielded three factors, explaining in total 62.1% of variance. Table 11 shows theoretical classifications and EFA results.

Table 11

Parental Expectations Scale: Theoretical Classifications and EFA Factor Loadings (3 factors)

Item	Domain	Complexity	Factor 1	Factor 2	Factor 3
Counting to 10	Numeracy	Basic	0.65		0.31
Counting to 100	Numeracy	Advanced			0.83
Counting to 1000	Numeracy	Advanced		0.41	0.61
Reading numbers up to 10	Numeracy	Basic	0.67		
Reading numbers up to 100	Numeracy	Advanced			0.80
Knowing simple sums	Numeracy	Advanced	0.44		
Calculating simple multiplications, like 2 x 6	Numeracy	Advanced		0.44	0.44
Identifying which group has more or fewer objects	Numeracy	Basic	0.74		
Identifying the position of a number (e.g., 4 comes after 3, 5 comes before 6, etc.)	Numeracy	Basic	0.78		
Knowing some letters of the alphabet	Literacy	Basic	0.79		
Writing their own name	Literacy	Basic	0.72		
Reading a few words	Literacy	Advanced		0.54	
Knowing all the letters of the alphabet	Literacy	Advanced	0.44	0.49	
Knowing how to write all the letters of the alphabet	Literacy	Advanced		0.66	
Identifying brand names on packages/billboards	Literacy	Basic		0.54	
Associating letters with the names of people/objects (e.g., A for Amanda)	Literacy	Basic	0.50	0.50	

Item	Domain	Complexity	Factor 1	Factor 2	Factor 3
Knowing the difference between letters and numbers	Mixed	Basic	0.79		
Reading simple books with pictures and words	Literacy	Basic		0.72	
Reading books with words only	Literacy	Advanced		0.78	
Eigenvalue			8.68	2.14	0.99
% Variance			27.2	20.0	14.9
Cronbach's α			0.91	0.90	0.87
McDonald's ω			0.91	0.90	0.88

Note. $N = 124$. Loadings < .30 suppressed. ML extraction, oblimin rotation. Domain and Complexity columns are theoretically attributed.

The parental expectations EFA yielded three factors. The first factor, Basic Expectations, merged basic literacy and numeracy skills into a single construct. The second factor, Advanced Literacy Expectations, captured more complex skills relating only to literacy. The third factor, Advanced Numeracy Expectations, represents advanced expectations in the numeracy domain.

The boundaries between factors are not as clean as these descriptions suggest. “Knowing all letters of the alphabet” cross-loads between Factor 1 and Factor 2 (.44 and .49), “Associating letters with names” splits evenly between Factors 1 and 2 (.50/.50), and “Calculating simple multiplications” loads equally on Factors 2 and 3 (.44/.44). These ambiguities are invisible once labels are assigned, which is precisely why factor labels alone are insufficient and the factor projection in theoretical axes that follows is needed.

We computed the loading-weighted theoretical coordinates for each factor, as shown in Table 12.

Table 12

Parental Expectations Scale: Loading-Weighted Factor Coordinates

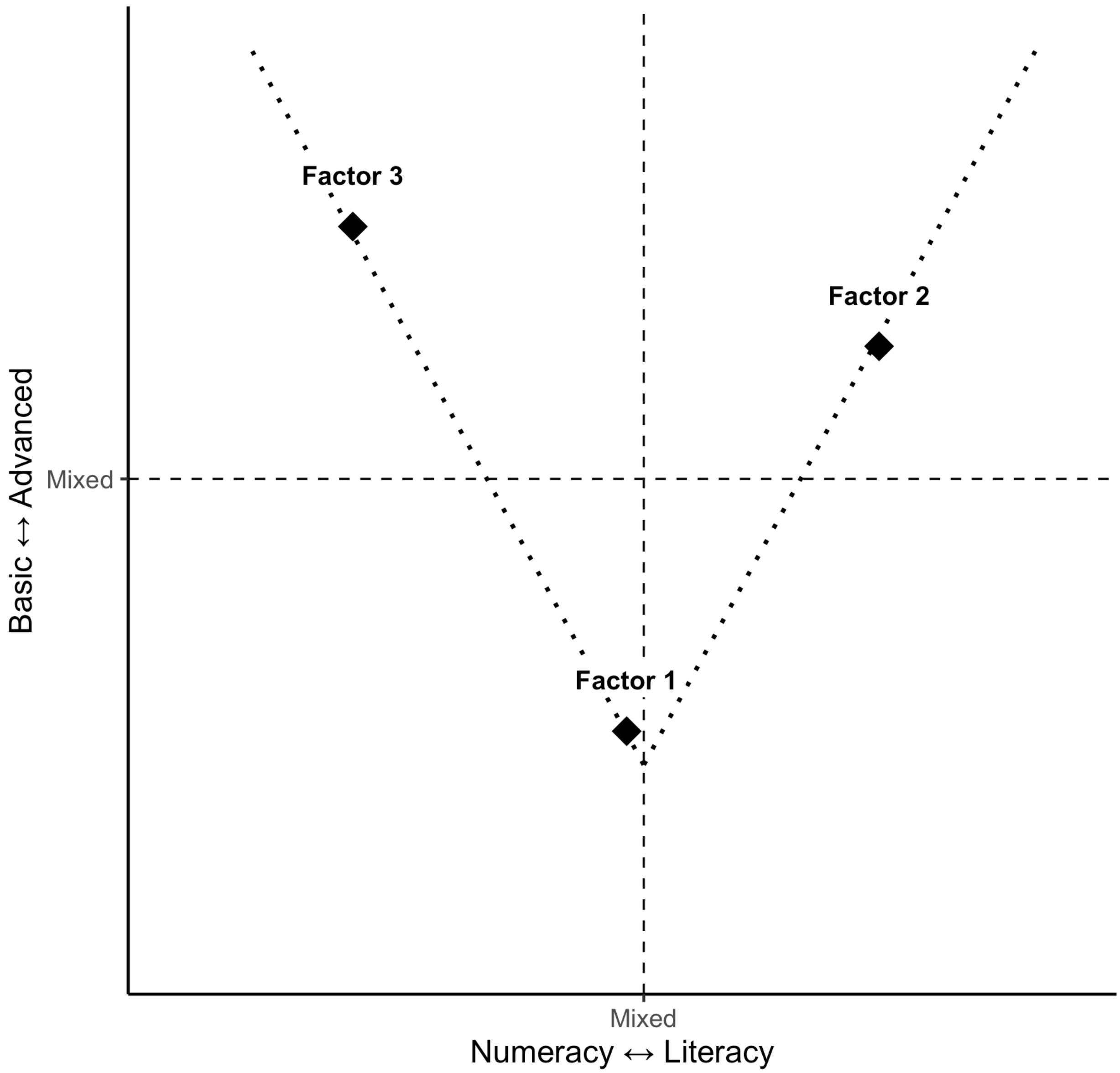
Factor	Domain	Complexity
Factor 1	-0.04	-0.59
Factor 2	0.55	0.31
Factor 3	-0.68	0.59

Factor 1 sits near the domain midpoint (D = -0.04) with a negative complexity coordinate (C = -0.59), confirming its position as a basic, domain-mixed factor. Factor 2 shows positive domain differentiation toward literacy (D = 0.55) with a positive complexity coordinate (C = 0.31). Factor 3 shows strong numeracy pull (D = -0.68) with the highest complexity coordinate (C = 0.59). Domain differentiation is near zero for the basic factor and substantial for the advanced factors, paralleling the engagement pattern.

Absolute domain differentiation was regressed on complexity at the factor level (r = .99, p = .023, slope = 0.55). The vertex fell at -0.67 on the complexity axis, beyond the most basic-positioned factor centroid (Factor 1 at C = -0.59) but within the theoretical range of -1. Figure 7 shows the plot of factor centroids with the regression slope.

Figure 7

Factor Centroids Projected onto Domain and Complexity Axes for Parental Expectations



Note. Diamonds represent factor centroids positioned on theoretical axes. Dotted lines represent the regression of absolute domain differentiation on complexity (r = .99, p = .023, slope = 0.55).

The complementary regression, absolute complexity differentiation on domain, yielded no significant result ($r = -.85$, $p = .35$, slope = -0.22): parents distinguish basic from advanced expectations regardless of domain.

Parental Attitudes Scale

The parental attitudes scale had 10 items categorized a priori on the domain dimension. EFA yielded two factors, explaining in total 44.9% of variance. Table 13 shows theoretical classifications and EFA results.

Table 13

Parental Attitudes Scale: Theoretical Classifications and EFA Factor Loadings (2 factors)

Item	Domain	Factor 1	Factor 2
I was good at math in school	Numeracy	0.81	
I was good at language and reading in school	Literacy		0.61
Writing and preparing reports are part of my job	Literacy	0.37	
I like math	Numeracy	0.94	
I like to read	Literacy		0.64
I like to write	Literacy		0.94
I avoid situations that involve writing	Literacy		0.59
Math is part of my job	Numeracy	0.52	
I avoid situations that involve math	Numeracy	0.49	
I feel capable of helping the child with their schoolwork	Mixed	0.30	
Eigenvalue		2.73	1.78
% Variance		23.4	21.5
Cronbach's α		0.75	0.78
McDonald's ω		0.77	0.79

Note. $N = 124$. Loadings < .30 suppressed. ML extraction, oblimin rotation. Domain column is theoretically attributed.

The parental attitudes EFA yielded two orthogonal factors. The first factor, Attitudes Towards Math, captured all math items and, additionally, the mixed item ("I feel capable of helping the child with their schoolwork"). The second factor, Attitudes Towards Literacy, captured all literacy items.

For the attitudes scale, the procedure to calculate loading-weighted theoretical coordinates for each factor was not computed. Since the attitudes scale had only one theoretical dimension (domain), the procedure, even though mathematically possible, would not be informative, as it would merely plot factor coordinates in a straight line (domain axis).

Discussion

Domain Differentiation Dissolves Under Low Formality in Engagement and Low Complexity in Expectations

Domain differentiation is so important in home learning environment literature that it is the main criterion to define the home numeracy/mathematics environment and the home literacy environment as research fields. While these categories make theoretical sense, ontologically reflecting what children learn (that is, math/numeracy or literacy), they may not necessarily reflect how parents understand learning activities or children's skills.

Domain Differentiation Dissolves Under Low Formality: The Parental Engagement Scale. This scale yielded four factors: Formal Literacy Engagement, Formal Numeracy Engagement, Informal Playful Engagement and Informal Applied Engagement.

While Factors 1 and 2 were clearly domain specific, both Factors 3 and 4 blended mixed-domain items. So factors that loaded formal items had clear domain differentiation, while factors that loaded informal items mixed numeracy and literacy. This observation suggests reduced domain differentiation under lower formality for home engagement frequency.

Our factor projection in theoretical axes showed a clear wedge pattern for the regression. Domain differentiation is clear when parents engage in formal learning activities and dissolves when parents engage in informal learning activities.

Another important fact is that Factors 1 and 2 were labelled according to our theoretical dimensions (that is, domain and formality), but Factors 3 and 4, as described, split by a criterion external to these dimensions, which can indicate the existence of another theoretical dimension that is not being considered. Factor 4 captured items that, besides being more applied, were more cognitively demanding, while Factor 3 captured simpler and more playful activities. This informal split between “basic” and “advanced” activities loosely parallels earlier findings. Skwarchuk et al. (2014) identified basic and advanced components within formal numeracy, and Mutaf-Yıldız et al. (2020) reported differential effects of advanced versus basic home numeracy interactions on child outcomes. Our results suggest that a similar distinction emerges in how parents organize informal engagement across both domains.

Still, this interpretation has to be taken carefully, since, for this work, these activities were not categorized a priori beyond domain (literacy and numeracy) and formality (formal and informal), and our method did not include any way to assess the level of cognitive demand of each activity. Also, “basic” and “advanced” are, indeed, relative predicates, and their concrete application must consider, at least, developmental factors (e.g., age, brain maturation, executive functions) and familiarity of each child with certain literacy or numeracy contents.

We should note that the engagement measured in the home learning activities scale refers to frequency, in line with common measures in the field (LeFevre et al., 2009; Skwarchuk et al., 2014), so our factor structure reveals how parents organize the frequency of home learning activities without relation to other potentially relevant factors, such as quality of interaction (Purpura et al., 2020) or even efficacy of said activities to improve child outcomes.

Domain Differentiation Dissolves Under Low Complexity: The Parental Expectations Scale. This scale yielded three factors: Basic Expectations, Advanced Literacy Expectations, and Advanced Numeracy Expectations.

EFA produced two domain-specific factors that represent higher complexity, while the low complexity factor merges literacy and numeracy skills. This finding parallels the parental engagement analysis. Factor projection in theoretical axes showed that domain differentiation is stronger when parents hold advanced expectations and weaker for foundational ones.

One important difference between this scale and engagement, though, is that all factors yielded in the expectations scale analysis could be labeled according to the pre-defined theoretical dimensions (domain and complexity). This may indicate two things. First, for this scale, parental understanding of categories approaches researcher-defined categories more closely than in parental engagement. Second, this scale may be more appropriately designed regarding theoretical dimensions.

What Domain Differentiation in Engagement and Expectations Tells Us. The slope of the regression reveals the rate at which domain differentiation dissolves in engagement and expectations. For engagement, the slope was .50 (r = .99, p = .006), and for expectations, the slope was similar, at .55 (r = .99, p = .023).

We consider that this cross-construct replication strengthens the finding, since the two scales have different objects (frequency of activities versus anticipated milestones) and the second axis differs (formality versus complexity), yet both produce a similar pattern and a similar rate.

One important observation is about the fit of the two regressions, showing both $r = .99$. This is because the regression operates on few data points. More generally, typical EFA solutions yield few factors. With two factors, correlation would be trivially perfect. With three or four, which is our case, the probability of the absolute values in domain aligning for a good fit is high, since the regression would have few degrees of freedom. However, the informative quantity is not the correlation itself but the slope, which captures how rapidly domain differentiation changes as a function of the second axis (e.g., formality and complexity). A steeper slope indicates faster collapse of domain differentiation; a flatter slope indicates more gradual change. The slope is interpretable regardless of sample size and is comparable across scales and applications.

A reader might reasonably ask why the projection procedure is needed when the pattern is visible from factor labels alone. The answer is twofold. First, factor labels are post hoc interpretations, not measurements. Factor 1 in expectations is labeled "basic mixed" because the researcher decided it looked mixed, but this judgment has no associated statistic. The projection replaces it with coordinates: Factor 1 lands at $D = -0.04$, confirming its near-mixed position, while Factors 2 and 3 sit at $D = 0.55$ and $D = -0.68$, quantifying the degree of domain separation at each complexity level. Second, labels cannot be compared across scales. The engagement slope (0.50) and the expectations slope (0.55) are directly comparable quantities, indicating similar rates at which domain differentiation dissolves. Without the projection, both would simply be described as "domain differentiation weakens," with no way to distinguish their magnitude or rate. The resulting pattern is wedge-shaped: domain differentiation fans out at one pole of the second axis and narrows toward the other.

The wedge-shaped pattern replicated across these two scales suggests that the dissolution of domain differentiation may be a general feature of how parents cognitively organize home learning when literacy and numeracy domains are involved, not limited to engagement or expectations. Indeed, complementary regressions on the alternate axes confirmed that the effect is directional: domain differentiation depends on formality and complexity, but formality and complexity differentiation do not depend on domain. In other words, regarding engagement items, for any given activity, lower formality means less differentiation between literacy and numeracy, but being “more literacy” or “more numeracy” does not mean to be more or less formal. Parents distinguish formal from informal activities, and basic from advanced expectations, regardless of whether the content is literacy or numeracy. Domain is the dimension that weakens while the second axis remains stable.

Parental Attitudes Scale. This scale yielded two orthogonal factors: Attitudes Towards Math and Attitudes Towards Literacy.

It seems that, when parents think about their attitudes toward these skills or knowledge, they separate cleanly between domains. The small imbalance between factors was the mixed item (“I feel capable of helping the child with their schoolwork”) that loaded on the Math factor, even though weakly (.30). This loading may be an indication that parents perceive that general homework competence is more dependent on math skills. This makes theoretical sense, considering how negative attitudes toward math are correlated to avoidance of math interactions (Elliott et al., 2020; Maloney et al., 2015). With only one cross-domain item, though, this is hard to assess.

The clear separation between literacy attitudes and math attitudes may indicate that there is no "trade-off", at least regarding attitudes, between being competent in these different areas. Parents who see themselves as "math people" are not necessarily more or less likely to see themselves as "literacy people". This may be consequential for planning communication with parents.

This orthogonal structure contrasts with the high cross-domain correlations observed for engagement (r = .67) and expectations (r = .78), suggesting that how parents perceive their own abilities in math versus literacy is psychologically more distinct than how they engage with or what they expect from their children across domains. This distinction proved consequential in our path analyses in Article 1. In the first model, only literacy attitudes predicted engagement, while math attitudes did not. In the second model, with finer-grained EFA-derived constructs, literacy attitudes still predicted literacy engagement, and math attitudes were a negative predictor of playful numeracy engagement. In this case, parents with lower attitudes engaged more in playful numeracy activities that were unrelated to child outcomes.

Weakening Domain Differentiation as a Cognitive and Behavioral Finding. The finding that domain differentiation depends on formality (or complexity) is a statement about parental cognition: parents distinguish literacy from numeracy more when they teach explicitly and less when they play, and they distinguish literacy from numeracy expectations more when the milestones are advanced and less when they are foundational.

In the case of engagement, this may happen because of how parents differentiate between "teaching" (that is, formal activities) and "playing" (informal activities). In the first case, the object that should be taught is explicitly important, so the parent clearly thinks that they are either teaching literacy skills or numeracy skills. When playing, this differentiation disappears, because the objective is not teaching, but playing itself, so informal literacy or numeracy activities fulfill this goal.

For expectations, one possibility is that basic skills are treated as a group of milestones that the child should achieve indistinctively, since they are the fundamentals for performing well in first grade. Indeed, it's reasonable to think that doing well only in basic letter knowledge but not in basic counting isn't enough. We would think that advanced literacy and advanced numeracy skills should group as another factor, but the fact that each formed its own construct indicates that parents attribute differential importance to these domains.

These patterns are not properties of the activities or milestones themselves but of how parents perceive and enact them. This distinction matters for interpretation: the wedge-shaped pattern in engagement does not mean that informal activities lack domain content, but that parents tend to differentiate less between domains, cognitively or behaviorally, when engaging informally. The activity may objectively involve numeracy, but when parents report how often they do it, the boundary with informal literacy activities blurs. Domain differentiation, in this sense, is a property of reported engagement frequency, not of the activities' content.

Limitations

Study limitations. Regarding cross-sectional design, sample size, sample not being representative of all socioeconomic strata, and inherent limitations of assessing home learning activities engagement through a self-reported frequency measure, the same limitations from Article 1 apply here, since both articles draw from the same dataset.

Additionally, all scales originated from the same questionnaire, with all answers from the same sample. The findings about factor structure can be specific to these specific parents. Generalizability of the main findings regarding domain differentiation depends on replication across a diversity of cultural settings. Parents from other cultures may show diverging patterns, but this would be a finding by itself.

Method-Specific Limitations. Since this study proposes factor projection in theoretical axes as a quantitative procedure to evaluate factor structure post-EFA, the methodology's limitations deserve specific consideration.

The validity of factor coordinates depends entirely on the quality of the a priori theoretical classifications. Misclassification directly displaces factor coordinates and can either reveal false patterns or mask true dynamics. In the present study, classifications were assigned by the first author without external validation. Reasonable disagreements are inevitable for certain items: "I help the child to weigh, measure, and compare quantities" is clearly numeracy, but could be classified as either formal or informal depending on how the parent enacts it; "counting to 100" could be considered advanced or intermediate on a trichotomous scale, especially compared to "counting to 1000". An independent panel of researchers rating each item would strengthen the classifications and, consequently, the precision of the results. Moreover, we used a trichotomous classification scale (-1, 0, +1) for theoretical classifications. Researchers may prefer finer-grained scales, such as a five- or seven-point continuum. The weighted-average formula is indifferent to scale granularity.

The number of factors, which is out of the researcher's control, constrains the analysis. With a single factor, only one centroid can be positioned on the theoretical axes. In this case, the procedure can still be used to assess factor labelling, but with potentially less utility. With two factors, the regression is trivially perfect ($r = 1$). Three or more factors allow a genuine statistical test, but still with a high probability of yielding a very high fit. Possibly, even with two factors the slope can be carefully interpreted as the rate of domain differentiation along a secondary dimension, but the fewer the number of points in a regression, the higher the probability of a mathematical artifact. In this context, *p*-values should be interpreted as sanity checks rather than as precise inferential tests, as the informative quantity is the slope itself.

The procedure assumes that loadings across factors are on comparable scales. Within a single standardized rotated solution, this assumption is reasonable (Fabrigar et al., 1999). The assumption would deserve explicit attention in future applications with non-standard extraction methods or when comparing results across separately estimated factor solutions.

Despite these limitations, this supplementary analysis' simplicity and potential for comparability are practical advantages. It permits an intuitive visualization of exploratory factor analysis results, and depends on weighted averages and linear regressions, which are computationally inexpensive procedures. Whether the approach proves useful beyond home learning research is an empirical question that future applications will answer.

Future Research Directions

We hypothesize that the home learning environment and, correspondingly, the factor structure of the parental variables we researched is dynamic. Longitudinal designs are necessary to capture whether these domain differentiation findings hold over time, or even exist before preschool. Indeed, it would be ideal to start collecting evidence from parental beliefs and engagement as early as possible in the child's life to understand how parental beliefs and engagement change with child's growth, the presence of siblings of different ages and other relevant variables.

The domain differentiation patterns were investigated for parents of 5-year-old preschoolers from a relatively affluent area of the country's capital. Indeed, the United Nations Development Programme classifies the Federal District as having "very high" (> .800) Human

Development Index (Programa das Nações Unidas para o Desenvolvimento et al., 2022). Naturally, factor structure of parental variables should be investigated in demographically representative samples, and in many more SES and cultural settings, within and outside of Brazil. The LeFevre et al. (2009) scale, widely adapted across contexts (Silva-Chelles, 2023), provides a natural starting point for cross-cultural replication, since its parallel literacy-numeracy structure is suited to the projection procedure.

Regarding the novel supplementary analyses presented in this work, the factor projections in theoretical axes method should be applicable wherever a researcher must compare a theoretical dimensional structure against an empirical factor solution, including in research not related to child education or learning. The home learning environment, with its evident structural symmetries between literacy and numeracy/math, and having these two perspectives fully developed as research fields in their own right, makes the use of such method particularly apt. Beyond engagement frequency and expectations, it can be used within the field for parental self-efficacy, anxiety, knowledge, engagement quality, responsiveness to interaction, developmental and prior knowledge appropriateness, initiative of interaction, long-term educational aspirations, school-subjects relative importance, importance and availability of educational resources and any other scale designed as a cross-domain assessment of parental variables or home learning conditions. With the correct mathematical adaptations, it is possible to consider additional domains, such as executive functions or socioemotional skills.

Indeed, the field should also expand beyond frequency as the primary unit of measurement for engagement. LeFevre et al. (2009) included parental knowledge measures (e.g., knowledge of children’s books, math games) that remain underexplored in terms of factor structure. Other measurement approaches, such as time diaries, experience sampling, or quality ratings of interactions (Hornburg et al., 2021; Purpura et al., 2020), would provide complementary evidence about how parents organize home learning. Whether the weakening of domain differentiation in engagement is specific to frequency reports or reflects a deeper cognitive pattern observable across measurement modalities is an open question.

The explored theoretical dimensions deserve further consideration. For the expectations scale, basic and advanced is an intuitive fit for the objective of the scale. Not only having higher expectations in each item, but having higher expectations in more basic or advanced items is directly consequential. For the engagement scale, the formal and informal categories are not as immediately evident, and it is worth reflecting on whether they serve a purpose beyond theoretical classification. First, the distinction between formal and informal activities can, in practice, be mitigated when parental intentionality in doing such activities is taken into consideration (Purpura et al., 2020). Also, researchers have not adopted a comparable distinction between informal and formal activities (Skwarchuk et al., 2014). Some research suggests formal and informal numeracy activities as two conceptual categories (LeFevre et al., 2009), and, regarding numeracy, formal activities have been investigated more commonly than informal activities (Mutaf-Yıldız et al., 2020). Still our finding about factor structure of home engagement suggests that there is a real underlying mechanism that justifies the distinction between formal and informal activities.

Finally, examining predictive relationships between parental variables factor structures and child outcomes was outside the scope of this descriptive work. A natural extension would be to test path models with observed variables or, preferably, latent factors that incorporate empirical cross-domain factor structures, rather than assuming domain-specific scales.

Policy Implications

Parents may not naturally differentiate between literacy and numeracy during informal activities or when considering basic expectations. This has practical consequences for how educators and policymakers communicate with families.

When engaging in formal learning, parents seem to intentionally “teach reading” or “teach math”. But when playing games, singing songs, or making everyday routines learning-rich, this distinction dissolves, and parents seem to intentionally engage regardless of whether it’s literacy or numeracy content. Educators who want parents to support child development in both domains cannot assume parents are actively aware of how their informal activities distribute between these categories, precisely because the domain distinction for these activities is weaker. This prompts more explicit guidance for parents, to achieve a better domain balance according to the specific needs of the child.

Parental expectations about knowledge they believe their child should acquire before first grade showed a similar pattern. Parents in our sample treated basic literacy and numeracy skills as a single package, while differentiating by domain only for advanced expectations. Holding undifferentiated basic expectations may mask, in the parental point of view, different cognitive processes and, therefore, instructional necessities and prerequisites between literacy and numeracy.

For this purpose, the parental assessment questionnaire and the parental curriculum, both proposed in Article 1, could serve as operational tools to aid educators. By mapping each family’s engagement and beliefs profile and understanding common activity grouping patterns (such as reduced domain differentiation under low formality), educators can assess if the home learning environment is sufficiently balanced. This can be translated into an actionable diagnostic that is enriched by the educator's privileged knowledge of that specific child's cognitive development. Importantly, incorporating belief variables to understand possible avoidance behaviors can help reframe parental expectations about child outcomes in a positive and developmentally informed way.

For policy makers, these findings stress the importance of promoting continued on-the-job training for educators regarding family inclusion in their children's education. Advancing the use of guidance or assessment instruments for the use of educators can make the family-school interface more informative. These policies would promote equity at home, the very first place where disparities across cultural and socioeconomic settings first emerge.

General Discussion

This dissertation investigated the home learning environment of Brazilian preschoolers, addressing complementary questions about the home learning environment. Article 1 aims to understand how parental variables associate with child outcomes, while Article 2's descriptive approach seeks to unveil how parents cognitively and behaviorally organize those same variables across literacy and numeracy domains. Both articles drew from the same dataset of 124 parent–child dyads from four public preschools in Brasília, Brazil.

Parental Expectations Outperform Home Engagement Frequency as Predictors of Preschoolers' Numeracy Skills

Parents reported on engagement frequency, expectations and attitudes. Article 1 used two observed-variable path models designed around diverse construct granularity: the first model used composites as defined in the questionnaire, while the second used factors derived from exploratory factor analyses. This study found that the proposed mediation model of beliefs predicting child outcomes through engagement was not supported.

Engagement frequency showed limited prediction of child outcomes, with Model 1 having only one significant path from numeracy engagement to number identification, and with Model 2 having none. Expectations, on the other hand, predicted all child numeracy outcomes in the first model, while basic numeracy expectations predicted three in the second model. For this sample, parental expectations are stronger predictors of early numeracy skills than home engagement frequency. In both models, parental attitudes predicted engagement, but not child outcomes. Model 2's finer-grained constructs showed that basic, or proximal, expectations were predictive of parental engagement, a relationship that did not appear on Model 1. Additionally, in the second model, lower math attitudes predicted more playful numeracy engagement, which may be explained as a possible math anxiety avoidance pattern. Overall, belief variables explained very little engagement variance: less than 1% in Model 1, and 1% to 8% in Model 2.

Even though Model 2 showed fewer engagement-to-outcome paths, it yielded a strikingly strong path. Children whose parents showed high developmentally appropriate expectations regarding what they should learn before first grade performed substantially better in the number writing task ($\beta = .41$, $p < .001$).

This study also employed, in an exploratory fashion, bivariate correlation analyses to compare different types of parental expectation and engagement variables. In both models, expectations were normative measures ("how important is it that the child…") about concrete skills. More abstract expectations (school subject importance and long-term educational aspirations) showed no correlations to child outcomes, while concrete expectations were strong predictors.

Bivariate analyses showed that, even though in Model 2 no engagement-to-outcome paths were significant, the Applied Numeracy Engagement construct correlated with all five numeracy outcomes. Included in the path model, however, the explained variance from these potential paths was captured by expectations.

Our study raised the question of whether engagement frequency was the best variable to measure parental home engagement. Indeed, this measure may obscure quality considerations, such as responsiveness, scaffolding, feedback, purpose of the interaction. Two variables measured in this study, parental game and book recognition, correlated more strongly with outcomes than engagement frequency, maybe indicating a quality proxy. These variables correlated negatively with expectations but positively with child outcomes, which could mean they represent distinct channels of influence on child performance.

Domain Differentiation Dissolves Under Low Formality in Engagement and Low Complexity in Expectations

Article 2 used the main scales from Article 1, that is, parental engagement frequency, expectations and attitudes, but as cross-domain composites. This permitted the evaluation of covariances between items of different domains through exploratory factor analyses.

The engagement scale EFA yielded four factors. Two included mainly formal items and were domain specific. The other two were mainly comprised of informal items and were domain-mixed. A similar pattern was observed in the expectations scale EFA, that yielded three factors: two included advanced skills and were domain specific, while the third was composed of items referring to basic skills, undifferentiated by domain. The attitudes scale EFA yielded two orthogonal factors with clean domain separation. Since this scale had, by design, only one theoretical dimension (that is, domain), no additional procedures were applied.

The results suggest that domain differentiation is conditional on formality level (for engagement frequency) or complexity (for expectations). Further analyses employing the factor projection in theoretical axes procedure calculated coordinates for each of the mentioned factors. Linear regressions of domain coordinates against formality and complexity confirmed the conditionality of domain differentiation on the secondary theoretical axes for engagement (slope = .50) and expectations (slope = .55). Directionality of this effect was also confirmed by complementary regressions: formality and complexity differentiation does not depend on domain.

In this sample, parents distinguish literacy from numeracy more when they teach explicitly and less when they play, and they distinguish literacy from numeracy expectations more when the milestones are advanced and less when they are foundational. These findings are not properties of the activities or skills themselves, but of parental cognition and behavior. The way parents think and act regarding learning activities, expectations and their own attitudes constitutes the actual structure of the home learning environment.

Factor Projection in Theoretical Axes as a Supplementary post-EFA Procedure

Article 2 described the factor structure of main parental variables. One practical problem that was faced is that even the best effort to assign post hoc factor labels carried inherent imprecisions. Some items that did not fit theoretically would load in a “mainly formal literacy” factor, and cross-loadings complicated the matters further. As a response to this, I developed a simple supplementary post-EFA procedure which I have termed “factor projection in theoretical axes”.

Basically, the procedure handles impurities that labels can’t by assigning, according to a priori researcher classifications of each items on theoretical dimensions, coordinates in theoretical axes to each factor. This provides an objective way to compare different factors regarding these dimensions.

By regressing the coordinates, it is possible to verify if coordinates from one dimension (say, domain) depend on coordinates from a secondary dimension (e.g., formality or complexity). This yields slopes that can be compared across scales, and a regression plot that produces intuitive visual patterns. If the slope is steep enough, a wedge result is produced, signaling a theoretical point in theory space where dimensional (in our case, domain) differentiation fully collapses.

This methodology is simple and requires, after EFA, only computationally inexpensive procedures: weighted averages and linear regressions. Its main constraints are the strict dependence on the quality of the a priori classifications, and the inherent limitation imposed by the number of factors produced.

Factor projection in theoretical axes being a procedure created in home learning environment research is no coincidence, since it is a field that constantly deals with cross-domain analyses. Symmetrical theoretical structures that are replicated in different domains are a strong use case for this procedure. Indeed, this method should be applicable wherever a researcher must compare a theoretical dimensional structure against an empirical factor solution, including in research not related to child education or learning. Beyond engagement frequency and expectations, it can be used within the field for virtually any cross-domain scales that measure home learning conditions (such as availability of educational resources) or parental beliefs (self-efficacy, anxiety, knowledge) and engagement (responsiveness, scaffolding, feedback) constructs.

Parents Think and Parents Do, but Not as Expected

Articles 1 and 2 are united by a common thread: the way parents think and parents act are, many times, presumed by researchers, policy makers, and educators according to theoretical categories, but these theoretical relationships do not necessarily hold.

In Article 1, it was most natural to think that the beliefs to child outcomes mediation through parental engagement would be the main driver, but in two models this mediation did not hold. Instead, beliefs (expectations, specifically) predicted child outcomes directly, most likely representing causal channels not yet recognized or measured.

In Article 2, the main expectation was to verify a clean separation of factors by the theoretical dimensions we defined a priori, or at least by domain. What happened in the engagement frequency and expectations scales was truly unforeseen: under low formality, parents differentiate weakly between literacy and numeracy engagement, and regarding low complexity skills, parents blend literacy and numeracy expectations.

So parents think, and parents do, but not as (we) expected. And we strive to learn from them, so we can better support each family in building the strong learning environments every child deserves.

References

Alcock, L., Ansari, D., Batchelor, S., Bisson, M.-J., De Smedt, B., Gilmore, C., Göbel, S. M., Hannula-Sormunen, M., Hodgen, J., Inglis, M., Jones, I., Mazzocco, M., McNeil, N., Schneider, M., Simms, V., & Weber, K. (2016). Challenges in mathematical cognition: A collaboratively-derived research agenda. *Journal of Numerical Cognition*, *2*(1), 20–41. https://doi.org/10.5964/jnc.v2i1.10

Angelini, A. L., Alves, I. C. B., Custódio, E. M., Duarte, W. F., & Duarte, J. L. (1999). *Matrizes Progressivas Coloridas de Raven: Escala Especial*. Centro Editor de Testes e Pesquisas em Psicologia.

Associação Brasileira de Empresas de Pesquisa. (2024). *Critério de classificação econômica Brasil*. Associação Brasileira de Empresas de Pesquisa.

Aunio, P., & Niemivirta, M. (2010). Predicting children's mathematical performance in grade one by early numeracy. *Learning and Individual Differences*, *20*(5), 427–435. https://doi.org/10.1016/j.lindif.2010.06.003

Ausubel, D. P. (1968). *Educational psychology: A cognitive view*. Holt, Rinehart and Winston.

Berkowitz, T., Schaeffer, M. W., Maloney, E. A., Peterson, L., Gregor, C., Levine, S. C., & Beilock, S. L. (2015). Math at home adds up to achievement in school. *Science*, *350*(6257), 196–198. https://doi.org/10.1126/science.aac7427

Cahoon, A., Cassidy, T., & Simms, V. (2017). Parents' views and experiences of the informal and formal home numeracy environment. *Learning, Culture and Social Interaction*, *15*, 69–79. https://doi.org/10.1016/j.lcsi.2017.08.002

Ciping, D., Silinskas, G., Wei, W., & Georgiou, G. K. (2015). Cross-lagged relationships between home learning environment and academic achievement in Chinese. *Early Childhood Research Quarterly*, *33*, 12–20. https://doi.org/10.1016/j.ecresq.2015.05.001

Claessens, A., Duncan, G., & Engel, M. (2009). Kindergarten skills and fifth-grade achievement: Evidence from the ECLS-K. *Economics of Education Review*, *28*(4), 415–427. https://doi.org/10.1016/j.econedurev.2008.09.003

Costello, A. B., & Osborne, J. (2005). *Best practices in exploratory factor analysis: Four recommendations for getting the most from your analysis*. *10*(7). https://doi.org/10.7275/JYJ1-4868

De Keyser, L., Bakker, M., Rathé, S., Wijns, N., Torbeyns, J., Verschaffel, L., & De Smedt, B. (2020). No Association Between the Home Math Environment and Numerical and Patterning Skills in a Large and Diverse Sample of 5- to 6-year-olds. *Frontiers in Psychology*, *11*, 547626. https://doi.org/10.3389/fpsyg.2020.547626

DeFlorio, L., & Beliakoff, A. (2015). Socioeconomic Status and Preschoolers' Mathematical Knowledge: The Contribution of Home Activities and Parent Beliefs. *Early Education and Development*, *26*(3), 319–341. https://doi.org/10.1080/10409289.2015.968239

Del Río, M. F., Susperreguy, M. I., Strasser, K., & Salinas, V. (2017). Distinct Influences of Mothers and Fathers on Kindergartners' Numeracy Performance: The Role of Math Anxiety, Home Numeracy Practices, and Numeracy Expectations. *Early Education and Development*, *28*(8), 939–955. https://doi.org/10.1080/10409289.2017.1331662

Diamond, A. (2013). Executive Functions. *Annual Review of Psychology*, *64*(1), 135–168. https://doi.org/10.1146/annurev-psych-113011-143750

Dunn, T. J., Baguley, T., & Brunsden, V. (2014). From alpha to omega: A practical solution to the pervasive problem of internal consistency estimation. *British Journal of Psychology*, *105*(3), 399–412. https://doi.org/10.1111/bjop.12046

Elliott, L., Bachman, H. J., & Henry, D. A. (2020). Why and How Parents Promote Math Learning with their Young Children: A Mixed-Methods Investigation. *Parenting*, *20*(2), 108–140. https://doi.org/10.1080/15295192.2019.1694830

Fabrigar, L. R., Wegener, D. T., MacCallum, R. C., & Strahan, E. J. (1999). Evaluating the Use of Exploratory Factor Analysis in Psychological Research. *Psychological Methods*, *4*(3), 272–299.

Fricke, S., Szczerbinski, M., Fox-Boyer, A., & Stackhouse, J. (2016). Preschool Predictors of Early Literacy Acquisition in German-Speaking Children. *Reading Research Quarterly*, *51*(1), 29–53. https://doi.org/10.1002/rrq.116

Froiland, J. M., Peterson, A., & Davison, M. L. (2013). The long-term effects of early parent involvement and parent expectation in the USA. *School Psychology International*, *34*(1), 33–50. https://doi.org/10.1177/0143034312454361

Heckman, J. J. (2008). Schools, Skills, and Synapses. *Economic Inquiry*, *46*(3), 289–324. https://doi.org/10.1111/j.1465-7295.2008.00163.x

Hooper, S. R., Roberts, J., Sideris, J., Burchinal, M., & Zeisel, S. (2010). Longitudinal predictors of reading and math trajectories through middle school for African American versus Caucasian students across two samples. *Developmental Psychology*, *46*(5), 1018–1029. https://doi.org/10.1037/a0018877

Horn, J. L. (1965). A Rationale and Test for the Number of Factors in Factor Analysis. *Psychometrika*, *30*(2), 179–185. https://doi.org/10.1007/BF02289447

Hornburg, C. B., Borriello, G. A., Kung, M., Lin, J., Litkowski, E., Cosso, J., Ellis, A., King, Y. A., Zippert, E., Cabrera, N. J., Davis-Kean, P., Eason, S. H., Hart, S. A., Iruka, I. U., LeFevre, J.-A., Simms, V., Susperreguy, M. I., Cahoon, A., Chan, W. W. L., … Purpura, D. J. (2021). Next directions in measurement of the home mathematics environment: An international and interdisciplinary perspective. *Journal of Numerical Cognition*, *7*(2), 195–220. https://doi.org/10.5964/jnc.6143

Huntsinger, C. S., Jose, P. E., Liaw, F.-R., & Ching, W.-D. (1997). Cultural Differences in Early Mathematics Learning: A Comparison of Euro-American, Chinese-American, and Taiwan-Chinese Families. *International Journal of Behavioral Development*, *21*(2), 371–388. https://doi.org/10.1080/016502597384929

Hwang, S. (2020). Examining the Effect of Students Early Numeracy Activities at Home on Later Mathematics Achievement via Early Numeracy Competencies and Self-Efficacy Beliefs. *Lnternational Electronic Journal of Elementary Education*, *13*(1), 47–56. https://doi.org/10.26822/iejee.2020.172

Hyde, J. S., Lindberg, S. M., Linn, M. C., Ellis, A. B., & Williams, C. C. (2008). Gender Similarities Characterize Math Performance. *Science*, *321*(5888), 494–495. https://doi.org/10.1126/science.1160364

Jordan, N. C., Kaplan, D., Locuniak, M. N., & Ramineni, C. (2007). Predicting First–Grade Math Achievement from Developmental Number Sense Trajectories. *Learning Disabilities Research & Practice*, *22*(1), 36–46. https://doi.org/10.1111/j.1540-5826.2007.00229.x

Kaiser, H. F. (1974). An Index of Factorial Simplicity. *Psychometrika*, *39*(1), 31–36. https://doi.org/10.1007/BF02291575

Kline, R. B. (2016). Principles and Practice of Structural Equation Modeling. *The Guilford Press*.

LeFevre, J.-A., Clarke, T., & Stringer, A. P. (2002). Influences of Language and Parental Involvement on the Development of Counting Skills: Comparisons of French- and English-speaking Canadian Children. *Early Child Development and Care*, *172*(3), 283–300. https://doi.org/10.1080/03004430212127

LeFevre, J.-A., Fast, L., Skwarchuk, S.-L., Smith-Chant, B. L., Bisanz, J., Kamawar, D., & Penner-Wilger, M. (2010). Pathways to Mathematics: Longitudinal Predictors of Performance. *Child Development*, *81*(6), 1753–1767. https://doi.org/10.1111/j.1467-8624.2010.01508.x

LeFevre, J.-A., Polyzoi, E., Skwarchuk, S.-L., Fast, L., & Sowinski, C. (2010). Do home numeracy and literacy practices of Greek and Canadian parents predict the numeracy skills of kindergarten children? *International Journal of Early Years Education*, *18*(1), 55–70. https://doi.org/10.1080/09669761003693926

LeFevre, J.-A., Skwarchuk, S.-L., Smith-Chant, B. L., Fast, L., Kamawar, D., & Bisanz, J. (2009). Home numeracy experiences and children's math performance in the early school years. *Canadian Journal of Behavioural Science / Revue Canadienne Des Sciences Du Comportement*, *41*(2), 55–66. https://doi.org/10.1037/a0014532

Lehrl, S., Evangelou, M., & Sammons, P. (2020). The home learning environment and its role in shaping children's educational development. *School Effectiveness and School Improvement*, *31*(1), 1–6. https://doi.org/10.1080/09243453.2020.1693487

Lin, J., Litkowski, E., Schmerold, K., Elicker, J., Schmitt, S. A., & Purpura, D. J. (2019). Parent–Educator Communication Linked to More Frequent Home Learning Activities for

Preschoolers. *Child & Youth Care Forum*, *48*(5), 757–772. https://doi.org/10.1007/s10566-019-09505-9

Lukie, I. K., Skwarchuk, S.-L., LeFevre, J.-A., & Sowinski, C. (2014). The Role of Child Interests and Collaborative Parent–Child Interactions in Fostering Numeracy and Literacy Development in Canadian Homes. *Early Childhood Education Journal*, *42*(4), 251–259. https://doi.org/10.1007/s10643-013-0604-7

MacKinnon, D. P., Lockwood, C. M., Hoffman, J. M., West, S. G., & Sheets, V. (2002). A comparison of methods to test mediation and other intervening variable effects. *Psychological Methods*, *7*(1), 83–104. https://doi.org/10.1037/1082-989X.7.1.83

Maloney, E. A., Ramirez, G., Gunderson, E. A., Levine, S. C., & Beilock, S. L. (2015). Intergenerational Effects of Parents' Math Anxiety on Children's Math Achievement and Anxiety. *Psychological Science*, *26*(9), 1480–1488. https://doi.org/10.1177/0956797615592630

Martini, F., & Sénéchal, M. (2012). Learning literacy skills at home: Parent teaching, expectations, and child interest. *Canadian Journal of Behavioural Science / Revue Canadienne Des Sciences Du Comportement*, *44*(3), 210–221. https://doi.org/10.1037/a0026758

Missall, K., Hojnoski, R. L., Caskie, G. I. L., & Repasky, P. (2015). Home Numeracy Environments of Preschoolers: Examining Relations Among Mathematical Activities, Parent Mathematical Beliefs, and Early Mathematical Skills. *Early Education and Development*, *26*(3), 356–376. https://doi.org/10.1080/10409289.2015.968243

Moura, R. (2026). Distinct ordinal representations mediate the influence of cardinal knowledge on Preschoolers' arithmetic performance. *Journal of Experimental Child Psychology*, *262*, 106392. https://doi.org/10.1016/j.jecp.2025.106392

Mutaf-Yıldız, B., Sasanguie, D., De Smedt, B., & Reynvoet, B. (2020). Probing the Relationship Between Home Numeracy and Children's Mathematical Skills: A Systematic Review. *Frontiers in Psychology*, *11*, 2074. https://doi.org/10.3389/fpsyg.2020.02074

National Early Literacy Panel. (2008). *Developing Early Literacy: Report of the National Early Literacy Panel*. National Institute for Literacy.

National Mathematics Advisory Panel. (2008). *The Final Report of the National Mathematics Advisory Panel*. U.S. Department of Education.

National Research Council. (1998). *Preventing reading difficulties in young children*. National Academy Press.

Neri, M. (2026). *Evolução das Classes Econômicas Brasileiras: 1976 a 2024*.

Niklas, F., & Schneider, W. (2014). Casting the die before the die is cast: The importance of the home numeracy environment for preschool children. *European Journal of Psychology of Education*, *29*(3), 327–345. https://doi.org/10.1007/s10212-013-0201-6

Organisation for Economic Co-operation and Development. (2025). *Results from TALIS 2024: The State of Teaching*. OECD Publishing. https://doi.org/10.1787/90df6235-en

Programa das Nações Unidas para o Desenvolvimento, Instituto de Pesquisa Econômica Aplicada, & Fundação João Pinheiro. (2022). *Atlas do Desenvolvimento Humano no Brasil*.

Purpura, D. J., King, Y. A., Rolan, E., Hornburg, C. B., Schmitt, S. A., Hart, S. A., & Ganley, C. M. (2020). Examining the Factor Structure of the Home Mathematics Environment to

Delineate Its Role in Predicting Preschool Numeracy, Mathematical Language, and Spatial Skills. *Frontiers in Psychology*, *11*, 1925. https://doi.org/10.3389/fpsyg.2020.01925

R Core Team. (2024). *R: A language and environment for statistical computing*. R Foundation for Statistical Computing. https://www.R-project.org/

Revelle, W. (2025). *Psych: Procedures for psychological, psychometric, and personality research*. Northwestern University. https://CRAN.R-project.org/package=psych

Rosseel, Y. (2012). lavaan: An R package for structural equation modeling. *Journal of Statistical Software*, *48*(2), 1–36. https://doi.org/10.18637/jss.v048.i02

Scarborough, H. S., & Dobrich, W. (1994). On the Efficacy of Reading to Preschoolers. *Developmental Review*, *14*.

Sénéchal, M. (2006). *The effect of family literacy interventions on children's acquisition of reading: From kindergarten to grade 3—A meta-analytic review*. National Center for Family Literacy.

Sénéchal, M., & LeFevre, J.-A. (2002). Parental Involvement in the Development of Children's Reading Skill: A Five-Year Longitudinal Study. *Child Development*, *73*(2), 445–460. https://doi.org/10.1111/1467-8624.00417

Sénéchal, M., & Young, L. (2008). The Effect of Family Literacy Interventions on Children's Acquisition of Reading From Kindergarten to Grade 3: A Meta-Analytic Review. *Review of Educational Research*, *78*(4), 880–907. https://doi.org/10.3102/0034654308320319

Silva-Chelles, E. de O. (2023). *Investigação do ambiente de literacia e numeracia familiar em famílias brasileiras* [Master's thesis]. Universidade Federal de Minas Gerais.

Skwarchuk, S.-L., Sowinski, C., & LeFevre, J.-A. (2014). Formal and informal home learning activities in relation to children's early numeracy and literacy skills: The development of a home numeracy model. *Journal of Experimental Child Psychology*, *121*, 63–84. https://doi.org/10.1016/j.jecp.2013.11.006

Starkey, P., & Klein, A. (2000). Fostering Parental Support for Children's Mathematical Development: An Intervention with Head Start Families. *Early Education & Development*, *11*(5), 659–680. https://doi.org/10.1207/s15566935eed1105_7

Susperreguy, M. I., Di Lonardo Burr, S., Xu, C., Douglas, H., & LeFevre, J.-A. (2020). Children's Home Numeracy Environment Predicts Growth of their Early Mathematical Skills in Kindergarten. *Child Development*, *91*(5), 1663–1680. https://doi.org/10.1111/cdev.13353

Sy, S., & Schulenberg, J. (2005). Parent beliefs and children's achievement trajectories during the transition to school in Asian American and European American families. *International Journal of Behavioral Development*, *29*(6), 505–515. https://doi.org/10.1080/01650250500147329

Tabachnick, B. G., & Fidell, L. S. (2013). *Using multivariate statistics* (6th ed.). Pearson.

Thompson, R. J., Napoli, A. R., & Purpura, D. J. (2017). Age-related differences in the relation between the home numeracy environment and numeracy skills. *Infant and Child Development*, *26*(5), e2019. https://doi.org/10.1002/icd.2019

Vandermaas-Peeler, M., Mischka, M., & Sands, K. (2019). "What do you notice?" Parent guidance of preschoolers' inquiry in activities at home. *Early Child Development and Care*, *189*(2), 220–232. https://doi.org/10.1080/03004430.2017.1310724

Vygotsky, L. S. (1978). *Mind in Society: The Development of Higher Psychological Processes*. Harvard University Press.

Appendix A

Supplementary Materials for Article 1: Data Screening and Quality Procedures

Of the 164 initial participants, three met exclusion criteria, yielding 161 potential dyads. From these, 29 parents (18.01%) did not answer the parental questionnaire, leaving 132 complete dyads. We investigated missing data patterns and, as described below, retained only attentive responders, with a final sample size of $N = 124$ dyads. School-level effects were also assessed.

Missing Data Patterns

We investigated whether missingness was related to child characteristics. Through bivariate logistic regressions that tested each child variable against missingness of the parental questionnaire, only sex ($p = .03$) reached significance. Also, a multivariate model including age, sex, fluid intelligence, Ordinality, Number Reading, and Rhyme Detection indicated that only age was a significant predictor of missingness ($\beta = 0.13$, $p = .040$), suggesting that older children were somewhat more likely to lack corresponding parent measures. The 132 dyad sample proceeded without imputation or weighting for missing data.

Attention Check Results

Parental response integrity was assessed through different types of attention checks.

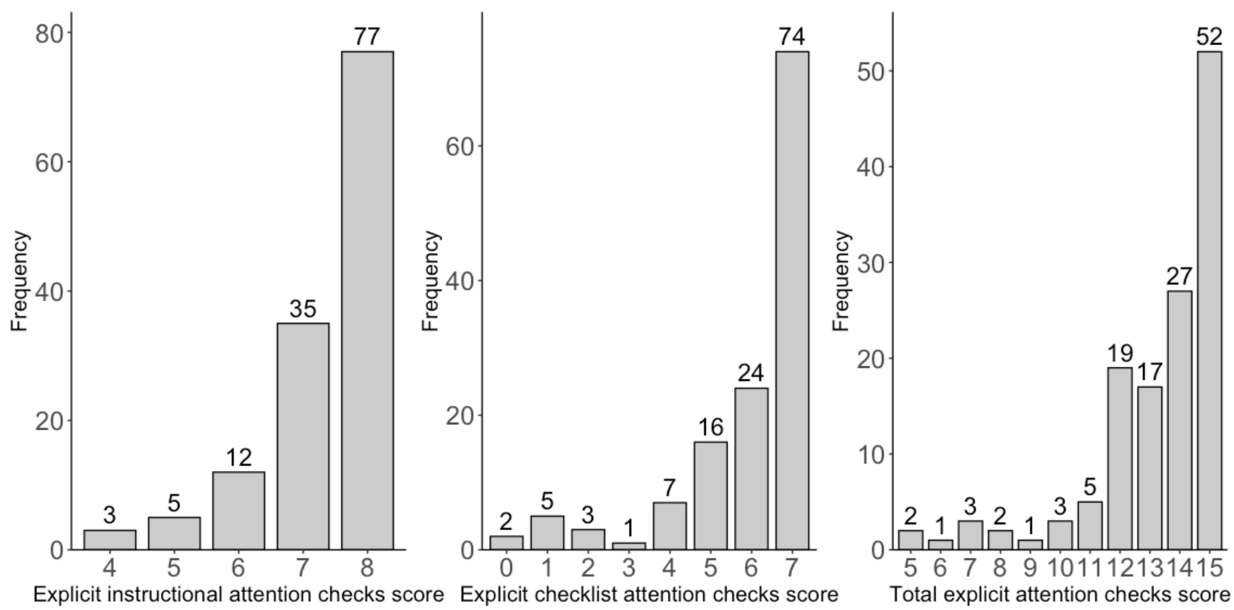
The first type, explicit instructional attention checks, prompted parents directly and could not be skipped (e.g., “Select the ‘Rarely or never’ option to show you are paying attention.”). In this category, 77 parents (58.3%) obtained the maximum score of 8 items, while 35 parents (26.5%) missed only one item. Scores of 6 or lower represented a combined 15.2% of respondents.

The second type, explicit checklist attention checks, also prompted parents directly (e.g., “Select this option to show you are paying attention.”). These checks were effectively skippable, since they were placed inside "check all that apply" questions. For this category, 74 (56%) of parents scored the ceiling 7 items, with 24 (18.2%) missing one item, 16 (12.1%) missing two items and 13.6% missing 3 or more items.

Figure A1 shows the distributions for these items. Combining both types of instructional items, as seen in the right panel of Figure A1, indicates that 52 parents (39.4%) obtained a perfect score of 15 and 115 parents (87.1%) had 3 or fewer misses.

Figure A1

Distribution of Explicit Attention Check Scores



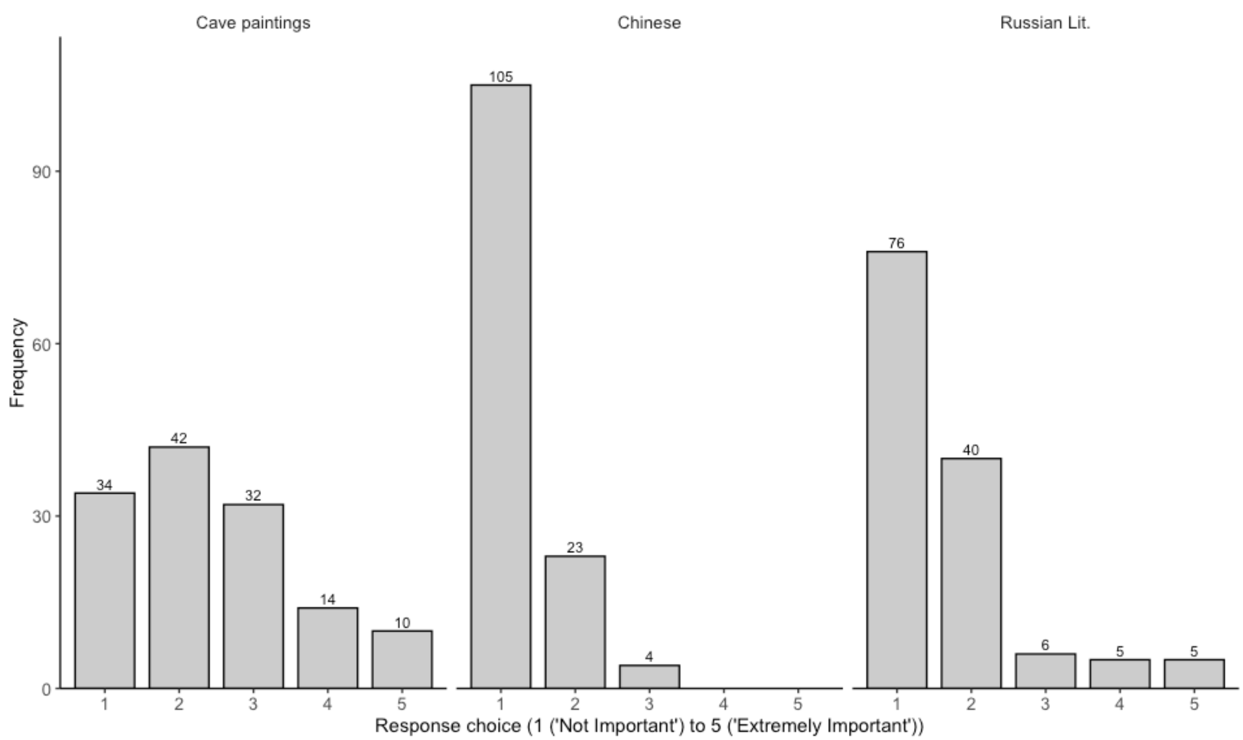
The explicit attention check items distribution gives indication that parental attentiveness was at sufficient levels, supporting the validity of the subsequent analyses.

Explicit instructional items distributions were used to exclude inattentive parents, considering a cutoff score of 5; that is, we consider making three or more errors on eight attention checks to be sufficient evidence of systematic inattention. This result roughly coincides with a conventional threshold for outlier removal of 5% (Tabachnick & Fidell, 2013); we removed 8 parents who scored 5 or less, corresponding to 6.1% of participants.

The third type consisted of semantic validity items, whose distributions are shown in Figure A2. These checks assumed that certain responses would be implausible for preschool contexts. Three items asked parents about recognizing cave wall art symbols, speaking Chinese, and Russian literature.

Figure A2

Responses to Semantic Validity Items



As shown in the figure, responses differed substantially across items. For Chinese, most parents selected “not important” (105 parents, 79.5%). A similar pattern was observed for

Russian literature (76 parents, 57.6%). These high frequencies of “not important” responses are consistent with the strong performance on explicit attention checks.

In contrast, only 34 parents (25.8%) selected “not important” for cave paintings, with responses broadly distributed across the scale. This pattern suggests the item may have been interpreted as a legitimate developmental goal rather than an implausible response.

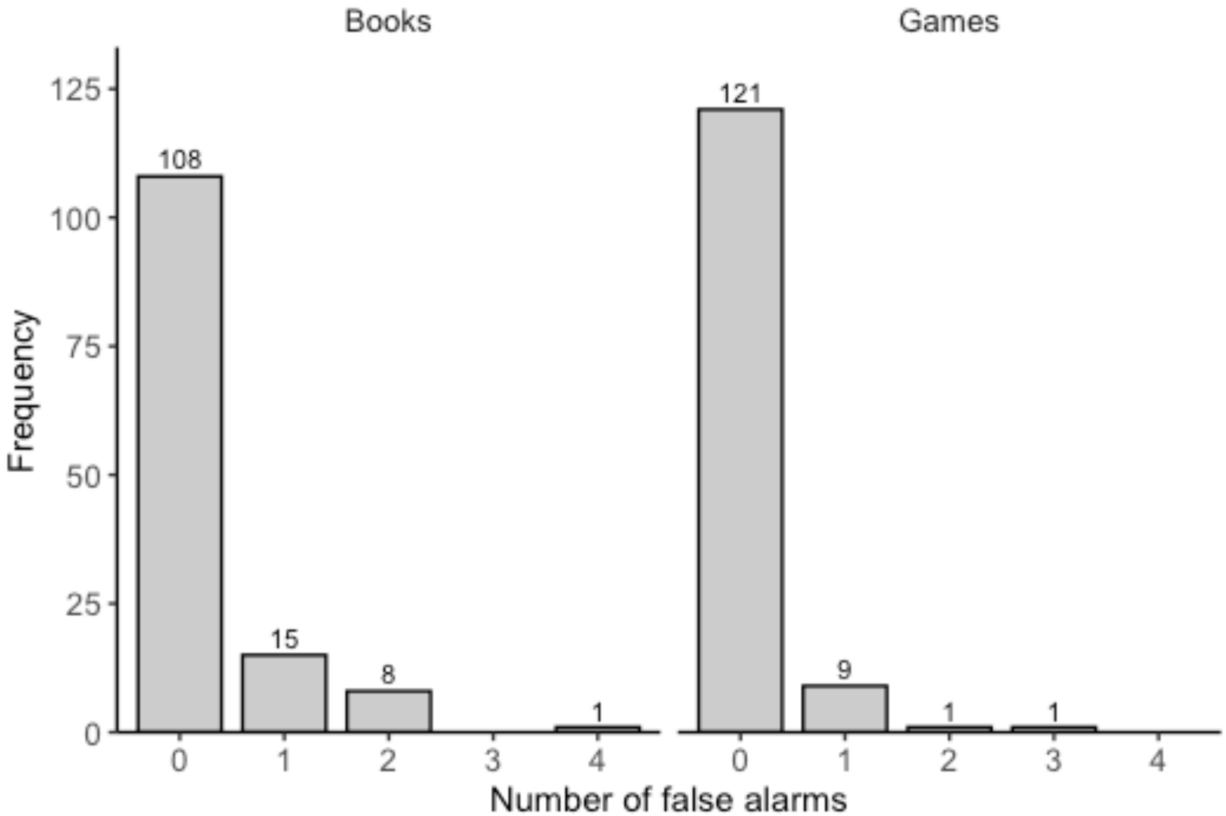
For these reasons, semantic validity items informed our understanding of response patterns but were not used for participant exclusion.

The final type was knowledge-check false alarms. These were derived from book and game recognition lists that mixed real and fictitious items. The games checklist contained 20 real games and 5 fictitious; the books checklist contained 43 real books and 17 fictitious. Parents were instructed to mark only items they truly recognized and not to guess.

These items were not mandatory, since parents unfamiliar with any items could simply skip the checklist. However, all 132 parents responded to both checklists, indicating genuine engagement with the task.

Figure A3

Distribution of False Alarm Scores on Knowledge Checklists



False alarm rates were low. For games, 121 parents (91.7%) made zero false alarms; for books, 108 parents (81.8%) made none. Maximum false alarms were 3 for games and 4 for books, with only one parent reaching each maximum.

Hit rates showed greater variability, reflecting genuine differences in familiarity with children's media. Parents achieved a median of 11 games (range: 2–18) and a median of 20 books (range: 2–30). The modal response for books (20 hits) and games (11 hits), matching the median in both cases, suggests that parents actively engaged with the full checklists rather than selecting minimally. These results provide additional evidence that participants read the lists carefully and followed the knowledge-check instructions as intended.

Low false alarm rates provide evidence that parents followed instructions and responded honestly rather than guessing, since false alarms could only occur when parents actively responded. All parents answered both book and game checklists, so nonresponse was not an issue for these instruments.

In summary, the four attention check types converged on a picture of generally attentive responding, with explicit instructional items providing the most reliable exclusion criterion. Having established the cutoff of ≤5 on instructional items, we now examine whether the 8 excluded participants differed systematically from the 124 retained.

To examine whether exclusions introduced bias in our sample, we compared attentive and inattentive responders on socioeconomic status (SES) and parental educational achievement variables. For this specific analysis, it is necessary to cautiously but reasonably consider parental data for these variables valid for inattentive parents, since these items appeared very early in the questionnaire and therefore less affected by fatigue or later inattentiveness.

Regarding SES, a Wilcoxon rank-sum test indicated that the inattentive group concentrated in lower SES categories ($W = 719.5$, $p = .033$). Mother's highest education differed significantly between attentive and inattentive parents ($W = 733.5$, $p = .020$). Differences for father's education ($W = 669.5$, $p = .092$) and head-of-household education ($W = 666.5$, $p = .053$) did not reach conventional significance thresholds.

School-level Effects

With respect to school-level effects, parental questionnaire response rates varied substantially across schools: School 1 had a 94% response rate; Schools 2 and 4 both had 77%; and School 3 had the lowest response rate, at 61%. The sequential assessment of schools, starting from School 1 and finishing with School 4 meant that child age and school affiliation were confounded.

To assess clustering, we calculated the intraclass correlation coefficients (ICCs) for all variables using linear mixed-effects models with school as a random intercept. Only age showed moderate clustering (ICC = 0.28), which is a finding consistent with the sequential data collection design.

One-way ANOVAs (or Kruskal-Wallis tests when test assumptions were violated) confirmed differences in age ($p < .001$), all cognitive outcomes (Ordinality ($p = .01$), Counting ($p < .001$), Number Reading ($p < .001$), Number Writing ($p < .001$), Number Identification ($p = .01$), Rhyme Detection ($p < .001$) and fluid intelligence ($p = .02$)), and parental expectations (concrete expectations for literacy ($p < .001$) and numeracy ($p = .03$), expectations regarding school subjects ($p = .006$) and long-term educational aspirations ($p = .01$)). Schools did not, however, differ significantly on socioeconomic status ($p = .20$), home numeracy engagement ($p = .10$), home literacy engagement ($p = .69$), or parental attitudes toward math, literacy, or general efficacy (all $p > .50$). Given low ICCs across the board, strong age confound that arises from data collection, and small number of clusters (k = 4), we did not employ multilevel modeling.

Finally, we proceeded to the main analyses with a final sample of 124 parent–child dyads.

Appendix B

Letter to School Principals, Teachers, and Parents

This appendix contains the informational letter intended for school principals, teachers, and parents or guardians of students at participating schools. The original Portuguese version is presented first, followed by an English translation.

The letter brings a summary in plain language of the main findings of this dissertation. This is important because these are the families and schools whose participation made the research possible in the first place. Its inclusion here reflects a commitment, articulated throughout both articles, of strengthening the home learning environment through the improvement of the interface between families, educators, and policy makers. And this improvement starts with making locally grounded evidence available to the audiences best positioned to act on it.

Aos pais e mães, professores e diretores dos Jardins de Infância que colaboraram com esta pesquisa

Ao longo dos anos de 2024 e 2025, os senhores e seus filhos participaram do projeto de pesquisa "Habilidades numéricas básicas em crianças e seu papel como preditores do desempenho na matemática: um estudo longitudinal", em que estudamos o ambiente de aprendizagem em casa e sua relação com o desenvolvimento das crianças na última etapa da educação infantil. Por isso, agradecemos profundamente.

A pesquisa envolveu um questionário respondido pelos pais sobre rotinas, atividades e expectativas em casa, e uma série de tarefas cognitivas realizadas pelas crianças na escola. Todo o trabalho foi conduzido com os cuidados éticos aprovados pelo Comitê de Ética em Pesquisa da Universidade de Brasília.

A pesquisa gerou, até aqui, diversos artigos científicos, dos quais dois, que compõem a dissertação de mestrado "Pais Pensam e Pais Agem, mas Não Como Esperado: Padrões na Estrutura do Aprendizado Doméstico e Desempenho Infantil", serviram de base para os achados apresentados nesta carta.

Acreditamos que esta pesquisa pertence não a nós, apenas, mas a todas as famílias e profissionais que cooperaram e à sociedade brasileira, que sustenta e conta com a Universidade de Brasília para produzir conhecimento relevante para nossa cidade e nosso País. Como forma de retorno a cada uma das 161 famílias participantes, bem como aos professores e diretores dos quatro Jardins de Infância públicos do Distrito Federal em que trabalhamos, reportamos abaixo os resultados que obtivemos.

- **As expectativas de hoje importam mais que as de amanhã.** Muitas pesquisas mostram que é importante ter altas aspirações de longo prazo para as crianças, como "quero que meu filho vá para a faculdade". Neste estudo, porém, vimos que o que mais se associou ao desempenho das crianças foram as expectativas concretas sobre o que elas devem aprender agora, nesta fase. Ou seja, pais que esperam que seus filhos aprendam a reconhecer letras, contar, identificar números e quantidades ou juntar palavras simples parecem exercer uma influência tal que se reflete em melhores resultados das crianças. São essas expectativas próximas, e não as abstratas, que parecem organizar, para crianças pequenas, o ambiente e as rotinas em casa.
- **Expectativas não dependem da renda.** Essas expectativas concretas não se associaram nem à renda nem à escolaridade dos pais. Ou seja, qualquer família, em qualquer contexto, pode cultivá-las e beneficiar a criança. É um recurso ao alcance de todos, o que nos anima muito como possibilidade de política pública.
- **Quantidade ajuda, mas qualidade e intenção pesam mais.** Como esperado, maior frequência com que pais e filhos se envolvem em atividades de leitura e matemática em casa tem, sim, relação com o desempenho das crianças. Porém, indicadores de expectativas e de atenção dos pais ao desenvolvimento do filho, como reconhecer livros e jogos adequados à idade da criança, se associaram de forma ainda mais forte. A mensagem, portanto, não é necessariamente "faça mais", e sim "faça com qualidade e intenção, entendendo o que seu filho precisa aprender agora".
- **A atitude do adulto com a matéria não precisa limitar a criança.** Pais que afirmam não gostar de matemática podem, ainda assim, sustentar expectativas altas e realistas sobre o que os filhos são capazes de aprender. Uma coisa não precisa contaminar a outra, e comunicar isso às famílias pode ser uma mensagem simples e valiosa.
- **Em atividades informais, os pais tendem a não separar leitura e matemática.** Quando os pais fazem atividades com a intenção clara de ensinar, como sentar com a criança para praticar letras ou treinar

continhas, eles separam bem o que é leitura e o que é matemática, tratando cada domínio como uma atividade distinta. Já em jogos, músicas, brincadeiras e rotinas do dia a dia, essa distinção desaparece, e os pais se engajam sem diferenciar se o conteúdo é de uma área ou de outra. Esse é um achado bastante interessante, e mostra que os professores podem orientar as famílias de forma mais explícita, ajudando-as a enxergar oportunidades de leitura e de matemática mesmo nas situações informais, de modo que nenhuma das duas áreas fique de fora do cotidiano da criança.

- **Para os pais, "aprender o básico" é uma coisa só.** Quando pensam em habilidades básicas que consideram ser essenciais para seu filho aprender na pré-escola (contar, reconhecer letras e números), os pais tratam leitura e matemática como um pacote único. A diferenciação entre as duas áreas só aparece quando as expectativas envolvem habilidades mais avançadas. Isso importa porque leitura e matemática exigem caminhos de aprendizado distintos desde o início, e tratá-las como a mesma coisa pode esconder necessidades específicas de cada criança.

E, na prática, o que fazer?

- **Para as famílias.** Sugerimos cultivar expectativas positivas e concretas sobre seus filhos e, na medida do possível, garantir tempo de qualidade com eles, em atividades que tenham a ver com o que a criança está aprendendo na escola. Sugerimos também conversar com os professores sempre, entendendo com eles o que seu filho já aprendeu e quais são os próximos passos. Essas expectativas se constroem junto com o tempo que se passa lendo, brincando, contando e conversando com a criança, e não no lugar desse tempo. E vale ser intencional: uma brincadeira curta de contar objetos ou apontar letras em placas pode render mais do que uma atividade longa feita sem atenção.
- **Para professores.** Os educadores ocupam uma posição única e privilegiada, pois conhecem e observam cada criança diariamente. Sugerimos orientar as famílias sobre quais brincadeiras e atividades realizar em casa, a partir do que sabem sobre o desenvolvimento de cada aluno. Como os pais tendem a não equilibrar naturalmente leitura e matemática em atividades informais, uma orientação explícita sobre esse equilíbrio pode fazer diferença real no cotidiano da criança.
- **Para a gestão escolar.** Sugerimos fortalecer o canal escola-família, buscando entender o que os pais têm feito em casa e promovendo palestras e formações voltadas para as famílias. Contem conosco para isso. São ações que podem potencializar o efeito da escola sobre o desenvolvimento da criança, sem aumentar a carga de trabalho dos professores.

Os resultados aqui resumidos só foram possíveis pela confiança e pelo tempo de cada criança, pai, mãe, professor e gestor que participou. Muito obrigado a cada um. Seguimos à disposição pelos contatos já compartilhados para quaisquer dúvidas ou interesse em saber mais sobre a pesquisa.

Atenciosamente,

Fábio de Barros Correia Gomes Filho – Mestre em Ciências do Comportamento, UnB
Orientação e chefia de pesquisa: Prof. Dr. Ricardo José de Moura

To the parents, teachers, and principals of the Kindergartens that collaborated with this research

Throughout the years 2024 and 2025, you and your children took part in the research project “Basic numerical skills in children and their role as predictors of mathematics performance: a longitudinal study,” in which we studied the home learning environment and its relationship with children’s development during the final stage of early childhood education. For this, we are deeply grateful.

The research involved a questionnaire answered by the parents about routines, activities, and expectations at home, as well as a series of cognitive tasks performed by the children at school. The entire work was conducted with the ethical care approved by the Research Ethics Committee of the University of Brasília.

So far, the research has generated several scientific articles, two of which, comprising the master’s dissertation "Parents Think and Parents Do, but Not as Expected: Patterns in Home Learning Structure and Child Outcomes", served as the basis for the findings presented in this letter.

We believe that this research belongs not only to us, but to all the families and professionals who cooperated and to Brazilian society, which sustains and relies on the University of Brasília to produce knowledge relevant to our city and our country. As a way of giving back to each of the 161 participating families, as well as to the teachers and principals of the four public Kindergartens in the Federal District where we worked, we report below the results we obtained.

- **Today’s expectations matter more than tomorrow’s.** Much research shows that it is important to hold high long-term aspirations for children, such as "I want my child to go to college." In this study, however, we saw that what was most associated with children’s performance were the concrete expectations about what they should learn now, at this stage. That is, parents who expect their children to learn to recognize letters, count, identify numbers and quantities, or put together simple words seem to exert an influence that is reflected in better outcomes for the children. It is these proximate expectations, and not the abstract ones, that appear to organize the home environment and routines for young children.
- **Expectations do not depend on income.** These concrete expectations were not associated with parents’ income or education. That is, any family, in any context, can cultivate them and benefit the child. It is a resource within everyone’s reach, which greatly encourages us as a possibility for public policy.
- **Quantity helps, but quality and intention weigh more.** As expected, greater frequency of parent-child engagement in reading and math activities at home does relate to children's performance. However, indicators of parents’ expectations and attention to the child’s development, such as recognizing books and games suitable for the child’s age, were associated even more strongly. The message, therefore, is not necessarily "do more," but rather "do it with quality and intention, understanding what your child needs to learn now."
- **The adult’s attitude toward the subject does not have to limit the child.** Parents who say they do not like math can, nonetheless, hold high and realistic expectations about what their children are capable of learning. One thing need not contaminate the other, and communicating this to families can be a simple and valuable message.
- **In informal activities, parents tend not to separate reading and math.** When parents engage in activities with the clear intention of teaching, such as sitting with the child to practice letters or work on simple sums, they clearly separate what is reading and what is math, treating each domain as a distinct

activity. In games, songs, play, and daily routines, however, this distinction disappears, and parents engage without differentiating whether the content belongs to one area or the other. This is a very interesting finding, and it shows that teachers can guide families more explicitly, helping them to see opportunities for reading and math even in informal situations, so that neither of the two areas is left out of the child's daily life.

- **For parents, "learning the basics" is one single thing.** When they think about the basic skills they consider essential for their child to learn in preschool (counting, recognizing letters and numbers), parents treat reading and math as a single package. Differentiation between the two areas only emerges when expectations involve more advanced skills. This matters because reading and math demand distinct learning pathways from the very beginning, and treating them as the same thing can hide each child's specific needs.

And, in practice, what to do?

- **For families.** We suggest cultivating positive and concrete expectations about your children and, as much as possible, ensuring quality time with them, in activities that relate to what the child is learning at school. We also suggest always talking with the teachers, understanding from them what your child has already learned and what the next steps are. These expectations are built together with the time spent reading, playing, counting, and talking with the child, and not in place of that time. And it is worth being intentional: a short game of counting objects or pointing out letters on signs can yield more than a long activity done without attention.
- **For teachers.** Educators occupy a unique and privileged position, as they know and observe each child every day. We suggest guiding families on which games and activities to carry out at home, based on what they know about each student's development. Since parents tend not to naturally balance reading and math in informal activities, explicit guidance on this balance can make a real difference in the child's daily life.
- **For school management.** We suggest strengthening the school-family channel, seeking to understand what parents have been doing at home and promoting talks and training sessions aimed at families. Count on us for this. These are actions that can amplify the school's effect on the child's development, without increasing the teachers' workload.

The results summarized here were only possible thanks to the trust and time of every child, parent, teacher, and administrator who participated. Many thanks to each of you. We remain available through the contacts already shared with you for any questions or interest in learning more about the research.

Sincerely,

Fábio de Barros Correia Gomes Filho – MSc in Behavioral Sciences, UnB
Research advisor and head of the research group: Prof. Dr. Ricardo José de Moura