

# **EFFECTS OF CONCRETE COMPRESSIVE STRENGTH AND THICKNESS ON THE NATURAL FREQUENCY OF A PRESTRESSED CONCRETE WIND TOWER**

## **LICENÇA**

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## **REFERÊNCIA**

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## Original Paper

# Effects of Concrete Compressive Strength and Thickness on the Natural Frequency of a Prestressed Concrete Wind Tower

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### **Abstract**

*In wind tower design, the natural frequency must remain within a safe working range to avoid resonance. This study investigates the effects of concrete thickness and compressive strength on the first natural frequency of a 100 m prestressed concrete tower supporting a 5 MW turbine. Sixty-four models were analyzed using finite element modal analysis. The tower was modeled using 4 m Class 1 3D Bernoulli beam segments, each subdivided into ten elements, and the turbine was represented by a concentrated mass at the top. Two studies were conducted for concrete classes C30, C40, C50, and C60: one varying both base and top thickness (study 1), and another varying only the base thickness (study 2). For a given concrete class, increasing thickness resulted in a higher natural frequency, but with decreasing impact as thickness grew, indicating a tendency toward stabilization. Likewise, higher concrete strength increased the natural frequency, although the effect diminished at higher strength levels. For all geometries, the natural frequencies relative to C50 were 85.6% for C30, 93.3% for C40, and 103.9% for C60.*

### **Keywords**

*Natural frequency, Concrete wind towers, Modal analyses, Turbine*

## **1. Introduction**

Some energy sources are classified as renewable because they are naturally replenished at a rate higher than that of consumption (Mubarak et al., 2024). Solar and wind energy are key examples, with the potential to support the achievement of the Sustainable Development Goals (SDGs) and mitigate the impacts of climate change (Sofian et al., 2024). Wind and solar photovoltaic energy are among the

main technologies for electricity generation capacity (Möllerström et al., 2025). Wind farms, in particular, represent a cost-effective solution with a promising future (Pons et al., 2017). Interest in wind energy increased significantly during the oil crisis of the 1970s, followed by a significant revival of the technology in the 1980s (Möllerström et al., 2025). Since then, continuous improvements have led to the current predominant three-blades design with upwind rotors (Möllerström et al., 2025). Nearly all wind turbines designed for electricity generation have horizontal-axis rotors, which are superior in design compared to other configurations (Hau, 2013). Wind energy is exploited to spin turbines that convert the kinetic energy of the wind primarily into electricity (Mubarak et al., 2024). The continued expansion of this energy source has been driven by the need to reduce greenhouse gas emissions, along with the advancement and economic maturity of wind energy technology (Möllerström et al., 2025). In this context, numerous studies have focused on wind turbine components, such as blades, rotors, foundations, and support towers, which are the focus of this study.

In 2020, the G20 countries accounted for 86% of final electricity demand and 87% of global energy-related CO<sub>2</sub> emissions (Miyake et al., 2024). Consequently, achieving the goals of the Paris Agreement will depend on the successful energy transition in these countries (Miyake et al., 2024). In a study based on Geographic Information Systems (GIS), Miyake et al. (2024) identified geographical areas with potential for solar and onshore wind power generation. The findings suggest that the G20's renewable energy potential is sufficient to meet the projected electricity demand for 2050.

Onshore wind towers are mainly constructed of steel or concrete (cast on situ or precast) with internal or external prestressing, or in isolated cases, steel lattice towers are also used (Grünberg & Göhlmann, 2013). Additionally, hybrid towers that combine a concrete lower with a tubular steel upper section have emerged as an alternative to take advantages of both materials (Pires, 2013; Chen et al., 2020). As wind turbine technology has advanced, larger support structures were required due to higher loads and stresses associated with the growing demand for clean energy (Pires, 2013; Von Der Haar & Marx, 2015). Modern wind turbines employ taller towers to access higher wind speeds and use longer blades to enhance energy generation capacity (Van Zyl & Van Zijl, 2015). Taller wind turbines not only access higher average wind speeds but also benefit from a more stable wind profile over the height of the rotor (Quilligan et al., 2012; Schmitz, 2013; Von Der Haar & Marx, 2015). Taller towers have the potential to significantly increase energy production (Schmitz, 2013). The logistical challenges associated with transporting large steel towers, particularly due to the limitations of road transport, have made concrete towers a viable alternative (Schmitz, 2013; Van Zyl & Van Zijl, 2015; Von Der Haar & Marx, 2015; Magar, 2017; Camargo et al., 2019). One advantage of replacing steel towers with prestressed concrete is that the increased mass of the structure provides greater stability against overturning and helps control the tower's natural frequency (Lana et al., 2021). Concrete towers can be either cast on-site or precast, and may incorporate either reinforced or prestressed concrete. Details of the construction processes for concrete towers are presented by Von Der Haar and Marx (2015).

Resonance is a fundamental problem for tall structures (Ercan et al., 2018). As the generator at the top

produces variable and continuous vibrations, the dynamic properties of the tower become more important than in other structures due to the resonance effect (Ercan et al., 2018). The main challenge in tower design is avoiding resonance, as a resonant response can significantly damage the structure and lead to failure conditions in the turbine (Huang et al., 2022). Every structure has one or more natural frequencies, which are related to its mass and elasticity (Camargo et al., 2019). When designing wind towers, it is essential not only to consider the requirements for limit states but also to ensure that the structure's natural frequency does not coincide with the operational frequency ranges. These operational frequencies include the rotor operating frequency and the blade passing frequency, referred to as 1P and 3P, respectively. The 1P and 3P frequencies are not single frequencies, but rather ranges of frequencies (Van Zyl & Van Zijl 2015). Based on experience with existing turbines, it is recommended to maintain a safety margin of at least 10% from the dominant excitation frequency (Hau, 2013). Other operating frequencies resulting from transient loading, such as startup conditions, are insignificant because they are applied only for a short period of time that will not cause resonance (Lofty, 2012). Tomczack (2021) describes three methods for designing a tower with frequencies outside the 1P and 3P ranges: (1) designing the tower with a first natural frequency greater than 3P (stiff-stiff structure), (2) designing the structure with a natural frequency between the 1P and 3P ranges (soft-stiff structure), or (3) designing the structure with a natural frequency below 1P (soft-soft structure). Current tower heights generally permit only a soft-stiff design, where the natural frequencies are balanced between the 1P and 3P excitation ranges (Harte & Van Zijl, 2007). A soft-stiff tower design can be achieved for all towers (LaNier, 2005). Careful attention to dynamic properties is essential to avoid the need to design hybrid and concrete 5 MW towers in seismic regions as stiff-stiff towers, which have a natural frequency higher than the 3P range (LaNier, 2005). LaNier (2005) defines the interval between the 1P and 3P bands as the working frequency range.

Despite the increasing use of concrete wind turbine towers, there is still a lack of scientific studies that systematically address the different stages of their structural design, particularly with regard to the evaluation of natural frequency and the parameters that influence it. Silva et al. (2008) present results illustrating how the tower's top diameter and conicity influence the natural frequency of an 82.5-meter-high reinforced concrete wind tower supporting a 2 MW turbine. Borges and Lameiras (2022) analyzed a 100 m prestressed concrete tower designed for a 5 MW turbine. They first examined how different aggregates, which modify the modulus of elasticity of 50 MPa concrete, influence the tower's natural frequency. They then analyzed 33 geometries using 50 MPa concrete with basalt aggregate to examine the effect of wall-thickness variation. The natural frequency was determined using a program based on the Finite Element Method utilizing three-dimensional beam elements (Borges & Lameiras, 2022). Zhang et al. (2024) studied the effects of thickness and mass at the top of the tower on the natural frequency of 120 m concrete wind turbine towers with a 3 MW generator. The authors employed a reduced-degree-of-freedom model to simplify the modal analysis and to calculate the theoretical natural frequency values. Huang et al. (2022) conducted a sensitivity analysis to

investigate the correlation between the geometry and natural frequency of a hybrid tower composed of concrete lower sections (class C50) and steel upper sections, with a turbine capacity between 3 and 4 MW. The study examined tower heights of 100, 120, 140, or 160 m. For each height, Latin hypercube sampling was used to obtain 5,000 groups of geometric design parameters (Huang et al., 2022). They fixed the upper sections of the concrete portion, the adapter ring, and the steel portion in the sampling to avoid a tapered profile with larger top sections and smaller bottom sections.

Concrete compressive strength is a fundamental parameter in the design of concrete structures. The compressive strength test on cylindrical specimens is the most widely used method for characterizing concrete in its hardened state. Despite its central role in structural design, its relevance to the dynamic behavior of concrete wind towers has received limited attention in the literature. Existing studies addressing this topic are scarce and often restricted in scope. For instance, Kenna and Basu (2015) investigated the effect of concrete compressive strength (30, 48, or 70 MPa) on the natural frequency of a 100 m tower supporting a 5 MW turbine. However, their analysis was limited to a single tower geometry without variations in wall thickness. Typically, higher-strength concrete in structural elements allows for reduced cross-sectional dimensions and lower structural weight. For wind turbine towers, the thickness of the concrete segments must be selected and optimized based on stress distribution to achieve the required mechanical performance (Zhang et al., 2024). In addition to mechanical demands, the natural frequency must also be considered when defining segment thickness. Within this context, the objective of this study is to assess the structural relevance of concrete compressive strength and wall thickness as governing parameters of the first natural frequency of a 100 m prestressed concrete wind turbine tower supporting a 5 MW turbine, thereby providing insights to support the preliminary dynamic design of such structures. For each characteristic compressive strength, different thickness values were simulated, and it was checked whether the tower with these thicknesses was within the turbine's working frequency range.

## 2. Modal Analyses

### 2.1 Reference Wind Turbine Tower

For the modal analyses conducted in this study, the full-height onshore post-tensioned concrete wind tower designed by LaNier (2005) was adopted as the reference model. The tower has a total height of 98 m, a hub height of 100 m, and supports a 5 MW wind turbine. The reference values for the tower height, base external diameter, and top external diameter were set to 100 m, 7.62 m, and 3.658 m, respectively. The wall thicknesses at the base and top of the tower were 76.2 cm and 45.7 cm, respectively. LaNier (2005) also defines the external diameter and wall thickness at mid-height of the tower as 5.639 m and 68.6 cm, respectively. Thus, the tower has two inclination angles: one from the base to mid-height, and another from mid-height to the top. The tower design is described in detail by LaNier (2005). The concrete used in the tower design had a compressive strength of 48 MPa. Some characteristics of the 5 MW turbine are shown in Table 1. For this turbine, towers with a first natural

frequency between 0.205 Hz and 0.485 Hz are considered to be dynamically safe.

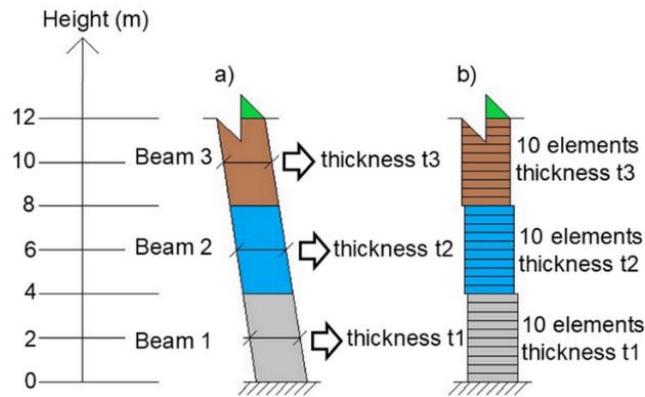
**Table 1. Main Specifications of the 5 MW Turbine**

Rotor diameter (m)	128
Total head weight (kg)	480076
1P frequency range – passing frequency (Hz)	0.187 to 0.205
3P frequency range – rotational frequency (Hz)	0.485 to 0.560
Working frequency range (Hz)	0.205 to 0.485

*Source:* Adapted from LaNier (2005).

### *2.2 Simulations – Finite Element Method*

A parametric study comprising sixty-four simulations was conducted to evaluate the influence of wall thickness and the characteristic compressive strength of concrete on the natural frequency of wind towers. Numerical analyses were performed using the DIANA 10.4 software, based on the Finite Element Method. The towers were considered to have a truncated cone shape, with the outer diameter and thickness decreasing linearly from the base to the top. The tower was divided into 4-meter-high beams, each with 10 elements, as shown in Figure 1. Gama (2015) employed a similar finite-element strategy in his analyses of prestressed concrete wind towers. The methodology adopted in this study is similar to that described by Borges and Lameiras (2022). The geometry of each beam (comprising 10 elements) was assumed constant and modeled as a predefined circular tube characterized by its outer diameter and wall thickness. For each beam, the diameter and thickness were defined as the average of the values of the two end nodes. A comparable discretization approach, dividing the tower into sections of constant diameter, was also used by Schaumann et al. (2011) to determine the natural frequencies of tubular steel towers using empirical formulas. A three-dimensional representation of the tower is shown in Figure 2.



**Figure 1. Discretization of the Tower for Natural Frequency Analysis: a) Division of the Tower into Beam Segments; b) Finite-Element Model in Which each Beam Contains 10 Elements with Constant outer Diameter and Wall Thickness**



**Figure 2. Three-dimensional Representation of the Tower**

Class-1 three-dimensional beam elements were employed. These are straight two-node elements based on Bernoulli's theory (Ferreira & Manie, 2020). In such elements, shear deformation and relative axial deformation are neglected (Chai, 2020). Each beam element was modeled as a circular pipe shape section, defined by its outer diameter and wall thickness. As a boundary condition, the base of the tower was considered fixed. To determine the first natural frequency, concrete was assigned to the entire cross-section. The concrete was defined as linear elastic isotropic material, characterized by its modulus of elasticity, Poisson's ratio, and mass density. A Poisson's ratio of 0.2 and a mass density of 2500 kg/m<sup>3</sup> were adopted. The secant modulus of elasticity specified in ABNT NBR 6118: 2023 (ABNT, 2023) was used. Different concrete strength classes were considered, and for each

characteristic compressive strength value, Equations 1 to 4 were applied to determine the corresponding secant modulus of elasticity. Concrete made with granite aggregate was considered. The initial modulus of elasticity of concrete (MPa) is calculated using Equation 3 for concrete with a characteristic compressive strength up to 50 MPa, and using Equation 4 for strengths exceeding 50 MPa. The secant modulus of elasticity should be used in elastic analyses and service limit state assessments (Cholfe & Bonilha, 2018). The natural frequency should be calculated using the secant modulus of elasticity of concrete, because the excitation is at the level of frequent actions (Grünberg & Göhlmann 2013).

$$E_{cs} = \alpha_i E_{ci} \tag{1}$$

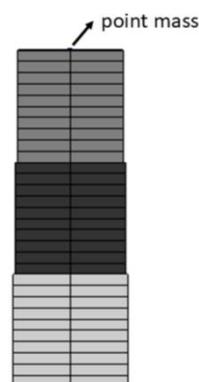
$$\alpha_i = 0.8 + 0.2 \frac{f_{ck}}{80} \leq 1.0 \tag{2}$$

$$E_{ci} = \alpha_E 5600 \sqrt{f_{ck}} \tag{3}$$

$$E_{ci} = 21.5 \times 10^3 \alpha_E \left( \frac{f_{ck}}{10} + 1.25 \right)^{1/3} \tag{4}$$

where  $E_{cs}$  is the secant modulus of elasticity, MPa;  $\alpha_i$  is a coefficient given by Equation 2;  $E_{ci}$  is the initial modulus of elasticity, MPa;  $\alpha_E$  is a coefficient depending on the mineralogical origin of the aggregate (1.0 for granite);  $f_{ck}$  is the characteristic strength of concrete, MPa.

The turbine mass, with 480,076 kg, was modeled as a concentrated mass at the top of the wind tower. A point mass was used for this purpose. These elements can introduce mass or damping into the model without affecting its stiffness (Ferreira & Manie, 2020). These elements do not have post-analysis results, such as stresses and strains, and are typically used to correct dead weight or to modify inertial mass in dynamic analyses (Ferreira & Manie, 2020). A representation of the upper tower elements and the point mass is shown in Figure 3.



**Figure 3. Upper Tower Elements and Point Mass Representation**

### 2.3 Model Validation

The proposed model was applied to determine the first natural frequency of the tower proposed by LaNier (2005). LaNier (2005), in Appendix B, employed the SAP2000 software also based on the finite element method. In his analysis, concrete was modeled with a modulus of elasticity of 34,972 MPa, resulting in a first natural frequency of 0.4125 Hz for the tower with a fixed base. Applying the methodology described in Section 2.2, a natural frequency of 0.4127 Hz was obtained. The difference of only 0.05% between the two results indicates excellent agreement.

### 2.4 Parametric Study: Variation in Concrete Strength and Thicknesses

Two parametric studies were conducted to evaluate the influence of concrete compressive strength and tower thicknesses on the first natural frequency. Four types of concrete, with characteristic compressive strengths of 30, 40, 50, and 60 MPa, were evaluated for each tower geometry. Granite aggregate was considered in all cases, and the secant modulus of elasticity was defined as previously described. In the first study, the thicknesses of the tower base and top were varied while maintaining the base and top diameters proposed by LaNier (2005), at 762 cm and 365.8 cm, respectively. The analyzed cases are summarized in Table 2. In the second study, the top thickness was fixed at 45.7 cm, as defined in the design by LaNier (2005), while the base thickness was varied, as shown in Table 3. For each tower configuration, the first natural frequency was evaluated to determine whether it remained within the working range defined in Section 2.1. The prestressing steel strands are positioned at mid-thickness. Although the strands were not included in the modal analyses, their inclinations from the base to the top are also reported in Tables 2 and 3.

**Table 2. Study 1—Variation in Thicknesses at the Tower Base and top. The Original Thickness Values Proposed by LaNier (2005) Are Indicated in Bold**

Base thickness (cm)	Top thickness (cm)	Inclinations of steel strands	Tower volume (m <sup>3</sup> )	concrete
7.62	4.57	88.87°	109.95	
11.43	6.85	88.88°	163.99	
19.05	11.42	88.89°	270.36	
38.10	22.85	88.91°	525.91	
57.15	34.27	88.93°	766.49	
<b>76.2</b>	<b>45.7</b>	88.95°	992.25	
95.25	57.125	88.97°	1203.08	
114.30	68.55	89.00°	1399.02	
133.35	79.97	89.02°	1580.04	
152.40	91.40	89.04°	1746.21	

**Table 3. Study 2 — Variation in the Thickness of the Tower Base. The Original Thickness Values Proposed by LaNier (2005) Are Indicated in Bold**

Base thickness (cm)	Top thickness (cm)	Inclinations of steel strands	Tower concrete volume (m <sup>3</sup> )
57.15	<b>45.7</b>	88.90°	839.47
<b>76.2</b>	<b>45.7</b>	88.95°	992.25
95.25	<b>45.7</b>	89.01°	1139.22
114.30	<b>45.7</b>	89.06°	1275.01
133.35	<b>45.7</b>	89.12°	1404.99
152.40	<b>45.7</b>	89.17°	1527.33

**3. Results**

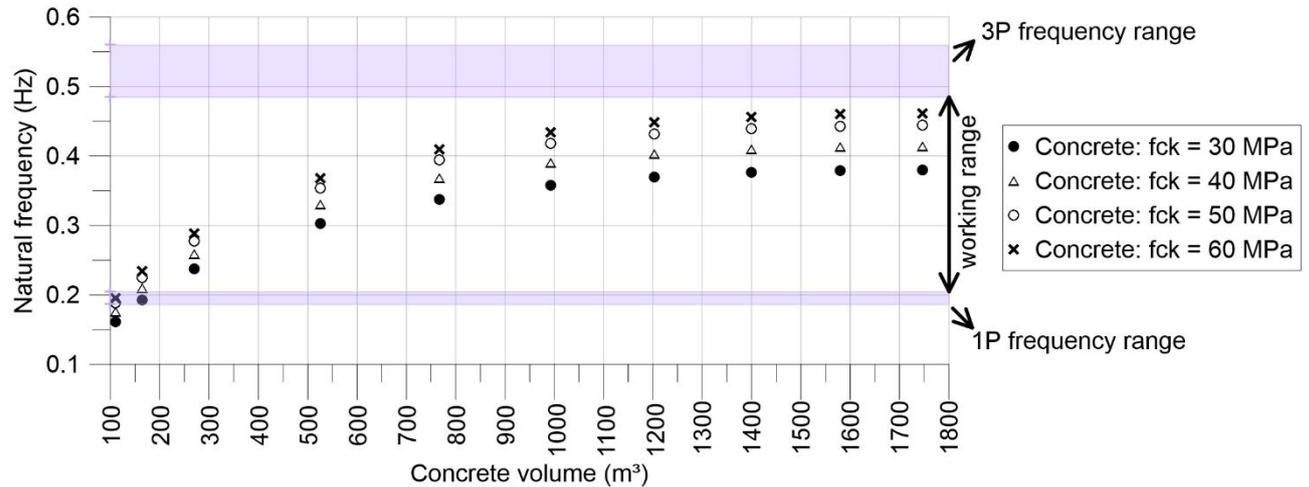
*3.1 Study 1 – Variation in Thicknesses at the Tower Base and Top*

Table 4 presents the natural frequencies for tower geometries with varying base and top thicknesses. The reported concrete volume corresponds to the gross concrete area; that is, the volumes occupied by active and passive reinforcement are not excluded. The table also indicates whether the natural frequency of each geometry falls within the working frequency range. Figure 4 illustrates the natural frequency as a function of tower concrete volume to facilitate the assessment of overall trends. The 1P, 3P, and working frequency ranges are highlighted in the figure.

**Table 4. Natural Frequency Results – Study of the Variation in Base and top Thicknesses**

Base thickness (cm)	% of the initial base thickness	Top thickness (cm)	% of the initial top thickness	Concrete volume (m <sup>3</sup> )	% of the initial concrete volume	Natural frequency (Hz)	% of the initial natural frequency	Verification: does it meet the frequency range?
Characteristic compressive strength of concrete: 30 MPa								
7.62	10	4.57	10	109.95	11.08	0.16105	45.02	no
11.43	15	6.85	15	163.99	16.53	0.19266	53.86	no
19.05	25	11.42	25	270.36	27.25	0.23773	66.45	yes
38.1	50	22.85	50	525.91	53.00	0.30289	84.67	yes
57.15	75	34.27	75	766.49	77.25	0.33763	94.38	yes
76.2	-	45.7	-	992.25	-	0.35773	-	yes
95.25	125	57.125	125	1203.08	121.25	0.36945	103.28	yes
114.3	150	68.55	150	1399.02	140.99	0.37595	105.09	yes
133.35	175	79.97	175	1580.04	159.24	0.37910	105.97	yes

152.4	200	91.4	200	1746.21	175.98	0.38001	106.23	yes
Characteristic compressive strength of concrete: 40 MPa								
7.62	10	4.57	10	109.95	11.08	0.17552	45.02	no
11.43	15	6.85	15	163.99	16.53	0.20996	53.86	yes
19.05	25	11.42	25	270.36	27.25	0.25908	66.45	yes
38.1	50	22.85	50	525.91	53.00	0.33009	84.67	yes
57.15	75	34.27	75	766.49	77.25	0.36796	94.38	yes
76.2	-	45.7	-	992.25	-	0.38986	-	yes
95.25	125	57.125	125	1203.08	121.25	0.40263	103.28	yes
114.3	150	68.55	150	1399.02	140.99	0.40972	105.09	yes
133.35	175	79.97	175	1580.04	159.24	0.41315	105.97	yes
152.4	200	91.4	200	1746.21	175.98	0.41415	106.23	yes
Characteristic compressive strength of concrete: 50 MPa								
7.62	10	4.57	10	109.95	11.08	0.18815	45.02	no
11.43	15	6.85	15	163.99	16.53	0.22507	53.86	yes
19.05	25	11.42	25	270.36	27.25	0.27772	66.45	yes
38.1	50	22.85	50	525.91	53.00	0.35384	84.67	yes
57.15	75	34.27	75	766.49	77.25	0.39443	94.38	yes
76.2	-	45.7	-	992.25	-	0.41791	-	yes
95.25	125	57.125	125	1203.08	121.25	0.43160	103.28	yes
114.3	150	68.55	150	1399.02	140.99	0.43920	105.09	yes
133.35	175	79.97	175	1580.04	159.24	0.44288	105.97	yes
152.4	200	91.4	200	1746.21	175.98	0.44394	106.23	yes
Characteristic compressive strength of concrete: 60 MPa								
7.62	10	4.57	10	109.95	11.08	0.19546	45.02	no
11.43	15	6.85	15	163.99	16.53	0.23382	53.86	yes
19.05	25	11.42	25	270.36	27.25	0.28852	66.45	yes
38.1	50	22.85	50	525.91	53.00	0.36760	84.67	yes
57.15	75	34.27	75	766.49	77.25	0.40976	94.38	yes
76.2	-	45.7	-	992.25	-	0.43416	-	yes
95.25	125	57.125	125	1203.08	121.25	0.44838	103.28	yes
114.3	150	68.55	150	1399.02	140.99	0.45628	105.09	yes
133.35	175	79.97	175	1580.04	159.24	0.46009	105.97	yes
152.4	200	91.4	200	1746.21	175.98	0.46120	106.23	yes



**Figure 4. Effect of Concrete Volume on the Natural Frequency Across Different Concrete Classes, Evaluated by Varying the Base and top Thicknesses of the Tower**

For a given characteristic compressive strength, increasing the wall thickness leads to higher natural frequencies, as the associated increase in concrete volume enhances tower stiffness. However, the increase in natural frequency is not proportional to the increase in thickness. Although the thickness and total concrete volume rise substantially, the corresponding increase in natural frequency becomes proportionally smaller. Figure 4 shows that, for each concrete strength, the initial data points increase sequentially and then tend toward stabilization. This behavior is also evident in Table 4, where the values in the last rows of the natural-frequency-percentage column are very similar. For some of the lower thickness values considered in this parametric study, the resulting natural frequencies fall outside the turbine’s working frequency range (0.205 Hz to 0.485 Hz). Such geometries would therefore not be suitable for practical design applications.

Table 4 contains a column presenting the natural frequency as a percentage of the reference value. For each concrete strength, the natural frequency of the tower with the base and top thicknesses defined by LaNier (2005) is used as the reference, and the corresponding percentages are calculated for all other tower geometries. The natural-frequency-percentage column is identical for all concrete strengths. This indicates that when the thickness is reduced or increased by the same proportion along the tower, the corresponding percentage change in natural frequency is independent of the concrete compressive strength. For example, if the tower thicknesses are reduced to 50% of their initial values along its full height, the natural frequency becomes 84.67% of the initial frequency, regardless of the concrete class (C30, C40, C50, or C60). This trend is also evident in Figure 4. The natural frequency versus concrete volume curves share the same shape, but their positions vary with concrete strength: the curve for lower-strength concrete is the lowest, while that for higher-strength concrete is the highest.

For identical thicknesses, an increase in the characteristic compressive strength of the concrete results in a higher natural frequency of the tower. Table 5 shows a comparison of natural frequencies for

different compressive strength classes. Concrete class C50 is used as the reference, and the percentage differences in natural frequency for the other classes are calculated. The strength of C30 concrete corresponds to 60% of the strength of C50, while the natural frequency of towers made with C30 concrete reaches 85.6% of that of towers made with C50 concrete for the same thicknesses. For C40 concrete, which has 80% of the strength of C50, the natural frequency corresponds to 93.3% of the value obtained for C50 concrete. The strength of C60 concrete corresponds to 120% of the strength of C50, and the natural frequency of towers made with C60 concrete reaches 103.9% of that of towers with C50 concrete for the same thicknesses. As concrete strength increases significantly, the corresponding percentage increase in natural frequency becomes smaller. This trend is also evident in Figure 4, where, for the same concrete volume, the point representing C60 concrete lies closer to that of C50 than do the lower-strength classes. It should be noted that a different equation is used to determine the modulus of elasticity for C60 concrete. Nevertheless, the comparison of C30 and C40 with C50 similarly shows that the percentage increase in natural frequency decreases as concrete strength increases.

**Table 5. Comparison of Natural Frequencies for Different Concrete Compressive Strength Classes Relative to C50 Concrete**

Base thickness (cm)	Top thickness (cm)	Concrete volume (m <sup>3</sup> )	% of natural frequency for C30 as a function of C50	% of natural frequency for C40 as a function of C50	% of natural frequency for C60 as a function of C50
7.62	4.57	109.95	85.6	93.3	103.9
11.43	6.85	163.99	85.6	93.3	103.9
19.05	11.42	270.36	85.6	93.3	103.9
38.1	22.85	525.91	85.6	93.3	103.9
57.15	34.27	766.49	85.6	93.3	103.9
76.2	45.7	992.25	85.6	93.3	103.9
95.25	57.125	1203.08	85.6	93.3	103.9
114.3	68.55	1399.02	85.6	93.3	103.9
133.35	79.97	1580.04	85.6	93.3	103.9
152.4	91.4	1746.21	85.6	93.3	103.9

Many studies have proposed the use of high-strength concrete in combination with reduced dimensions of structural elements. Particularly in the case of concrete wind turbine towers, a reduction in dimensions is advantageous with respect to the transportation of the structural components. Figure 4 and the results in Table 5 indicate that the concrete strength can be further increased by reducing its

thickness, while still maintaining the tower’s natural frequency within the working range for the 5 MW turbine considered in this study.

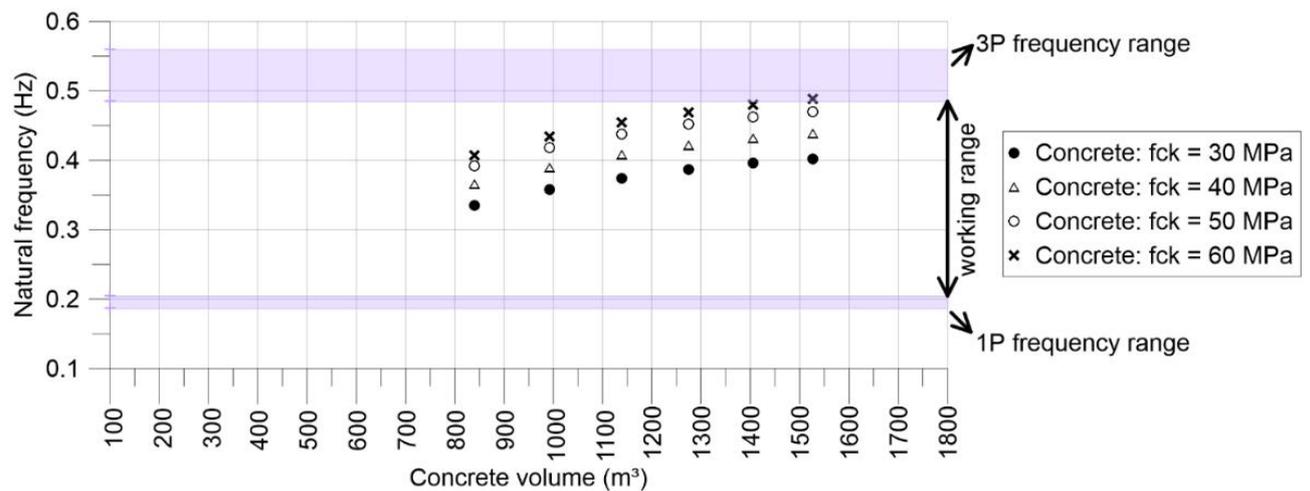
3.2 Study 2 – Variation in Base Thickness

In the previous parametric study, the same percentage variation in wall thickness was applied simultaneously to the base and the top of the tower. The thicknesses at the top reached excessively low or very high values in some cases. To refine the analysis, a new natural-frequency study was performed in which the top thickness was held constant while only the base thickness was varied. The wall thickness was assumed to vary linearly from the base to the top. Base thickness values ranging from 75% to 200% of the reference thickness proposed by LaNier (2005) were examined. The results are presented in Table 6 and Figure 5 for the same concrete strength classes adopted previously: 30 MPa, 40 MPa, 50 MPa, and 60 MPa.

**Table 6. Natural Frequency Results – Study of the Variation in Base Thickness**

Base thickness (cm)	% of the initial base thickness	Top thickness (cm)	% of the initial top thickness	Concrete volume (m <sup>3</sup> )	% of the initial concrete volume	Natural frequency (Hz)	% of the initial natural frequency	Verification: does it meet the frequency range?
Characteristic compressive strength of concrete: 30 MPa								
57.15	75	45.7	100	839.47	84.60	0.33532	93.74	yes
76.2	-	45.7	-	992.25	-	0.35773	-	yes
95.25	125	45.7	100	1139.22	114.81	0.37431	104.63	yes
114.3	150	45.7	100	1275.01	128.50	0.38669	108.10	yes
133.35	175	45.7	100	1404.99	141.60	0.39587	110.66	yes
152.4	200	45.7	100	1527.33	153.93	0.40255	112.53	yes
Characteristic compressive strength of concrete: 40 MPa								
57.15	75	45.7	100	839.47	84.60	0.36544	93.74	yes
76.2	-	45.7	-	992.25	-	0.38986	-	yes
95.25	125	45.7	100	1139.22	114.81	0.40793	104.63	yes
114.3	150	45.7	100	1275.01	128.50	0.42142	108.10	yes
133.35	175	45.7	100	1404.99	141.60	0.43143	110.66	yes
152.4	200	45.7	100	1527.33	153.93	0.43871	112.53	yes
Characteristic compressive strength of concrete: 50 MPa								
57.15	75	45.7	100	839.47	84.60	0.39173	93.74	yes
76.2	-	45.7	-	992.25	-	0.41791	-	yes
95.25	125	45.7	100	1139.22	114.81	0.43728	104.63	yes
114.3	150	45.7	100	1275.01	128.50	0.45174	108.10	yes

133.35	175	45.7	100	1404.99	141.60	0.46247	110.66	yes
152.4	200	45.7	100	1527.33	153.93	0.47027	112.53	yes
Characteristic compressive strength of concrete: 60 MPa								
57.15	75	45.7	100	839.47	84.60	0.40696	93.74	yes
76.2	-	45.7	-	992.25	-	0.43416	-	yes
95.25	125	45.7	100	1139.22	114.81	0.45428	104.63	yes
114.3	150	45.7	100	1275.01	128.50	0.46930	108.10	yes
133.35	175	45.7	100	1404.99	141.60	0.48044	110.66	yes
152.4	200	45.7	100	1527.33	153.93	0.48855	112.53	no



**Figure 5. Effect of Concrete Volume on the Natural Frequency Across Different Concrete Classes, Evaluated by Varying the Base Thickness of the Tower**

As in the previous study, increasing the base thickness leads to an increase in the tower’s natural frequency, although the increase becomes less pronounced as the thickness becomes larger. Again, the initial natural frequency percentage column is the same for all concrete strengths. Thus, the percentage change in natural frequency is independent of concrete strength. In Study 2, the same natural frequency percentages observed in Study 1 were obtained when comparing different concrete classes: 85.6% for C30, 93.3% for C40, and 103.9% for C60, all relative to towers made with C50 concrete.

Table 7 presents a comparison between the results of studies 1 and 2. In the first and third rows, which correspond to thickness values closest to those originally proposed by LaNier (2005), the difference between the concrete-volume percentages (sixth column) is relatively small. As the base thickness increases significantly, this difference also increased, reaching a maximum of 22.05% for a base thickness of 152.4 cm. In this case, the top thickness was 91.4 cm in study 1 and 45.7 cm in study 2. For cases with smaller differences in concrete-volume percentages, the difference in natural-frequency

percentages was also smaller. This relationship can be observed by comparing the sixth and ninth columns of Table 7. Finally, a comparison of the last points in the seventh and eighth columns, which represent the percentage of the initial natural frequency in studies 1 and 2, shows that the natural frequency values in study 1 exhibited a more pronounced tendency to stabilize. This trend is also evident in the comparison of Figures 4 and 5.

**Table 7. Comparison of Natural Frequency Percentages between Studies 1 and 2**

Base thickness (cm)	Top thickness (cm) study 1	Top thickness (cm) study 2	% of the initial concrete volume study 1	% of the initial concrete volume study 2	Difference between the two concrete volume percentage values		% of the initial natural frequency study 1	% of the initial natural frequency study 2	Difference between the two natural frequency percentage values	
76.2	34.27	45.7	77.25	84.60	7.35		94.38	93.74	0.64	
-	45.7	45.7	-	-	-		-	-	-	
95.25	57.125	45.7	121.25	114.81	6.44		103.28	104.63	1.35	
114.3	68.55	45.7	140.99	128.50	12.49		105.09	108.10	3.01	
133.35	79.97	45.7	159.24	141.70	17.54		105.97	110.66	4.69	
152.4	91.4	45.7	175.98	153.93	22.05		106.23	112.53	6.30	

**4. Conclusions**

Parametric studies were conducted to investigate the effects of concrete thickness and characteristic compressive strength on the first natural frequency of wind towers. The reference structure was the prestressed concrete wind tower proposed by LaNier (2005) for 5 MW turbines with a hub height of 100 m. Two separate studies examined variations in tower thickness: in the first, both the base and top thicknesses were varied, while in the second, only the base thickness was modified, with the top thickness kept constant. Both studies were performed for concrete classes C30, C40, C50, and C60.

For towers constructed from concrete of the same compressive strength, an increase in thickness led to a higher natural frequency, as the greater concrete volume increases the stiffness of the structure. However, as the thickness increased, the increase in natural frequency became less pronounced, indicating a tendency for this value to stabilize. Similarly, for towers of the same thickness, increasing the concrete strength also resulted in higher natural frequencies, although the effect became less pronounced at higher strengths. Several studies have suggested the use of high-strength concrete in combination with reduced dimensions of certain structural elements. From a design perspective, the present analysis of tower thickness variation suggests that it is possible to increase concrete strength

while reducing tower thickness, maintaining the natural frequency within the working range suitable for the 5 MW turbine considered in this study.

Overall, the study highlights that beyond certain geometric and material limits, further increases in thickness or concrete strength have limited effectiveness in adjusting the natural frequency, emphasizing the importance of identifying governing parameters and sensitivity regimes in the preliminary dynamic design of prestressed concrete wind towers.

As a continuation of this study, future research is recommended to extend the structural analyses by explicitly considering soil–structure interaction in concrete wind turbine towers, replacing the fixed-base assumption with a foundation model represented by spring systems. This approach allows a more realistic assessment of the influence of tower wall thickness and concrete strength on the natural frequencies when foundation stiffness is finite. In addition, the development of parametric models incorporating different foundation configurations and geotechnical parameters representative of various soil types is suggested, with particular attention to regions with a high concentration of onshore wind farms.

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