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Monetary and Fiscal Interactions: a Behavioral DSGE Model for Brazil

Brasília, DF, Brasil

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Aos meus pais, Maria Beatriz & Maurício.
Aos meus avós paternos, Iria & Antônio Mário.
Aos meus avós maternos, Carmem & Décio.
Aos meus irmãos, Maria Olívia e Gabriel.
À minha namorada, Carolina.
Amo vocês.

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"Essentially, all models are wrong, but some are useful."
George E.P. Box, British Statistician.

Abstract

This dissertation builds a behavioral dynamic stochastic general equilibrium - DSGE - model for the Brazilian economy to study monetary and fiscal policies interactions, under different policy regimes. Behavioral in this case means economic agents such as consumers and firms are rationally bounded or inattentive to future events. The model is inspired by Gabaix [2018], who constructed a micro founded behavioral New Keynesian model. In order to incorporate Brazil's economic policy, fiscal and monetary authorities are designed to follow the country's current rules of government primary budget surplus and inflation targeting as done in the SAMBA model by Castro et al. [2011]. A Bayesian estimation is then conducted to find proper parameters based on Brazilian historical data. In particular, Gabaix's cognitive discount factors for consumers and firms, M and M^f, respectively, are estimated for the country's economy for the first time under different model specifications and briefly compared with the author's results. In the main specification, M and M^f estimations are 0.7580 and 0.8649, respectively. The model then is set to simulate monetary and fiscal dominance situations by changing certain parameter values. Based on the model's variance results for product and inflation, which are used as proxies for social welfare, this dissertation's results also include: (i) under monetary dominance, the central bank should fiercely pursue its targets while the fiscal authority should consider its objectives carefully not to cause instability in the economy's product and prices; and (ii) under fiscal dominance, the central bank should accommodate for fiscal policy and public debt trajectory while it carefully chooses its responses to inflation, so that it does not cause higher inflation variances.
Key words: DSGE, behavioral, monetary, fiscal, dominance, Bayesian estimation.

Resumo

Esta dissertação constrói um modelo de equilíbrio geral estocástico - DSGE - comportamental para a economia brasileira com a intenção de estudar as interações entre políticas monetária e fiscal, sob diferentes regimes de política econômica. Comportamental nesse caso significa que agentes econômicos como consumidores e firmas são racionalmente limitados ou desatentos aos eventos futuros. O modelo é inspirado em Gabaix [2018], que construiu um modelo Novo Keynesiano comportamental micro fundamentado. Para incorporar a política econômica vigente no Brasil, as autoridades monetária e fiscal são desenhadas de modo a seguir as regras de superávit primário do governo e o regime de metas para a inflação conforme feito no modelo SAMBA elaborado por Castro et al. [2011]. Uma estimação Bayesiana é então conduzida para encontrar parâmetros com base em dados históricos brasileiros. Em particular, os fatores de desconto cognitivo propostos por Gabaix para consumidores e firmas, M e M^f, respectivamente, são estimados para a economia do país pela primeira vez sob diferentes especificações e brevemente comparados com os resultados do autor. Na especificação principal, os fatores M e M^f são estimados em 0.7580 e 0.8649, respectivamente. O modelo então é usado para simular situações de dominância monetária e fiscal ao mudar valores de alguns parâmetros. Baseado nos resultados do modelo para as variâncias do produto e da inflação, que são usados como *proxies* para o bem-estar social, as conclusões desta dissertação incluem: (i) sob dominância monetária, o banco central deve perseguir sua meta de inflação enquanto a autoridade fiscal deve considerar seus objetivos com cuidado para não causar instabilidade no produto da economia e nos preços; e (ii) sob dominância fiscal, o banco central deve acomodar a política fiscal e a trajetória da dívida pública enquanto escolhe com cuidado a intensidade de sua reação à inflação, para não causar maiores variabilidades na mesma.
Key words: DSGE, comportamental, monetária, fiscal, dominância, estimação Bayesiana.

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1 Introduction

Brazil's fiscal conditions have deteriorated continuously over the past five years. According to Treasury official data [2017], government expenditures have grown in average 5.4% p.a. in the last twenty years (from 1997 to 2017)[1]. Additionally, since 2014 the Brazilian government has experienced systemic primary budget deficits (-0.4% of GDP in 2014, -2% in 2015, -2.6% in 2016 and -1.9% in 2018). As a consequence, government gross debt[2] has increased from 52% pf GDP in 2013 to around 74% of GDP in the end of 2017. Worse, debt trajectory is ascending and there is no room for stabilization without politically burdensome fiscal reforms.

In this context, important questions surrounding the interaction between fiscal and monetary policies arise under an increasingly fiscally unbalanced environment. Sargent and Wallace [1981] were pioneers in exploring the theoretical consequences of an unbalanced budget in a monetarist economy, with no exciting results for monetary authorities desiring to fight inflation in such conditions. In particular, the authors conclude that sooner or later budget deficits cause the central bank to issue currency, thus generating inflation.

Woodford [2001] also studied the interactions between fiscal and monetary policies reaching similar conclusions, though with different causalities. According to the author, as it shall be exposed in the following chapter, monetary policy has fiscal effects and fiscal policy has monetary effects - and both of them cannot be ignored. Otherwise, the interaction between policies is not fully understood. Furthering his argument, Woodford shows a possible direct link between the price level and government debt. Hence, he concludes that fiscal policy determines the efficacy of monetary policy.

From these theoretical problems discussed by Sargent and Wallace and Woodford, under the fiscally unbalanced context being experienced by Brazil, this dissertation studies the interaction between fiscal and monetary policies through a dynamic stochastic general equilibrium (DSGE) model for Brazil. In particular, the DGSE model incorporates micro founded inattention behavior by consumers and firms. It also incorporates elements of Brazilian economic institutions, such as a rule for government primary budget surplus and an inflation targeting rule. The focus is on trying to understand how the policy interactions may affect product and inflation variances under monetary and fiscal dominance regimes.

There are several definitions for such regimes. In this dissertation, the dominant policy is determined independently by the responsible authority. Therefore, monetary

[1] In prices of 2017.
[2] In Portuguese: Dívida Bruta do Governo Geral (DBGG).

dominance implies the monetary authority sets its policy independently from the fiscal authority. On the other hand, fiscal dominance implies that the fiscal authority sets policy autonomously and that the monetary authority is left with the task of adjusting accordingly. In the next chapter some bibliography on authors that analyze monetary and fiscal dominance situations is reviewed. It becomes clear that they generally do not take into account behavioral aspects of economic agents, such as cognitive limitations. In particular, the notion of bounded rationality is not considered when studying the interaction between monetary and fiscal policies - not to say fiscal dominance contexts. Loyo [1999], for instance, studied fiscal dominance in Brazil by building a general equilibrium model for the 1970s and 1980s context without taking in consideration behavioral biases (e.g., cognitive limitations) of economic agents. His focus was to show that under such conditions the monetary authority generates more inflation when it tries to fight it.

To better understand how the interactions of fiscal and monetary policies can generate results in terms of economic performance and welfare, this dissertation takes into account psychological aspects for consumers and firms in the construction and estimation of a general equilibrium model for the Brazilian economy - a behavioral New Keynesian model, so to speak. In particular, it uses the developments of Gabaix [2018], who micro founded the notion of bounded rationality through inattention for economic agents in a dynamic stochastic general equilibrium model. In addition, this dissertation also simulates monetary and fiscal dominance situations and assesses the best policy options for a range of parameter values for the fiscal and monetary authorities, given the country's inflation targeting regime and its government primary budget surplus rule.

Apart from this brief introduction, this dissertation is organized as follows: chapter 2 reviews relevant literature on monetary and fiscal interactions, from the canonical Sargent and Wallace [1981] paper to the developments of Gabaix [2018]; the subsequent chapter specifies the general equilibrium model, followed by a chapter on its estimation, calibration and comparison with Gabaix's results; chapter 5 discusses the results and chapter 6 concludes. In the appendix one finds the transformed data, impulse response functions and the code used in the estimation procedures.

2 Literature Review

In this chapter, a review of the most relevant literature is conducted. The goal is to set the stage and the motivation for building the behavioral general equilibrium model and the subsequent analysis of monetary and fiscal interactions.

2.1 Sargent and Wallace's Contribution

Sargent and Wallace [1981] were pioneers in raising the plausibility of optimal monetary policy under the dominance of fiscal policy. The authors desired to demonstrate the limits of monetary policy even in a regime which satisfied monetarist assumptions and showed that under certain circumstances monetary policy is not effective against inflation.

They classify a monetarist economy as one in which the monetary base is directly connected to the price level and the monetary authority (e.g. central bank) has the power of seigniorage - that is, to issue money. The key to understand their reasoning stands in the fiscal and monetary policies coordination and in the demand for government bonds. The latter restraints government in two ways: (i) there is a limit for real public debt demand in relation to the size of the economy; and (ii) the size of public debt influences the amount of interest paid over it.

These limits affect the monetary authority's capacity in fighting inflation as the monetary and fiscal policy mix changes. In the case in which monetary policy dominates, that is, when the central bank sets its policy independently[1], it then determines how much revenue from seigniorage will be needed. The fiscal authority becomes constrained in financing itself with a mix of seigniorage (determined by the monetary authority) and public bonds (which itself is constrained by demand). In such a situation - which this dissertation too defines as monetary dominance - the central bank can successfully control inflation.

On the other hand, if the fiscal authority is independent to set its budget, by announcing its current and future deficits, then it is up to the central bank to accommodate the financing of these deficits. They will have to be financed by a certain mix of public debt issuance and seigniorage. In the case of public debt issuance, the monetary authority has a demand constraint, since economic agents will demand public bonds only up to a certain quantity relative to the size of the economy. If deficits are bigger than what can be financed by debt issuance, then the difference will have to be backed by seigniorage.

[1] The authors mention as an example the case in which the central bank determines the monetary base's expansion for the current period and all others.

The demand for public bonds will then determine whether the central bank can control inflation: if the interest rate is higher than GDP growth, then the monetary authority can still fight inflation by letting the real public debt stock increase; given that interest payments on this stock of debt is financed by issuance of more public debt, the real stock of public debt will increase faster than the economy (once interest rates are higher than GDP growth); since there is a limit to demand for public debt, eventually deficits will have to be financed by seigniorage. In a monetarist economy, this results in inflation.

Therefore, the authors show that under monetarist conditions, if there is no monetary dominance, inflation is the most likely result. This conclusion sets the stage and is part of the motivation for the model of chapter 3. Additionally, as it is shown in the sections below, one of the model's results is similar to this one: under fiscal dominance, the central bank is not likely to successfully fight inflation.

2.2 Leeper's Active and Passive Policies

Leeper [1991] also studies the interaction between fiscal and monetary policies through what he calls their passive and active behaviors. In his stochastic model, which serves as an inspiration for this dissertation's own definition of fiscal dominance in the model of chapter 3, the active authority does not worry about public debt trajectory and is free to determine its preferred controls. The passive authority thus becomes responsible for answering to public debt trajectory and shocks - being constrained by the actions of the active authority.

The author constructed models to understand the financing of public debt shocks with a focus on marginal revenue sources. Since the intertemporal budget constraint of government requires that shocks on real debt value cause changes in some future tax, the question comes down to whether the shocks will result in primary surplus or money creation. Two groups of models were created by the author.

In the first one, monetary policy is active and fiscal policy is passive. The monetary authority is responsible for keeping inflation under control, choosing its instrument independently, while the fiscal authority should guarantee public debt stability and that the government intertemporal budget constraint is respected. If one assumes a simple Taylor rule for monetary policy and a simple lump-sum tax rule for the fiscal authority, then inflation targeting via interest rates causes the fiscal authority to always adjust via lump-sum taxes. A fiscal shock, for instance, shall be totally accommodated by the fiscal authority.

In the second group, fiscal policy is active and monetary policy is passive. In this case, it is up to the monetary authority to guarantee the financing of public debt so that the intertemporal budget constraint of government is respected. Also, it is assumed the central bank holds interest rates constant, choosing as a control variable only the money supply. The fiscal authority is independent and does not respond to public debt trajectory in this case. Under such conditions, a negative fiscal shock[2] will be financed by increasing the money supply either today or in the future. It is a similar conclusion to that of Sargent and Wallace reviewed above, in which the inflation-fighting monetary authority is forced to raise interest rates today, keeping agents' demand for public bonds alive while it can. However, in Leeper's problem there is an interest-rate peg - rates are fixed. Then the monetary authority only is able to issue money, generating inflation.

In such a situation, a negative fiscal shock ended up generating inflation[3], since the central bank had to issue money to finance the fiscal authority. An underlying assumption is that, with interest rates fixed, more money in the economy means a higher price level. In the case of an interest rate hike, agents seek to acquire more public debt and less cash money, which makes debt service increase in the next period. Since there is no guarantee that there will be revenues enough for affording debt service in the next period, the monetary authority will have to accommodate by issuing money. This generates inflation. Hence, raising interest rates today increases inflation, under an active fiscal policy and passive monetary policy.

Though with a different rationale, Sargent and Wallace and Leeper arrive at similar conclusions: an independent fiscal policy will eventually generate inflation even if the monetary authority tries to control it.

Leeper's contribution serves as an inspiration for the model built in chapter 3. In this sense, the author's active and passive policies are denominated dominant and non-dominant policies in this dissertation - like Sargent and Wallace do. For instance, active monetary policy for Leeper means monetary dominance for this dissertation. The model also takes into consideration public debt in a similar way, making it the responsibility of the non-dominant policy - in Brazil's current institutional environment, the non-dominant policy is fiscal policy. When simulations of fiscal dominance are conducted, public debt trajectory becomes the responsibility of the monetary authority.

2.3 Woodford and the Fiscal Theory of the Price Level

Woodford [2001] argues that price stability requires not only good monetary policy rule but also a good fiscal policy rule. He says that from the 1990s onward, with the rise of inflation targeting regimes and the independence of central banks, it appeared to have become appropriate to think monetary policy decisions as separate from other government policies, such as fiscal policy. Such thinking is underpinned by two main

[2] For example, a shock that lowers government revenues.
[3] The stochastic models do not change the allocation of resources, only the price level.

hypotheses according to him: (i) that fiscal policy has no significant impact on inflation; and (ii) that monetary policy does not influence government's budget. Woodford affirms both hypotheses are false because they ignore the channel by which monetary policy can have fiscal effects - by changing the real value of government debt, the economy's price level, public bonds' values and the real service of public debt.

Additionally, they also ignore the fiscal policy effects on inflation[4] due to the common shared understanding of inflation as a monetary phenomena or due to the Ricardian equivalence. However, the author argues that fiscal policy affects inflation directly because of disturbances caused on the private sector's budget constraints and thus on aggregate demand. Such effects are neutralized in rational expectations models only if it is understood that the fiscal authority always adjusts its budget to the disturbances, in present value. If fiscal policy does not have this component, it is non-Ricardian.

As an implication of this line of reasoning, the monetary authority cannot ignore fiscal policy if its objective is to fight inflation. However, even in a context of monetary dominance, in which the central bank sets its policy independently, there is a possibility it cannot control inflation, depending on the fiscal policy regime.

This is a fiscalist approach to inflation, from which emerged what is known by the Fiscal Theory of the Price Level (FTPL). This theory argues that the price level is given by the government intertemporal budget constraint, in which prices work as the adjustment mechanism between the nominal public debt level and the expected values of government primary budget surpluses. Mathematically, one can write:

$$\frac{NPD}{PL} = PSEV \tag{2.1}$$

Where NPD = Nominal Public Debt, PL = Price Level, and PSEV = Primary Surpluses Expected Values. This means that any adjustments needed between nominal public debt and the primary surpluses expected values are given by the price level.

Cochrane [1999] is a strong advocate of the Fiscal Theory of the Price Level. The author argues that interest rates and monetary aggregates seem to have little to do with inflation. He places that the quantitative theory of money (QTM), which is mainly based on money demand for transactions, is not sufficient for explaining inflation in the U.S. post-war period - when an increasing number of transactions have been conducted electronically or via credit/debit cards. Additionally, several liquid-interest-bearing private financial instruments have been created - outside Federal Reserve control. The QTM is founded under the assumption of no monetary frictions[5] and its equation $MV = PQ$ states that transactions determine the price level.

4 Generally because developed countries such as the USA and the UK do not have the apparent objective of obtaining seigniorage with monetary policy.

5 For example: no monetary innovations nor liquidity constraints.

The FTPL equation above allows for monetary frictions, innovations and even an economy with no currency. Government debt is what changes the price level and thus causes inflation. So does the expected values of government primary budget surpluses. In this sense, the intertemporal government budget constraint backs the value of money which is apparently *fiat* in the same way the FTPL treats money and public debt as claims on the expected government primary budget surpluses. These sources of value are independent of financial structure or liquidity and are adjusted by the price level. Thus, Cochrane concludes that inflation is not caused by monetary aggregates nor can be fought via interest rates.

The FTPL inspired several authors to research on fiscal issues. It is certainly an inspiration for this dissertation, which treats fiscal and monetary interactions with the utmost importance. The model of chapter 3 relates interest rates, public debt and government primary budget surpluses in the public debt's law of motion. It certainly has inspirations in Woodford's contribution and partially in the FTPL - though following a different specification. The main message is the same: it is best to take monetary and fiscal policies together into account when trying to understand their effects on the economy.

2.4 Fiscal and Monetary Interactions in Brazil

Several authors have contributed to the Brazilian literature on monetary and fiscal interactions. This section reviews two of their studies, emphasizing the role of fiscal dominance contexts.

Inspired by the Fiscal Theory of the Price Level, Loyo [1999] built a dynamic stochastic general equilibrium model to identify evidences of fiscal dominance in Brazil during the 1980s. The author reasons that, in a fiscalist economy, prices are determined by the economic agent's public-debt-denominated wealth and not by liquid money. Government deficits, under non-Ricardian equivalence, add to the stock of wealth. Inflation then becomes a symptom of too much nominal wealth for too little goods and services. Therefore, a change in the price level - inflation - erodes the value of this wealth, bringing demand in line with supply again. In such a situation, inflation is to be understood as a fiscal phenomena.

Additionally, it is plausible for monetary policy to cause inflation in a fiscalist regime - just as fiscal policy can cause inflation in a monetarist regime. Loyo argues that monetary policy ends up determining the nominal growth of net private wealth, by changing interest rates and the interest-bearing portion of government liabilities. The *Tight Money Paradox* then emerges: given primary deficits, higher interest rates cause private wealth to increase more and, with it, inflation accelerates. Therefore, fiscalist hyperinflations can only be sustained if the monetary authority persistently raises interest rates to fight inflation, generating more of it. While this happens, budget deficit trajectory does not systemically affect inflation. He concludes that hyperinflations will always have higher interest rates causing higher inflation.

The Brazilian case of the 1980s can be explained by this logic. The country had a liquid market for public bonds, which makes hyperinflation more probable. In addition, with the introduction of the *Plano Cruzado*[6] the country changed its monetary policy significantly, with the monetary authority increasing its attention to fighting inflation. Loyo emphasizes that, in a non-fiscalist model, it would be difficult to clearly capture the reasons for the rising inflation of the 1970s and 1980s - mainly because such a result usually derives from seigniorage in traditional models. With his fiscalist model, the author concludes that Brazil in the 1970s and 1980s is a good example of a fiscal dominance context - as Woodford would say.

Blanchard [2005] also studied a Brazilian fiscal dominance situation. Unlike Loyo, who used a closed economy model, the author built an open-economy model to study the impacts of having an inflation targeting regime under an unstable macroeconomic context in which there is strong exchange rate depreciation.

It is common knowledge in the open-macroeconomics field that when a country raises its interests rates, its public bonds usually become more attractive and its currency often appreciates. However, there are contexts in which that may not occur. Blanchard argues that Brazil underwent such a context during the 2002 presidential race, when then candidate Luiz Inácio Lula da Silva was leading in the polls. Financial markets panicked with his chances of becoming president and the Brazilian currency started to depreciate when investors began sending money abroad. The Brazilian Central Bank (BCB[7]) initially raised interest rates to try to contain depreciation - and thus, its main objective, inflation. Brazil's public debt to GDP ratio was then (as is today) at a considered high level for emerging markets, and such an interest rate hike augmented the probability of an eventual default. As a consequence, public bonds became less attractive instead of more attractive after the interest rate hike. This resulted in further depreciation and inflation eventually increased. Realizing this phenomena, the BCB stopped raising interest rates even though inflation was rising and its objective was to keep it under control.

Blanchard's model justifies this type of decision making under adverse macroeconomic conditions. His modeling advocates that the proper remedy in such a context is fiscal, not monetary. Only fiscal policy could contain inflation in the Brazil of 2002-3. In the model he builds, the relationship between interest rates, exchange rates and default probability in high risk economies generate a result in which pursing an inflation target can end up causing more inflation - and thus worsen the real economy's situation. It

[6] An economic policy plan implemented in 1986 to fight hyperinflation.
[7] Banco Central do Brasil.

follows that fiscal adjustments are the best remedy, specially if the country has a high proportion of foreign currency-denominated debt and economic agents are highly risk averse.

Blanchard's model offers insights on fiscal dominance contexts. His logic certainly applies to a closed-economy model with debt denominated in its own currency - just as Loyo showed. The important lesson remains that one cannot ignore what is happening on the fiscal side when analyzing inflation. Fiscal policy is important on its determination, though the channels through which inflation is affected by fiscal policy are not always clear.

2.5 Behavioral Contributions

The reviewed literature so far has focused mainly on the interactions between fiscal and monetary policies and applications to the Brazilian economy, with an emphasis on fiscal dominance contexts. In this section, behavioral economics literature is briefly analyzed to underpin the motivation behind including human behavioral biases into the DSGE model of chapter 3.

Relevant contributions have been made to the behavioral economics literature over the past 3 decades. Daniel Kahneman [2003] presents a summary of his and Amos Tversky's main contributions in introducing psychological elements into economic decision making analysis, such as intuitive beliefs and bounded rationality[8]. In particular, the authors' research focused on obtaining a bounded rationality map to explore systemic biases that separate people's beliefs and choices from the optimal beliefs and decision making of fully rational agents. In particular, some of the results from their research are: (i) the utilization of heuristics, that is, rule of thumb decision making for individuals under uncertainty; (ii) Prospect Theory, which models choice under risk and loss aversion in choices without risk; and (iii) the framing effects and its consequences.

These contributions are relevant for economics. Rule of thumb decision making, for instance, is not usually fully compatible with intertemporal maximization (though it can be, depending on the modeling). Prospect theory, on its account, has interesting results for economic theory: one of its main conclusions is that economic agents are risk lovers when faced with a context of loss. That is, if the agent is "losing', she will choose to take risks to improve her condition. The famous result is that the utility curve is convex on the loss area[9]. Finally, framing effects have impacts on real decision making: the way contexts are presented matter for the final result.

8 Herbert Simon [1955] was the first to propose bounded rationality for decision makers in a model that substituted utility maximization for satisfaction.

9 However, it is not clear what the agents choose if the risk they take to improve their condition offers the chance of being worse off afterwards.

A classical example of behavioral bias found by Kahneman and Tversky is that fully rational individuals should be indifferent between having (a) USD 2,00 or (b) a 50% chance of USD 0,00 and 50% chance of USD 4,00. However, the authors encountered evidence that people often choose differently when faced with such options: a higher percentage of them generally goes for the certain choice.

Based on these contributions, Xavier Gabaix [2014] builds a model in which economic agents have bounded rationality. The author utilizes a similar reasoning as Kahneman, arguing that human beings have limited cognitive capacities, which cause economic agents to be rationally bounded - or inattentive. For instance, in the act of buying a good, a consumer generally considers only a few variables in her decision making process. Suppose the consumer is buying a bottle of wine, for instance. She will likely place higher attention on the price, quality and origin of the wine at first; she then places a lower attention weight on the impact the purchase has on her future income; finally, she most likely will not even consider an infinity of other variables such as future interest rates and the trade deficit with China. Rational expectations models usually take economic agents as fully rational, being able to process all that available information. This is not the case with inattentive agents, who are rational but not able to process all available information due to limited cognitive capacities[10].

It is important to emphasize Gabaix's model is not a total departure from rational expectations, though he argues that smoothing this microeconomics hypothesis of fully rational agents is doable and desirable, since it gives a more psychological realistic tone to economic theory. However, the way the author pursues this objective is through the rational expectations hypothesis itself.

By introducing the idea of "sparse" maximization (the sparse max operator), the author is able to write a behavioral version of the traditional microeconomics framework - consumer theory and competitive equilibrium, for example. The meaning of sparse, in this case, is the same of a sparse vector or matrix, full of zeroes. Mathematically, this makes the behavioral agent pay less attention to certain variables in comparison with the fully rational agent[11].

Gabaix argues "sparsity" encompasses many psychologically realistic features of life. It is able to incorporate limited attention and defaults (when people do not pay attention to a variable, they rely on defaults - just as Kahneman and Tversky found in their research). It is also able to keep the math tractable and the derivation follows a similar track to the one of the traditional framework.

[10] For further knowledge in the subject, see Gabaix [2019], in which the author extensively explains behavioral inattention.

[11] To understand this in a simple way is to think of the fully rational agent as having a vector full of ones (that is, she pays attention to all variables) while the sparse agent has many zero values in it (she does not pay attention to all variables).

On the latter, Gabaix [2014] argues consumer theory is developed on the basis of the following maximization problem:

$$\max_{c_1,...,c_n} u(c_1, ..., c_n) \tag{2.2}$$

subject to a budget constraint $p_1c_1 + ... + p_nc_n \leq w$, where $u(.)$ is the utility function, c_n is the consumption of good n, and p_n is the price of good n.

In the sparse maximization problem, the agent maximizes her utility the same way, like the fully rational agent. However, she does so based on her perceived prices for goods - and the agent does not pay full attention to all prices, because she has limited cognitive capacities to process information (even if it is fully available). The agent then faces an objective reality - which encompasses the way reality actually behaves - and a perceived reality, which encompasses the perception of reality by the inattentive agent. This is possible to capture by rewriting the problem above as:

$$s\max_{a} u(a, x) \tag{2.3}$$

subject to $b(a, x) \geq 0$, where $u(.)$ is the utility function, b is a constraint and s stands for sparsity. This maximization problem is less than the fully attentive version of the max operator, as the author puts it. Here one can notice that the departure from rational expectations is subtle. The agent stills maximizes intertemporally, though with her perception of prices. Perception prices, as mentioned above, might not be the same as the actual prices - indeed, they will often differ, since the agent does not pay attention to all prices. This makes the agent be partially imperceptive to her objective reality. As it shall be shown further, although the differences are small, the results will differ - sometimes substantially.

There are many behavioral principles condensed in the sparse max: inattention, disproportionate salience, and the use of defaults being among them. The way to represent sparsity by few parameters that are nonzero or differ from the usual state of affairs makes it possible to calibrate a behavioral vector m such that $m = 0$ represents zero attention while $m = 1$ represents a full rational agent, as Gabaix [2014] shows. Thus, rational expectations can be understood as a particular case of the sparsity model.

Additionally, sparse maximization underpins the micro foundations for the behavioral DSGE model built in the next chapter. It is used to derive macro parameters of inattention for the Euler equation and the Phillips curve, thus making consumers and firms behavioral in the DSGE model. As already mentioned, the model is then estimated via Bayesian methods and used to better understand monetary and fiscal policy interactions.

3 The Model

In this chapter the behavioral DSGE model is constructed inspired by the literature above. There are four players in this model: households, firms, monetary authority, and fiscal authority. The model is first derived in the traditional way, following the New Keynesian model presented in Gali [2008]. The behavioral transformation is subsequently applied, using the "sparse" max foundation laid in the previous chapter with the DSGE transformation given by Gabaix [2018].

Households act as consumers and receive their incomes from firms. They pay lump-sum taxes to government. They can also invest in one-period public bonds and receive interest in the next period. Firms utilize labor to make final goods, which will then be consumed by households and government. The monetary authority follows a rule for setting interest rates, and its parameters change depending on the policy regime (monetary or fiscal dominance). The government collects taxes and sells bonds, paying for its consumption and its debt. It follows a primary budget surplus rule, so that its final objective is to stabilize public debt trajectory.

3.1 Households

Households are intertemporal optimizers. Let us assume there is a continuum of households indexed by $j \in [0,1]$. Consumers have the same preferences and utility functions, which allows for the use of a representative household. This section follows closely on the developments of Gali [2008], with minor changes.

Let $C_{j,t}$ and $N_{j,t}$ represent consumption and supply of labor for the representative consumer j at period t. Utility is represented by $U(.)$. The agent seeks to maximize the following:

$$E_0 \sum_{t=0}^{\infty} \beta^t U(C_{j,t}, N_{j,t}) \tag{3.1}$$

subject to:

$$P_t C_{j,t} + R_t^{-1} B_{j,t} \leq B_{j,t-1} + W_t N_{j,t} + D_{j,t} + T_{j,t} \tag{3.2}$$

where $\beta \in (0,1)$ is the agent's discount factor, E_0 is the expectancy operator, $U(.)$ is the utility, P_t is the price level at period t, $B_{j,t}$ are the one-period public bonds held by agent j at period t, R_t is the nominal interest rate paid on public bonds at period t defined by the monetary authority, W_t represents wages received by households at period t, $D_{j,t}$ represents profits received by consumer j from firms at period t, and $T_{j,t}$ are lump-sum taxes on agent j at period t. In this case, agent j chooses $C_{j,t}$, $N_{j,t}$, and $B_{j,t}$ in order to maximize equation 3.1.

There is also the classic Non-Ponzi condition, which dictates that the representative agent j will not hold any bonds in the afterlife:

$$\lim_{t->\infty} E_t[B_{j,t}] \geq 0 \quad \forall \quad t \tag{3.3}$$

Following Gali [2008], C_t is a consumption index given by $C_t = (\int_0^1 C_t(i)^{1-\frac{1}{\epsilon}} di)^{\frac{\epsilon}{\epsilon-1}}$ where there exists a continuum of goods $i \in$ [0,1] and ϵ represents the demand elasticity. At every period, agent j is required to maximize $C_{j,t}$ for any expenditure level $\int_0^1 P_t(i)C_{j,t}(i)di$. Solving the problem yields $C_{j,t}(i) = [\frac{P_t(i)}{P_t}]^{-\epsilon} C_{j,t}\ \forall\ i, j \in$ [0,1]. Notice that since the model works with a representative agent it is possible to aggregate all of them as $C_t(i) = [\int_0^1 C_{j,t}(i)dj]$. Considering this aggregation, one arrives at $C_t(i) = [\frac{P_t(i)}{P_t}]^{-\epsilon} C_t\ \forall$ $i \in$ [0,1] where $P_t = [\int_0^1 P_t(i)^{1-\epsilon} di]^{\frac{1}{1-\epsilon}}$.

The utility function for representative agent j is given by:

$$U(C_{j,t}, N_{j,t}) = \frac{(C_{j,t})^{1-\sigma}}{1-\sigma} Z_t^C - \frac{(N_{j,t})^{1+\phi}}{1+\phi} \tag{3.4}$$

where Z_t^C is a stochastic shock in consumption, σ is the consumer's intertemporal elasticity of substitution and ϕ is the Frisch elasticity of labor supply.

The first order conditions (FOCs) for this problem are:

$$C_{j,t}: \quad \lambda_t = \frac{\beta^t (C_{j,t})^{-\sigma}}{P_t} Z_t^C \tag{3.5}$$

$$N_{j,t}: \quad \lambda_t = \frac{\beta^t (N_{j,t})^{\phi}}{W_t} \tag{3.6}$$

$$B_{j,t}: \quad 1 = \beta R_t E_t[\frac{(C_{j,t+1})^{-\sigma}}{(C_{j,t})^{-\sigma}} \frac{Z_{t+1}^C}{Z_t^C} \frac{P_t}{P_{t+1}}] \tag{3.7}$$

where λ_t is the Lagrangian multiplier at time t. Notice that substitutions are already made in equation 3.7 so that it gives the intertemporal substitution of consumption by the representative household - her Euler equation.

3.2 Firms

This section also closely follows Gali [2008]. Assume there is a continuum of firms indexed by $i \in$ [0,1] with the production function below:

$$Y_t(i) = A_t N_t(i)^{1-\alpha} \tag{3.8}$$

where technology is commonly shared and represented by A_t. Firms have the following demand schedule $C_t(i) = [\frac{P_t(i)}{P_t}]^{-\epsilon} C_t$ and a portion of them readjusts prices each period, according to Calvo [1983]. Firms are identical, which allows the model to work with a representative firm.

First, it is important to understand the price setting mechanism of firms. The way companies seek to maximize their current expected value in this model is via price adjustments. Following Calvo, since $1-\theta$ of firms re-optimize their prices each period, the aggregate price level is given by:

$$P_t = [\int_{S(t)} P_{t-1}(i)^{1-\epsilon} di + (1-\theta)(P_t^*)^{1-\epsilon}] \tag{3.9}$$

which can be rewritten as:

$$P_t = [\theta P_{t-1}(i)^{1-\epsilon} + (1-\theta)(P_t^*)^{1-\epsilon}] \tag{3.10}$$

where $S(t) \subset [0,1]$ is the set of firms which do not readjust their prices at period t. P_t^* is the reset price at t. Dividing both sides of the above equation by P_{t-1}:

$$\Pi_t^{1-\epsilon} = \theta + (1-\theta)[\frac{P_t^*}{P_{t-1}}]^{1-\epsilon} \tag{3.11}$$

where $\Pi_t = \frac{P_t}{P_{t-1}}$. Notice that in the zero-inflation steady state $P_t^* = P_{t-1} = P_t$. Equation 3.11 describes the inflation dynamics in the model.

Let us now turn attention to the firm's problem. As mentioned before, firms seek to maximize their current market value of expected profits by choosing its prices. Thus their problem can be expressed as:

$$\max_{P_t^*} \sum_{k=0}^{\infty} \theta^k E_t[Q_{t,t+k}(P_t^* Y_{t+k|t} - \Psi_{t+k}(Y_{t+k|t}))] \tag{3.12}$$

subject to

$$Y_{t+k|t} = [\frac{P_t^*}{P_{t+k}}]^{-\epsilon} C_{t+k} \tag{3.13}$$

for $k \geq 0$ and where $Q_{t,t+k} = \beta^k [\frac{C_{t+k}}{C_t}]^{-\sigma} [\frac{P_t}{P_{t+k}}]$ is the discount factor for nominal payoffs, $Y_{t+k|t}$ represents output in $t+k$ for a firm that reset its price at t, and $\Psi(.)$ is the cost function. Rewriting the problem it is possible to arrive at:

$$\max_{P_t^*} \sum_{k=0}^{\infty} \theta^k E_t[Q_{t,t+k}(P_t^* [\frac{P_t^*}{P_{t+k}}]^{-\epsilon} C_{t+k} - \Psi_{t+k}([\frac{P_t^*}{P_{t+k}}]^{-\epsilon} C_{t+k}))] \tag{3.14}$$

Taking the first order condition in relation to the reset price P_t^*:

$$\sum_{k=0}^{\infty} \theta^k E_t[Q_{t,t+k}[(1-\epsilon)(\frac{P_t^*}{P_{t+k}})^{-\epsilon} C_{t+k} - \Psi'_{t+k|t}(-\epsilon)(\frac{P_t^*}{P_{t+k}})^{-\epsilon} C_{t+k} \frac{1}{P_t^*}]] = 0 \tag{3.15}$$

It is possible to rewrite the above as:

$$\sum_{k=0}^{\infty} \theta^k E_t[Q_{t,t+k}[(1-\epsilon) Y_{t+k,t} + \Psi'_{t+k|t}(\epsilon) \frac{Y_{t+k,t}}{P_t^*}]] = 0$$

which yields:

$$\sum_{k=0}^{\infty} \theta^k E_t[Q_{t,t+k} Y_{t+k,t}[(1-\epsilon) + (\epsilon)\Psi'_{t+k|t} \frac{1}{P_t^*}]] = 0$$

Finally, by multiplying both sides by $\frac{-P_t^*}{\epsilon - 1}$ and making $\bar{M} = \frac{\epsilon}{\epsilon - a}$ it is possible to arrive at:

$$\sum_{k=0}^{\infty} \theta^k E_t[Q_{t,t+k} Y_{t+k,t}(P_t^* + \bar{M}\Psi'_{t+k|t})] = 0$$

where $\Psi'_{t+k|t} = \Psi'_{t+k}(Y_{t+k|t})$ is the nominal marginal cost at $t+k$ for a firm which reset its price at period t, $\bar{M}$ represents the desired mark-up by firms in the absence of frictions on the frequency of price adjustment. The above equation, as mentioned before, follows closely Gali [2008]. Let us do further transformations in it. Diving it by P_{t-1} and making $\Pi_{t,t+k} = P_{t+k}/P_t$ gives:

$$\sum_{k=0}^{\infty} \theta^k E_t[Q_{t,t+k} Y_{t+k,t}(\frac{P_t^*}{P_{t-1}} + \bar{M}\Pi_{t-1,t+k} MC_{t+k|t})] = 0$$

where $MC_{t+k|t} = \frac{\Psi'_{t+k|t}}{P_{t+k}}$ is the real marginal cost at $t+k$ of a firm which reset its price at t. Finally, it is possible to rewrite the equation above after some transformations as:

$$\frac{P_t^*}{P_{t-1}} = (1-\beta\theta) \sum_{k=0}^{\infty} \theta^k \beta^k E_t[\frac{MC_{t+k|t}}{\bar{M}} \frac{P_{t+k}}{P_{t-1}}] \tag{3.16}$$

The equation above rules how firms will readjust their prices. Together with equation 3.11, it is possible to arrive at the price dynamics for this economy.

3.3 Monetary Policy

Monetary policy follows a rule that is meant to describe the way it works in Brazil. Since 1999, Brazil has had an inflation targeting regime in which the central bank pursues a target for inflation over the course of a year. The target is defined two years before, to allow for forward-guidance. In this model, the monetary authority is allowed to try to stabilize public debt trajectory under fiscal dominance contexts. Under usual circumstances (i.e., monetary dominance), the central bank worries only about inflation and output volatility.

Following a similar to rule to that of Castro et al. [2011], and adding public debt to the rule as in Kumhof et al. [2007] and Furtado [2017], the central bank policy becomes:

$$R_t = R_{t-1}^{\Gamma_R}[\frac{1}{\beta} E_t[\Pi_{t+1}]^{\Gamma_\Pi} (\frac{Y_t}{Y})^{\Gamma_Y} (\frac{B_t^y}{B^y})^{-\Gamma_{B^y}}]^{1-\Gamma_R} Z_t^R \tag{3.17}$$

where R_t is the nominal interest rate set by the monetary authority, Π_t is the inflation rate at time t, Y_t is the economy's product at period t, Y is the economy's steady state product, B_t^y is public debt at time t as a portion of current product, B^y is the steady state public debt as a portion of the steady state product, $\Gamma_R \in (0,1)$ is an interest rate smoothing parameter, $\Gamma_\Pi \geq 0$ is the monetary authority's reaction to inflation, $\Gamma_Y \geq 0$ is the monetary authority's reaction to output gap, $\Gamma_{B^y} \geq 0$ is the monetary authority's reaction to public debt's deviation from the steady state, and Z_t^R is a monetary policy shock. In the log-linearized version of the model, it is assumed that the target for inflation is zero.

Following Kumhof et al. [2007], monetary dominance contexts are given by $\Gamma_\Pi \geq 1$ and $\Gamma_{B^y} = 0$ while fiscal dominance regimes are defined by $\Gamma_\Pi \geq 0$ and $\Gamma_{B^y} > 0$. In words, in a monetary dominance regime the central bank does not worry about the trajectory of public debt and fights inflation often following the Taylor Principle[1]. This resembles the rules established by Leeper [1991], according to which the passive authority is the one responsible for accommodating public debt. When a fiscal dominance context is established, the central bank incorporates public debt dynamics in his policy rule - taking into account the impact of interest rates in the country's debt to decide on its policy.

3.4 Fiscal Policy

Fiscal policy in Brazil has had a rule of annual targets for the non-financial public sector primary surplus as a proportion of GDP. According to Castro et al. [2011], the government's ultimate goal is to stabilize public sector debt to GDP ratio, from which one concludes the primary budget surplus is an intermediate target and the real objective is to stabilize debt trajectory. Fiscal policy modeling accompanies closely the one proposed by the authors of the SAMBA model, with minor changes.

Firstly, an equation is defined where the actual primary surplus responds to the announced targets:

$$S_t^y = \bar{S}^y + \phi_S(S_{t-1}^y - \bar{S}^y) + \phi_{\bar{S}}(\bar{S}_t^y - \bar{S}^y) \tag{3.18}$$

where ϕ_S belongs to the interval [0,1] and represents the inertia of the primary surplus as a proportion of GDP, $S_t^y = \frac{S_t^n}{P_t Y_t}$ is the actual primary budget surplus as a proportion of GDP (where y represents this proportionality), $\bar{S}^y$ is the primary surplus to GDP ratio at the steady state, $\bar{S}_t^y$ is the adjustable target for the primary surplus as a ratio of GDP, and $\phi_{\bar{S}} > 0$ represents the weight of the adjustable primary surplus target as a deviation from the steady state.

It is worth mentioning that the tax rate is exogenous in this model, which implies that the fiscal policy instrument is government spending. This implies that any deviations of the primary surplus from target are corrected via government consumption. The

[1] According to which the percentage change of interest rates set by the central bank in response to a percentage change in the inflation rate is higher than 1.

primary surplus target follows:

$$\bar{S_t^y} = \bar{S^y} + \rho_{\bar{S}}(\bar{S_{t-1}^y} - \bar{S^y}) + \phi_B(B_{t-1}^y - B^y) \tag{3.19}$$

where $\rho_{\bar{S}} \in [0,1]$ is the smoothing parameter for the primary surplus deviation from steady state, $B_t^y = \frac{B_t}{P_t Y_t}$ is government debt as a proportion of GDP, B^y is government debt as proportion of GDP at the steady state, and ϕ_B is the parameter that captures changes in the primary surplus target due to deviations of public debt from its steady state value.

As for government aggregate consumption, it is assumed that government demands the same variety of goods as households. Then for a continuum of goods indexed by $i \in [0,1]$:

$$G_t(i) = [\frac{P_t(i)}{P_t}]^{-\epsilon} G_t \tag{3.20}$$

where $G_t = (\int_0^1 G_t(i)^{1-\frac{1}{\epsilon}} di)^{\frac{\epsilon}{\epsilon-1}}$. This follows the same consumption logic as in the households' problem. It basically means government consumes goods the same way households do. This assumption simplifies the aggregation procedures conducted in the next section.

The nominal primary surplus is given by the difference between non-interest government revenues and expenses. It is assumed revenues are proportional to nominal output and an AR(1) stationary process is defined for the difference between the average tax rate and its steady state value. The rules follow (similar to Furtado [2017]):

$$S_t^n = \tau_t(P_t Y_t) - P_t G_t \tag{3.21}$$

$$\tau_t = \tau_{ss} + \rho_\tau(\tau_{t-1} - \tau_{ss}) + \varepsilon_t^T \tag{3.22}$$

where $\tau_t = \frac{T_t^n}{P_t Y_t}$ is the average tax rate, T_t^n represents nominal lump-sum revenues, G_t is real government expenditures (consumption), and τ_{ss} is the average tax rate steady state value. Rewriting 3.21 for G_t, it is possible to arrive at:

$$G_t = Y_t(\tau_t - S_t^y) \tag{3.23}$$

To complete the model, it is important to have in mind the law of motion of government debt. The government finances its expenditures by tax revenues and one-period non-contingent bonds issuance at the rate of return R_t. Those revenues are used both for consumption and for paying debt. Hence, the government budget constraint is given by:

$$P_t G_t + B_{t-1} = \frac{B_t}{R_t} + \tau_t(P_t Y_t) \tag{3.24}$$

And using current nominal GDP and 3.23 it is possible to arrive at the following law of motion:

$$B_t^y = R_t[\frac{B_{t-1}^y}{\Pi_t}\frac{Y_{t-1}}{Y_t} - S_t^y] \tag{3.25}$$

Let us notice that this last equation sheds light on the relationship between monetary and fiscal policies. This is the equation which links them in the model. Once the monetary authority decides to raise interest rates to fight inflation, for example, the fiscal authority will have to increase the primary surplus target, and as a consequence reduce government consumption. On chapter 2 this relationship of monetary and fiscal policies was reviewed in light of the literature.

Additionally, under monetary dominance one has $\phi_B > 0$ and $\phi_{\overline{S}} > 0$, which means that higher government debt will result in higher primary surplus targets and an increasingly higher actual primary surplus. In other words, fiscal policy seeks to stabilize public debt to GDP ratio. This is the final objective of the fiscal authority in this model, following the rules established in Brazil.

On the other hand, in a fiscal dominance context one should have $\phi_{\overline{S}} = 0$. In this theoretical case, the actual primary surplus equation 3.18 will not depend on a primary surplus target anymore. The central bank then tries to control inflation at the same time as it seeks to stabilize public debt trajectory, according to the established rule $\Gamma_{B^y} > 0$ for fiscal dominance regimes in the previous section. It is also assumed government spending is partially financed by public debt issuance when debt deviates from its steady state value.

3.5 Aggregation and Equilibrium

Households have the same utility and face the same budget constraint. Therefore, all of them arrive at the same solution for their control variables. Aggregation follows from the representative agent as:

$$\int_0^1 C_{j,t} dj = C_t \quad \int_0^1 N_{j,t} dj = N_t \quad \int_0^1 B_{j,t} dj = B_t \tag{3.26}$$

Where C_t is aggregate consumption, N_t represents labor, and B_t is aggregate demand for one-period government bonds. From the firm's problem, labor market clearing requires:

$$N_t = \int_0^1 N_t(i) di \tag{3.27}$$

Using the representative firm's production function it is possible to rewrite the above as:

$$N_t = \int_0^1 [\frac{Y_t(i)}{A_t}]^{\frac{1}{1-\alpha}} di \tag{3.28}$$

$$N_t = (\frac{Y_t(i)}{A_t})^{\frac{1}{1-\alpha}} \int_0^1 [\frac{P_t(i)}{P_t}]^{-\frac{\epsilon}{1-\alpha}} di \tag{3.29}$$

This aggregation is less obvious due to the term $\int_0^1 [\frac{P_t(i)}{P_t}]^{-\frac{\epsilon}{1-\alpha}} di$, which represents a measure of price dispersion across the economy. It is worth noting that the steady state price dispersion is close to 0, so that when a first-order Taylor expansion is conducted, price dispersion can be ignored at the steady state.

As mentioned in the previous section, government consumption is given by G_t - following the same pattern as household consumption for final goods. Additionally, goods market clearing requires:

$$Y_t = C_t + G_t \tag{3.30}$$

The economy's resource constraint henceforth is respected and all the resources used are either consumed or saved in the form of government bonds. As a consequence, all markets clear.

3.6 The Log-Linearized Model

From the households' first order conditions, one uses conditions 3.5 and 3.6, apply log on the resulting equation and on equation 3.7 to arrive at:

$$w_t - p_t = \sigma c_t + \phi n_t + z_t^C \tag{3.31}$$

$$c_t = E_t(c_{t+1}) - \frac{1}{\sigma} E_t[r_t - E_t(\pi_{t+1})] + \frac{1}{\sigma}[z_t^C - z_{t+1}^C] \tag{3.32}$$

where $R = \frac{1}{\beta}$ is the steady state value for R. Equation 3.32 is the Euler equation[2], which dictates the intertemporal substitution for consumption by the household.

The log-linearization of 3.11 gives the inflation dynamics in this economy:

$$\pi_t = (1 - \theta)(p_t^* - p_{t-1}) \tag{3.33}$$

Applying a first-order Taylor expansion around the zero inflation steady state on the firm's problem result equation 3.16 it is possible to arrive at:

$$p_t^* - p_{t-1} = (1 - \beta\theta) \sum_{k=0}^{\infty} (\theta\beta)^k E_t[\widehat{mc}_{t+k} + (p_{t+k} - p_{t-1})] \tag{3.34}$$

From the labor market equilibrium condition 3.29, one can take logs and arrive at:

$$y_t = a_t + (1 - \alpha)n_t - d_t \tag{3.35}$$

[2] Using the fact that $R_t = R + \hat{R}_t$, the derivation becomes:
$1 = \beta R_t E_t[\frac{(C_{j,t+1})^{-\sigma}}{(C_{j,t})^{-\sigma}} \frac{Z_{t+1}^C}{Z_t^C} \frac{P_t}{P_{t+1}}] = \beta R(1 + \frac{\hat{R}_t}{R}) E_t[\frac{(C_{j,t+1})^{-\sigma}}{(C_{j,t})^{-\sigma}} \frac{Z_{t+1}^C}{Z_t^C} \frac{P_t}{P_{t+1}}]$, applying log on both sides and using the convention $r_t \simeq log(1 + \frac{\hat{R}_t}{R})$ it is possible to arrive at:
$0 = log(1 + \frac{\hat{R}_t}{R}) - \sigma E_t(c_{t+1}) + \sigma c_t - E_t(\pi_{t+1}) + z_{t+1}^C - z_t^C \rightarrow c_t = E_t(c_{t+1}) - \frac{1}{\sigma} E_t[r_t - E_t(\pi_{t+1})] + \frac{1}{\sigma}[z_t^C - z_{t+1}^C]$.

as shown by Gali [2008], where $d_t = (1-\alpha)log[\int_0^1 (\frac{P_t(i)}{P_t})^{-\frac{\epsilon}{1-\alpha}}$ represents the price dispersion across firms. In the neighborhood of the zero-inflation steady state, following a first-order Taylor expansion, d_t equals 0 - as mentioned before. This allows to rewrite 3.35 as:

$$y_t = a_t + (1-\alpha)n_t \tag{3.36}$$

Let us remember the linear model proposed here expresses its variables as deviations from the steady state at 0 for all variables. Next, the model's Phillips curve is derived. First, as in Gali [2008], an expression for a firm's marginal cost is given by:

$$mc_t = (w_t - p_t) - mpn_t$$

Notice that mpn_t is the derivative of equation 3.8 so that $mpn_t = \frac{dY_t(i)}{dN_t(i)}$. Then one can rewrite the equation above as:

$$mc_t = (w_t - p_t) - (a_t - \alpha n_t) - log(1-\alpha)$$

which leads to:

$$mc_t = (w_t - p_t) - \frac{1}{1-\alpha}(a_t - \alpha y_t) - log(1-\alpha) \tag{3.37}$$

The above equation expresses the marginal cost as a function of real wages and marginal productivity. The following is a trivial consequence:

$$mc_{t+k|t} = (w_{t+k} - p_{t+k}) - \frac{1}{1-\alpha}(a_{t+k} - \alpha y_{t+k|t}) - log(1-\alpha) \tag{3.38}$$

After making substitutions in the above the result below emerges:

$$mc_{t+k|t} = m_{t+k} - \frac{\alpha}{1-\alpha}(y_{t+k|t} - y_{t+k}) \tag{3.39}$$

The log-linearization of the equilibrium condition in the goods market 3.30 is given by[3]:

$$y_t = s_C c_t + s_G g_t \tag{3.40}$$

where $s_C = \frac{C}{Y}$ and $s_G = \frac{G}{Y}$ represent the proportions of C and G relative to output Y at the steady state, respectively. It is assumed $s_C + s_G = 1$, since the model is designed for a closed economy. By applying logs on $C_t(i) = (\frac{P_t(i)}{P_t})^{-\epsilon} C_t$ and $G_t(i) = (\frac{P_t(i)}{P_t})^{-\epsilon} G_t$ the following equations are derived:

$$c_t(i) = -\epsilon(p_t - p_t(i)) + c_t$$
$$g_t(i) = -\epsilon(p_t - p_t(i) + g_t$$

And using the log-linearized equilibrium condition 3.40 above it is possible to arrive at:

$$s_C c_t(i) + s_G g_t(i) = s_G \epsilon(p_t - p_t(i)) + s_C \epsilon(p_t - p_t(i)) - s_C c_t - s_G g_t$$

[3] Following Castro et al. [2011] and adjusting for a closed economy with government.

which then can be rewritten as:

$$y_t(i) = \epsilon(p_t - p_t(i)) + y_t \implies y_t(i) - y_t = \epsilon(p_t - p_t(i))$$

The equation above implies:

$$y_{t+k|t} - y_{t+k} = \epsilon(p_t - p_{t+k}) \tag{3.41}$$

By replacing the above equation 3.41 into 3.39, an expression for the marginal cost at $t + k$ for a firm that last reset its price at t can be found as:

$$mc_{t+k|t} = mc_{t+k} + \frac{\epsilon\alpha}{1-\alpha}(p_t^* - p_{t+k}) \tag{3.42}$$

Equation 3.42 is then replaced into equation 3.34 which gives:

$$p_t^* - p_{t-1} = (1 - \beta\theta) \sum_{k=0}^{\infty} (\theta\beta)^k E_t[\Theta\widehat{mc_{t+k}} + (p_{t+k} - p_{t-1})]$$

Rearranging some terms yields:

$$p_t^* - p_{t-1} = (1 - \beta\theta)\Theta \sum_{k=0}^{\infty} (\theta\beta)^k E_t[\widehat{mc_{t+k}}] + \sum_{k=0}^{\infty} (\theta\beta)^k E_t[(p_{t+k} - p_{t-1})]$$

where $\Theta = \frac{1-\alpha}{1-\alpha+\epsilon\alpha} \leq 1$. Rewriting the above gives:

$$p_t^* - p_{t-1} = \beta\theta E_t[(p_{t+k} - p_{t-1})] + (1 - \beta\theta)\Theta\widehat{mc_t} + \pi_t \tag{3.43}$$

And combining 3.33 with 3.43 yields:

$$\frac{\pi_t}{1-\theta} - \pi_t = \beta\theta E_t[\frac{\pi_{t+1}}{1-\theta}] + (1 - \beta\theta)\Theta\widehat{mc_t}$$

where the Phillips Curve finally emerges as:

$$\pi_t = \beta E_t[\pi_{t+1}] + \lambda\widehat{mc_t} \tag{3.44}$$

with $\lambda = \frac{(1-\theta)(1-\beta\theta)}{\theta}\Theta$.

A relation coming from the consumer's linearized first-order condition 3.31 and the economy's marginal cost is derived next. The goal is to arrive at equations for the marginal cost and the economy's natural output. Let us use an expression already used:

$$mc_t = (w_t - p_t) - mpn_t \tag{3.45}$$

And using the fact that $w_t - p_t = \sigma c_t + \phi n_t + z_t^C$ yields:

$$mc_t = (\sigma c_t + \phi n_t + z_t^C) - (y_t - n_t) - log(1 - \alpha)$$

From the linearized goods market equilibrium condition, it is possible to rewrite the above as:

$$mc_t = [\frac{\sigma}{s_C} + \frac{1+\phi}{1-\alpha} - 1]y_t - \frac{1+\phi}{1-\alpha}a_t + \frac{s_G}{s_C}\sigma g_t - log(1-\alpha) + z_t^C \quad (3.46)$$

Notice that a greater a_t diminishes the current real marginal cost of the economy, while greater g_t increases it. Then, let us denote y_t^n as the equilibrium level of output under flexible prices so that:

$$mc = [\frac{\sigma}{s_C} + \frac{1+\phi}{1-\alpha} - 1]y_t^n - \frac{1+\phi}{1-\alpha}a_t + \frac{s_G}{s_C}\sigma g_t - log(1-\alpha) + z_t^C \quad (3.47)$$

From the above, the natural output y_t^n can be written as:

$$y_t^n = \psi_a a_t - \psi_g g_t + v_y^n \quad (3.48)$$

where $\psi_a = \frac{s_C(1+\phi)}{\sigma(1-\alpha)+s_C(1+\phi)-s_C(1-\alpha)}$, $\psi_g = \frac{s_G\sigma(1-\alpha)}{\sigma(1-\alpha)+s_C(1+\phi)-s_C(1-\alpha)}$, and $v_y^n = mc + log(1-\alpha) - z_t^C$.
Also, subtracting 3.47 from 3.46 yields:

$$\widehat{mc_t} = [\frac{\sigma}{s_C} + \frac{1+\phi}{1-\alpha} - 1](y_t - y_t^n) \quad (3.49)$$

which is needed to complete the Phillips Curve derivation in terms of output gap $(y_t - y_t^n)$ and where $\widehat{mc_t} = mc_t - mc$.

Next, the monetary and fiscal policies are linearized. Following Castro et al. [2011] with the debt term added the linearized monetary policy rule is given by:

$$r_t = \Gamma_R * r_{t-1} + (1-\Gamma_R)[\Gamma_\Pi \pi_t + \Gamma_y * (y_t - y_t^n) - \Gamma_{B^y} * b_t^y] + z_t^R \quad (3.50)$$

Finally, similar to Castro et al. [2011] again, the government debt law of motion, the primary surplus, and the primary surplus target rules are the following:

$$b_t^y = r_t + R(b_{t-1}^y - \pi_t + y_{t-1} - y_t) - (R-1)s_t^y \quad (3.51)$$

$$s_t^y = \phi_S s_{t-1}^y + \phi_{\bar{S}} \bar{s_t^y} \quad (3.52)$$

$$\bar{s_t^y} = \rho_{\bar{S}} \bar{s_{t-1}^y} + \phi_B \frac{R}{R-1} b_{t-1}^y \quad (3.53)$$

where government consumption follows:

$$g_t = y_t + \frac{1}{s_G}(\tau_{ss}\tau_t - S^Y s_t^y) \quad (3.54)$$

And the lump-sum tax process is AR(1) given by:

$$\tau = \rho_\tau \tau_{t-1} + \frac{1}{\tau_{ss}}\varepsilon_t^T \quad (3.55)$$

Additional to the shock above, the remaining shocks are also AR(1) processes such that:

$$a_t = \rho_a a_{t-1} + \varepsilon_a \tag{3.56}$$

$$\nu_t = \rho_\nu \nu_{t-1} + \varepsilon_\nu \tag{3.57}$$

$$z_t^C = \rho_C z_{t-1}^C + \varepsilon_{z_C} \tag{3.58}$$

The above equations summarize the linear model. All variables are treated as deviations from the steady state, which is zero for all variables. The model is coded and run in MATLAB, using Dynare.

3.6.1 Behavioral Transformation

In this section a behavioral transformation for the Euler equation and Phillips curve is conducted to add inattention (bounded rationality) to the model. Notice that the model derived so far takes into account rational expectations and it is fully traditional, with Brazil's rules for monetary and fiscal policies.

These behavioral biases are added based on the "sparsity" model cited in the last chapter. As mentioned, the sparse max operator makes it possible to included bounded rationality and other behavioral human characteristics into the model. It is also the way Gabaix [2018] includes inattention in his behavioral New Keynesian model.

The author follows a similar pattern to the model derived in the section above, but notation is different. In this section, his notation is used - similarities with last section's notation should be ignored. The goal is arrive at the important lemmas used to transform the Euler and Phillips curves into behavioral equations, which take into account inattention by the behavioral agents (consumers and firms in this case). In the author's framework, agents face an objective reality similar to the rational expectations agent and a subjective reality - which represents the way economic agents perceive reality. The introduction of the inattention parameter occur in the latter reality. Below, Gabaix's derivation is reviewed followed by the transformation of the Euler and Phillips curves from last section.

Households. Gabaix uses a representative agent. She faces the following objective reality:

$$U = E \sum_{t=0}^{\infty} \beta^t u(c_t, N_t) \tag{3.59}$$

where c_t represents her consumption at time t and N_t her labor supply at time t. The agent's real wage is w_t, the interest rate is r_t, her income is given by $y_t = w_t N_t + y_t^f$, in which y_t^f is income from firms. The agent's financial wealth is k_t and evolves according to the following law of motion:

$$k_{t+1} = (1 + r_t)(k_t - c_t + y_t) \tag{3.60}$$

The agent's objective problem consists in $max_{(c_t,N_t)}U$ subject to 3.60 and to the transversality condition $\lim_{t\to\infty} \beta^t c_t^{-\gamma} k_t = 0$. The economy's aggregate production function is given by $c_t = e^{\zeta_t} N_t$ where ζ_t follows an AR(1) process with mean 0.
There exists a vector state X_t for all the agent's relevant variables[4] which evolves in equilibrium as:

$$X_{t+1} = G^X(X_t, \epsilon_{t+1}) \tag{3.61}$$

where G^X is a transition function, with mean 0 and ϵ_{t+1} innovations. This vector rules how the agent's objective reality in fact evolves. Thus, it is possible to rewrite 3.60 as:

$$k_{t+1} = G^k(c_t, N_t, k_t, X_t) = (1 + r(X_t))(k_t + y(N_t, X_t) - c_t) \tag{3.62}$$

Now, the agent's maximization problem becomes $max_{(c_t,N_t)}U$ subject to 3.61 and 3.62.
The agent faces a subjective reality just as in the "sparsity" model mentioned in the last chapter. This reality, as already mentioned, represents the one perceived by the inattentive rational agent. The latter does not necessarily pay attention to all relevant variables. This representation is captured in the micro foundation of the sparse vector. In the case of the behavioral New Keynesian model the vector state evolves according to the following assumption:
Assumption 1 (Perceived Reality by Households) X_t evolves in equilibrium as:

$$X_{t+1} = \bar{m}G^X(X_t, \epsilon_{t+1}) \tag{3.63}$$

where $\bar{m} \in [0,1]$ is a cognitive discount factor for the representative agent. This discount factor basically represents how much the agent pays attention to her objective reality. The behavioral consumer is not usually able to process all the information she has access to - even if information was perfectly symmetric - and this is why the discount factor is often less than 1. If $\bar{m} = 1$ then perceived reality is equal to the objective reality, thus resembling the fully rational agent. As a consequence, the rational expectations case is a particular result of this model. The cognitive discount factor also summarizes the presented literature on bounded rationality and inattention in the Literature Review chapter, incorporating aspects of human psychology.
The household's problem then becomes $max_{(c_t,N_t)}U$ subject to 3.62 and 3.63. In order to interpret $\bar{m}$, it is useful to linearize 3.63 as:

$$X_{t+1} = \bar{m}G^X(\Gamma X_t + \epsilon_{t+1}) \tag{3.64}$$

4 Among them: productivity ζ_t and government policies, for example.

as shown by Gabaix. Thus, applying the expectation operator for the behavioral agent gives $E_t^{BR}[X_{t+1}] = \bar{m}\Gamma X_t \implies E_t^{BR}[X_{t+k}] = \bar{m}^k\Gamma^k X_t$ (after iterating forward), where E_t^{BR} is the expectation for the rationally bounded agent[5]. Since the rational expectations expectation iteration results in $E_t[X_{t+k}] = \Gamma^k X_t$, the author arrives at:

$$E_t^{BR}[X_{t+k}] = \bar{m}^k E_t[X_{t+k}] \tag{3.65}$$

From this result above, Lemma 2.2 can be written as:
Lemma 2.2 (Cognitive Discounting of all Variables) For any variable $z(\mathbf{X}_t)$ with $z(0) = 0$ the beliefs of the behavioral agent satisfies, for all k:

$$E_t^{BR}[z(\mathbf{X}_{t+k})] = \bar{m}^k E_t[z(\mathbf{X}_{t+k})] \tag{3.66}$$

where E_t^{BR} is the subjective expectation operator and E_t is the rational one. This makes it clear the behavioral agent (with $\bar{m} < 1$) does not discount intertemporally the same way the fully rational agent does. The former places less weight onto future events, mainly due to her inability to process information. The next step is to derive the behavioral Euler equation by applying this lemma. This is done in the next section.

Firms. Gabaix utilizes a representative firm which wishes to maximize its current value in a similar way Gali [2008] does. Firms also face an objective and a subjective reality. In the latter case, as happens with behavioral consumers, firms do not process all the available information relative to macro variables - such as the interest rates, GDP growth, marginal costs, among others.
Gabaix divides firms in a competitive final sector and a monopolistically competitive intermediate sector with pricing frictions a la Calvo. There is a Dixit-Stiglitz continuum of firms where firm i produces $Y_{it} = N_{it}e^{\zeta_t}$ and sets its price P_{it}, where Y_{it} is the production of firm i at period t, N_{it} is the labor employed by firm i at period t and ζ_t is the shared common technology at time t.
In the final sector, quantity produced is given by $Y_t = (\int_0^1 Y_i t^{(\varepsilon-1)/\varepsilon} di)^{\varepsilon/(\varepsilon-1)}$ and the price level is given by:

$$P_t = (\int_0^1 P_{it}^{\varepsilon-1} di)^{1/(1-\varepsilon)} \tag{3.67}$$

where ε is the demand elasticity and P_{it} is the price of firm i at time t. In the intermediate sector, a proportion $1-\theta$ of firms readjust their prices every period. Having this in mind, consider firm i and let $q_{i\tau} = ln\frac{P_{i\tau}}{P_\tau} = p_{i\tau} - p_\tau$ be the log of the real price in period τ. The firm's real profit is given by:

$$\upsilon_\tau = (\frac{P_{i\tau}}{P_\tau} - MC_\tau)(\frac{P_{i\tau}}{P_\tau})^{-\varepsilon} c_\tau \tag{3.68}$$

where $(\frac{P_{i\tau}}{P_\tau})^{-\varepsilon} c_\tau$ is total demand for the firm's products, c_τ is aggregate demand, $MC_\tau = (1-\tau f)\frac{w_t}{e^{\zeta_t}} = (1-\tau f)e^{-\mu_t}$ is the firm's real marginal cost and $-\mu_t = ln(w_t) - \zeta_t$ is the

[5] BR stands for bounded rationality.

social marginal cost. Then, Gabaix shows υ_τ can be rewritten as:

$$\upsilon(q_{i\tau}, \mu_\tau, c_\tau) = [e^{q_{i\tau}} - (1-\tau_f)e^{-\mu_\tau}]e^{-\varepsilon q_{i\tau}} c_\tau \tag{3.69}$$

Given these transformations, it is possible to incorporate a state vector in the profit function of the representative firm. Consider X_τ as this vector such as $X_\tau = (X_\tau^M, \Pi_\tau)$, where Π_τ is the inflation rate between period τ and t and X_τ^M a macro state vector for period τ, comprising variables such as factor productivity and policy announcements. If the firm does not reset its price between τ and t, its real price will be $q_{i\tau} = q_{it} - \Pi_\tau$ such that the profit equation can be rewritten as:

$$\upsilon^r(q_{it}, X_\tau) = \upsilon(q_{it} - \Pi(X_\tau), \mu(X_\tau), c(X_\tau)) \tag{3.70}$$

where $\Pi_\tau = \Pi(X_\tau)$ represents future aggregate inflation and the superscript r denotes full rationality. Firms desire to reset their prices $q_{i\tau}$ at t in order to maximize their profits, given by the equation below:

$$E_t \sum_{\tau=t}^{\infty} (\beta\theta)^{\tau-t} \frac{c(X_\tau)^{-\gamma}}{c(X_t)^{-\gamma}} \upsilon^r(q_{it}, X_\tau) \tag{3.71}$$

where $\frac{c(X_\tau)^{-\gamma}}{c(X_t)^{-\gamma}}$ is the adjustment in the stochastic discount factor between t and τ (approximately equal to 1, so that after the linearization procedure it is approximately 0).
Under the subjective reality, just like behavioral consumers, behavioral firms cannot process all the available information due to cognitive limitations. Thus, cognitive discount factors are also included in the firm's maximization problem. Firms then perceive their current and future profits, for $t \geq \tau$, as:

$$\upsilon^{BR}(q_{it}, X_\tau) = \upsilon(q_{it} - m_\pi^f \Pi(X_\tau), m_x^f \mu(X_\tau), c(X_\tau)) \tag{3.72}$$

where the superscript BR stands for bounded rationality, m_π^f and $m_x^f \in [0,1]$ are cognitive discount factors for inflation and costs, respectively. The fully rational case is given by $m_\pi^f = 1$ and $m_x^f = 1$, being a particular case again. For $m_\pi^f < 1$ and $m_x^f < 1$, the firm is rationally bounded or inattentive.
The behavioral firm then chooses $q_{i\tau}$ so that it maximizes its profits according to:

$$E_t^{BR} \sum_{\tau=t}^{\infty} (\beta\theta)^{\tau-t} \frac{c(X_\tau)^{-\gamma}}{c(X_t)^{-\gamma}} \upsilon^{BR}(q_{it}, X_\tau) \tag{3.73}$$

which follows the same logic as the rational firm with the perceived law of motion given in 3.63. The nominal price that a firm chooses is $p_t^* = q_{it} + p_t$. Its value is given by the lemma below, which is derived by Gabaix [2018]:
Lemma 2.4 (Optimal Price for a Behavioral Firm Resetting its Price) A behavioral firm resetting its price at time t will set it to a value p_t^* following the same logic as equation 3.34, though with cognitive discount factors as given below:

$$p_t^* - p_{t-1} = (1-\beta\theta) \sum_{k=0}^{\infty} (\theta\beta\bar{m})^k E_t[m_\pi^f(\pi_{t+1} + ... + \pi_{t+k}) - m_x^f \mu_{t+k}] \tag{3.74}$$

where m_π^f and m_x^f represent the cognitive discount factors to inflation and macro disturbances, respectively, and $\bar{m}$ is the overall cognitive discounting factor.

Next, the results above are used to derive the behavioral Euler equation and Phillips curve.

3.6.2 Behavioral Euler Equation and Phillips Curve

Gabaix's *Lemma 2.2* and *Lemma 2.4* are applied on equations 3.32 and 3.44, respectively. By doing this, it is possible to transform the log-linearized equations into behavioral equations, in which consumers and firms have bounded rationality.
Therefore, equation 3.32 transformed by *Lemma 2.2* results in:

$$c_t = ME_t[c_{t+1}] - \frac{1}{\sigma}[r_t - E_t[\pi_{t+1}]] + \frac{1}{\sigma}[z_t^C - z_{t+1}^C] \tag{3.75}$$

where $\bar{m}^k$ becomes M, a macro parameter of inattention to future consumption.
Equation 3.44 transformed by *Lemma 2.4* is given by:

$$\pi_t = M^f \beta E_t[\pi_{t+1}] + \lambda_m \hat{mc}_t \tag{3.76}$$

where M^f is the cognitive discounting factor for future prices - it is a function of m_π^f - and $\lambda_m = m_x^f \lambda$, where m_x^f is the cognitive discount factor for marginal costs. It is assumed $m_x^f = 1$, in order to simplify the model. Even so, bounded rationality is still very well captured by the M and M^f parameters.

The interesting lemma proofs can be found in Gabaix [2018]. It is relevant to emphasize that the rational expectations result is a particular case of this model, when $M = M^f = m_x^f = 1$. In the next sections, Bayesian estimations are conducted followed by a discussion on the results.

4 Calibration and Estimation

In this section a calibration of some structural parameters is conducted followed by an estimation of selected parameters. The model to be estimated is the log-linearized version of chapter 3 with the behavioral Euler and Phillips curves derived in the last section. The calibration takes into account similar values to the ones considered by Castro et al. [2011] and Gabaix [2018]. The estimation utilizes Bayesian methods, which allows for available information to be used in the calculation of the posterior means and distributions of the estimated parameters. In addition, a section is included in the end of this chapter with an estimation following the same model specification as in Gabaix [2018] for two different sets of data and the results are compared with those of the author.

4.1 Empirical Evidence

The model is estimated with Brazilian quarterly data from 2000Q1 to 2017Q4. There are four observable variables in the model (thus accounting for four exogenous shocks). Data was obtained in the official websites of the Instituto Brasileiro de Geografia e Estatística (IBGE) and of the Banco Central do Brasil (BCB).

Table 1 – Data Used in the Estimation.

Variable	Description	Source
Product (Y_t)	GDP - seasonally adjusted (s.a.) (1995 values)	IBGE
Government Consumption (G_t)	Government Spending - s.a. (1995 values)	IBGE
Inflation (Π_t)	Quarterly IPCA-E (12 months)	IBGE
Interest Rates (R_t)	Quarterly SELIC (12 months)	BCB

Data received a close treatment to the one used in Castro et al. [2011] for the SAMBA model. Quarterly GDP and government consumption series underwent the application of the first-log difference. The resulting series was demeaned to eliminate differences in the trends across growing variables. Following Castro et al. [2011], the inflation target was subtracted from the actual quarterly IPCA-E series. The resulting series was then demeaned. For the interest rate series, SELIC, the sample average was subtracted from the actual series. These are the same procedures utilized by Castro et al. [2011] and the resulting series became stationary with mean 0.

4.2 Calibrated Parameters

There are nine calibrated parameters, following closely Castro et al. [2011]. Aggregate shares and steady state values which appear as parameters in the linear model are calibrated. The table below summarizes the values:

Table 2 – Calibrated Parameters.

Variable	Description	Value
β	Time discount factor	0.989
ϕ	Frisch elasticity of labor supply	1.0
α	Capital share in the product	0.448
ϵ	Demand elasticity	6.0
Γ_B	Central Bank reaction to debt	0
s_G	Government spending/GDP at the steady state	0.2
s_Y	Primary Surplus/GDP at the steady state	0.022
B^y	Net public debt/GDP at the steady state	2
τ_{ss}	Tax revenues/GDP at the steady state	0.35

Castro et al. [2011] calculate the GDP shares based on the sample average from the national accounts. They also calculate β based on Brazilian GDP growth, the average domestic interest rate and the inflation target over the period 2005Q1 to 2010Q2. The authors set ϕ based on existing literature and match the average capital income share in GDP with α. Demand elasticity ϵ is taken from Gali [2008]. Finally, Γ_B is initially calibrated to be 0, simulating a situation in which the monetary authority does not intend to influence debt trajectory with its interest rate policy. In a fiscal dominance context, when the fiscal authority ceases to follow a rule compatible with a sustainable debt trajectory, $\Gamma_B > 0$ meaning the central bank will take into account such trajectory when deciding interest rates.

4.3 Prior Distributions

In this section the priors for the parameters to be estimated are set. Again, since the model is supposed to fit with Brazilian data, Castro et al. [2011] is closely followed, with some modifications in a few parameters. The table below summarizes the priors, their distributions, their means and standard deviations:

Table 3 – Prior Means, Distributions and Standard Deviations 1.

Parameter	Description	Distribution	Mean	SD
σ	Intertemporal elasticity of substitution	Normal	1.3	0.05
θ	Calvo parameter	Beta	0.65	0.1
M^f	Firm's inattention parameter	Beta	0.8	0.05
M	Consumer's inattention parameter	Beta	0.8	0.05
Γ_R	Monetary policy smoothing parameter	Beta	0.75	0.05
Γ_Π	Monetary policy response to inflation	Normal	2.0	0.35
Γ_y	Monetary policy response to output gap	Gamma	0.25	0.1
ϕ_S	Primary surplus inertia	Beta	0.4	0.05
$\phi_{\bar{S}}$	Primary surplus reaction to target deviation	Normal	0.35	0.05
$\rho_{\bar{S}}$	Primary surplus target inertia	Beta	0.5	0.15
ϕ_B	Primary surplus reaction to public debt	Inv-Gamma	0.05	0.05
ρ_ν	Monetary policy autocorrelation	Beta	0.5	0.25
ρ_a	Technology shock autocorrelation	Beta	0.5	0.25
ρ_τ	Tax rate shock autocorrelation	Beta	0.5	0.25
ρ_c	Consumption shock autocorrelation	Beta	0.5	0.25
ϵ_ν^σ	Monetary policy shock variance	Inv-Gamma	1.0	inf
ϵ_a^σ	Technology shock variance	Inv-Gamma	1.0	inf
ϵ_τ^σ	Tax rate shock variance	Inv-Gamma	1.0	inf
ϵ_c^σ	Consumption shock variance	Inv-Gamma	1.0	inf

In the table above, the mean and standard deviation values of σ and θ are the same as in the SAMBA model. M^f and M, the inattention parameters, have their means and standard deviations close to the ones used by Gabaix [2018]. The monetary policy parameters Γ_Π and Γ_y follow the same values of Castro et al. [2011]. The fiscal parameters ϕ_S, $\phi_{\bar{S}}$ and ϕ_B follow the SAMBA model exactly, as do all remaining autocorrelation parameters and shocks. The parameter $\rho_{\bar{S}}$ has the same prior but not the same standard deviation, which is lower in this model. The only parameter that differs from the priors established in Castro et al. [2011] is Γ_R, whose value is inspired in the posterior distribution found in the Bayesian estimation of the SAMBA model.

4.4 Estimation Results

The estimation uses Bayesian methods to arrive at the posterior means and distributions of the selected parameters. It is conducted in MATLAB with the Dynare extension. The results are summarized in the table below:

Table 4 – A Posteriori Means and Confidence Intervals 1.

Parameter	Mean	Lower Bound	Higher Bound
σ	1.3823	1.3608	1.3998
θ	0.7093	0.6788	0.7541
M^f	**0.8649**	0.8310	0.9126
M	**0.7580**	0.7410	0.7790
Γ_R	0.7105	0.7010	0.7208
Γ_Π	2.3555	2.2555	2.4191
Γ_y	0.3965	0.3549	0.4371
ϕ_S	0.4739	0.4440	0.5086
$\phi_{\bar{S}}$	0.4802	0.4480	0.5197
$\rho_{\bar{S}}$	0.5540	0.5280	0.5874
ϕ_B	0.0304	0.0179	0.0448
ρ_ν	0.2532	0.2019	0.3008
ρ_a	0.5471	0.4472	0.6722
ρ_τ	0.6047	0.4737	0.7505
ρ_c	0.5361	0.5246	0.5531
ϵ_ν^σ	0.1225	0.1176	0.1293
ϵ_a^σ	0.1779	.1340	0.2193
ϵ_τ^σ	0.1215	0.1176	0.1265
ϵ_c^σ	0.1256	0.1146	0.1347

The lower and higher bounds fall within the 90% confidence interval level. The acceptance ratios were 33.673% and 34.083% for the Metropolis Hasting algorithm, which was replicated 100.000 times in two blocks. This falls within the 20%-40% interval that is well accepted by the literature, as seen in Gelman et al. [1996] and Fernández-Villaverde and Rubio-Ramirez [2004]. All parameters converged. The results are close to those of the SAMBA model constructed by Castro et al. [2011], which is reasonable since the data used and the model specification are similar - though much simpler in this model and with a wider time span.

Some interesting estimations are found. It is the first time a Bayesian estimation for the cognitive factors M and M^f is conducted for the Brazilian economy. Their posterior means, 0.7580 and 0.8648, respectively, make theoretical sense once they are lower than 1 but still allow for enough rationality. In addition, firms have a higher cognitive discount factor meaning they pay more attention to macro variables - this too makes theoretical sense since firms can hire consultants and information services to back their decisions.

The parameters that guide monetary policy also make theoretical sense, given the current policy regime in the country. Parameters $\Gamma_\Pi > 1$ and $\Gamma_y < 1$ account for the inflation targeting regime, in which the central bank has a strong reaction function towards inflation - following the Taylor principle. As for fiscal policy, parameters $\phi_{\bar{S}} > 0$, $\rho_{\bar{S}} > 0$, and $\phi_B > 0$ indicate a rule in which the primary surplus target matters for fiscal policy decisions and so does public debt trajectory. This certainly leads one to conclude that over the past years Brazil has generally operated under a monetary dominance regime.

In addition, the theoretical moments generated by the model are exposed in the table below:

Table 5 – Theoretical Moments for the Estimated Model.

Variable	Mean	SD	Variance
y	0.0000	0.2212	0.0489
g	0.0000	1.0468	1.0958
c	0.0000	0.2346	0.0550
n	0.0000	0.4739	0.2246
r	0.0000	0.1422	0.0202
π	0.0000	0.0581	0.0034
r_{real}	0.0000	0.1641	0.0269
b	0.0000	0.9217	0.8495
s_y	0.0000	4.7672	22.7265
ν	0.0000	0.1266	0.0160
a	0.0000	0.2125	0.0451
τ	0.0000	0.4359	0.1900
z_c	0.0000	0.1487	0.0221

In the table above, SD stands for Standard Deviation. Since the model is linear and all data used in the estimation was stationary, it is natural all the variables' mean values would equal 0. One can also observe the standard deviation and variance values above. In the next chapter, in which product and inflation variances are compared under different parameter calibrations to simulate monetary and fiscal dominance policy regimes, this table can prove a good benchmark.

4.5 A Brief Comparison

In order to analyze the consistency of the model proposed in the section above, a similar version of Gabaix's model is explored in this section and estimated using Brazilian data. The idea is compare the values of the cognitive discount factors that are estimated and check if they have approximate values to Gabaix's results. This is the purpose of this section: to briefly compare the estimation results of the models.

The proposed model, following Gabaix [2018], is given as:

$$\pi_t = \beta M^f E_t[\pi_{t+1}] + \kappa x_t + \eta_t^s \tag{4.1}$$

$$x_t = M E_t[x_{t+1}] - \sigma(r_t - E_t[\pi_{t+1}]) + \eta_t^d \tag{4.2}$$

$$r_t = \rho_r r_{t-1} + (1 - \rho_r)(\phi_\pi \pi_t + \phi_x x_t) + \eta_t^m \tag{4.3}$$

Notation follows Gabaix (it is different from last section's model). Equation 4.1 is the Phillps Curve, 4.2 is the IS curve, and 4.3 is the monetary policy rule. The variable x_t represents output gap at time t, π_t is inflation at period t and i_t is the nominal interest rate set by the monetary authority at time t. The exogenous shocks at period t are given by η_t^j, with $j \in \{d, s, m\}$. They follow AR(1) processes such that: $\eta_t^j = \rho_j \eta_{t-1}^j + \varepsilon_t^j$, with ε_t^j i.i.d Gaussian N(0, σ_j^2).

A Bayesian estimation is next conducted using two sets of data: (i) the same data utilized in the last section (data set 1 - DS1); and (ii) analogous Brazilian data to those used by Gabaix in his estimation for the American economy (data set 2 - DS2). In the first data set, government spending data is not used because there is no fiscal authority in this model specification and there are only three shocks - thus necessitating only three observable series. These are: quarterly GDP, interest rates (SELIC) and consumer price index (IPCA), transformed as in the last section.

Gabaix uses the unemployment gap as a proxy for output gap, making $x_t = \bar{u} - u_t$, where u_t is current unemployment rate and $\bar{u}$ is the sample mean. He also detrends data using the HP filter with a smoothing parameter $\lambda = 1600$. In addition to the unemployment rate, Gabaix uses the Federal Funds rate and the CPI inflation rate. The second set of Brazilian data used here follows a similar pattern: unlike last section's data set, the unemployment rate (from IBGE) is utilized here in the same way used by Gabaix. Interest rates (SELIC) and the consumer price index (IPCA) are also used, but all data is detrended using the HP filter with $\lambda = 1600$ (using statistical software R), the same way Gabaix does. The second data set uses quarterly data from 2002Q2 to 2015Q4 (a shorter time span than last section's model).

The priors used here are defined in the table below and follow a similar pattern as before, with minor changes due to the new model specification:

Table 6 – Prior Means, Distributions and Standard Deviations 2.

Parameter	Description	Distribution	Mean	SD
M^f	Firm's inattention parameter	Beta	0.8	0.05
M	Consumer's inattention parameter	Beta	0.8	0.05
σ	Inverse of the elasticity of substitution	Normal	0.76	0.05
κ	Inflation response to output	Normal	0.15	0.10
ϕ_π	Monetary policy response to inflation	Normal	2.0	0.35
ϕ_x	Monetary policy response to output gap	Gamma	0.25	0.1
ρ_d	Demand autocorrelation	Beta	0.5	0.25
ρ_s	Cost-push autocorrelation	Beta	0.5	0.25
ρ_m	Monetary policy shock autocorrelation	Beta	0.5	0.25
ε_d^σ	Demand shock variance	Inv-Gamma	1.0	inf
ε_s^σ	Cost-push shock variance	Inv-Gamma	1.0	inf
ε_m^σ	Monetary policy shock variance	Inv-Gamma	1.0	inf

In the table above, SD stands for Standard Deviation. The parameter σ is to be understood as the inverse of the one considered in the last section (its prior is the same as before, with an inverse value). The main difference is parameter κ, which follows the same values for mean, distribution and standard deviation as in Gabaix [2018] - the other parameters follow the mean and standard deviation values defined in the previous section. Also, let us emphasize there is no fiscal policy in this model; it is assumed deficits are zero. After running the Bayesian estimations with both data sets (DS1 and DS2) one arrives at the following results:

Table 7 – A Posteriori Means and Confidence Intervals 2.

	Data Set 1			Data Set 2		
Parameter	Mean	L. B.	H. B.	Mean	L. B.	H. B.
M^f	**0.8125**	0.7348	0.8948	**0.8141**	0.7355	0.8920
M	**0.8140**	0.7414	0.8912	**0.8117**	0.7341	0.8857
σ	0.8373	0.7712	0.9134	0.8339	0.7487	0.9231
κ	0.6467	0.5359	0.7845	0.5775	0.4394	0.7253
ϕ_π	3.1828	2.6735	3.7502	2.8865	2.3655	3.4748
ϕ_x	0.4846	0.2024	0.8079	0.4420	0.1440	0.7000
ρ_d	0.0583	0.0004	0.1238	0.0501	0.0000	0.1006
ρ_s	0.0602	0.0013	0.1219	0.0564	0.0003	0.1167
ρ_m	0.5546	0.2448	0.8600	0.4268	0.0431	0.7583
ε_d^σ	0.1224	0.1176	0.1283	0.1247	0.1176	0.1336
ε_s^σ	0.1221	0.1176	0.1277	0.1241	0.1176	0.1322
ε_m^σ	0.1218	0.1176	0.1267	0.1243	0.1176	0.1318

L. B. and H. B. stand for lower bound and higher bound, respectively. They fall within the 90% confidence level for the interval. The acceptance ratios for the first data set were 36.57% and 36.375% for the Metropolis Hasting algorithm, which was replicated 20.000 times in two blocks. This is within the 20%-40% interval that is well accepted in the literature. All parameters converged. The acceptance ratios for the second data set were 32.885% and 33.405% for the Metropolis Hasting algorithm, which was also replicated 20.000 times in two blocks. Again, all parameters converged.

Finally, the table below compares the values above for the cognitive discount factors M^f and M with the ones found by Gabaix [2018], who firstly conducted a similar estimation for the American economy using quarterly data from 1960Q1 to 2016Q3.

Table 8 – Dissertation's Estimations vs. Gabaix's Results.

Variables	Last Section	Data Set 1	Data Set 2	Gabaix's Results
M	0.7580	0.8140	0.8117	0.668
M^f	0.8649	0.8125	0.8141	0.803

In the table above, Last Section denotes the mean values found in the estimation conducted in the section before (with the full model, also accounting for fiscal policy); Data Set 1 gives the values for the model specification of this section estimated with data from the last section; Data Set 2 gives the values for the model specification of this section with data transformed following Gabaix [2018]; and Gabaix's Results gives the values found by the author in his estimation[1]. It is important to emphasize the latter is not exactly comparable with Last Section's values, because the model specifications are different. It is worth mentioning Gabaix uses a longer set with more than 4 decades of data while Brazil's data account for less than two decades.

The results do not differ considerably, specially if one considers the 90% confidence level for the lower and higher bounds. All models in the table above have their confidence intervals intersect at some point of the 90% level range. In addition, Data Set 1 and Data Set 2 yielded close posterior mean values and intervals which suggests data transformation does not cause relevant differences for the estimates conducted in this section.

The values found for Brazil cause the impression the country's consumers and firms are less rationally bounded than their colleagues in the U.S.A. This could well be the case; however, since the data used by Gabaix takes into account a longer time span, it is difficult to draw such a conclusion without further considerations.

[1] Specifically, the Behavioral version of Table 4 found in Gabaix [2018] on page 31.

5 Simulations

This dissertation now goes back to the complete model, with fiscal policy. The model is run to simulate monetary dominance situations (the usual rules that apply in Brazil nowadays, as seen above) and fiscal dominance contexts (a hypothetical that still uses most of the values for the parameters estimated in the last section while changing the values of other parameters), like Furtado [2017] - which serves as a benchmark for this section. Since the objective is to analyze monetary and fiscal interactions with a behavioral model, the variances of output and inflation are used as proxies for social welfare - that is, the monetary and fiscal authorities' goals include pursuing their targets while keeping inflation and product volatility (variances) low. In other words, higher variance values for output and inflation leads to lower social welfare. Variance results are given after the model is run for all possible shocks considered in the model; that is, monetary, technology, tax rate, and consumption shocks. Under such assumptions, the model and its variations are analyzed next.

First, let us look at the results under monetary dominance - which can be understood as the country's current context, at least under the rules and laws that apply today. Last chapter's estimation suggests the country has lived under such a regime over the past seventeen years. The model is run considering different responses the central bank can exert when fighting inflation, the parameter Γ_{Π}, and different responses the fiscal authority can weigh on public debt trajectory, the parameter ϕ_B. Those are the parameters to be tested for different values while keeping the other parameters constant. It is important to emphasize that the rules established when deriving the model are similar to the ones run in this context. Parameter values for the simulations were chosen based on the last chapter's estimation and their neighborhoods. Additionally, results from the rational expectations case, in which $M = M^f = 1$, are also analyzed for the same circumstances and the results briefly compared to those of the behavioral model.

The table below summarizes the variances of output y and inflation π for different responses by the central bank and the fiscal authority according to the behavioral case:

Table 9 – Behavioral Case ($M = 0.7580$, $M^f = 0.8649$) - Monetary Dominance.

	Γ_Π							
	1.5		2.0		2.5		3.0	
ϕ_B	y	π	y	π	y	π	y	π
0.01	0.0623	0.0038	0.0549	0.0030	0.0495	0.0025	0.0453	0.0022
0.02	0.0625	0.0045	0.0541	0.0035	0.0483	0.0029	0.0439	0.0024
0.05	0.0641	0.0064	0.0535	0.0048	0.0463	0.0038	0.0412	0.0032
0.10	0.0761	0.0103	0.0605	0.0077	0.0503	0.0060	0.0431	0.0049
0.15	0.2023	0.0337	0.1256	0.0199	0.0911	0.0137	0.0716	0.0103

Let us notice that when the central bank fights inflation fiercely, the variance of inflation lowers for all cases. Let us also notice that for certain values of ϕ_B, especially for those that belong to the [0.01, 0.05] interval, product variance can be lowered in most cases (for instance, when $2 \leq \Gamma_\Pi \leq 3$). This means the fiscal authority can pursue its objective (debt stability) with care, otherwise it could generate too much product volatility.

Next, the same is done for the rational expectations case - in which $M = M^f = 1$. The other parameter values are kept constant, following the same pattern as above. The table below summarizes the results:

Table 10 – Rational Expectations Case ($M = 1$, $M^f = 1$) - Monetary Dominance.

	Γ_Π							
	1.5		2.0		2.5		3.0	
ϕ_B	y	π	y	π	y	π	y	π
0.01	0.0683	0.0035	0.0601	0.0029	0.0540	0.0025	0.0491	0.0022
0.02	0.0616	0.0038	0.0542	0.0030	0.0488	0.0025	0.0446	0.0022
0.05	0.0475	0.0057	0.0408	0.0041	0.0365	0.0032	0.0334	0.0026
0.10	0.0447	0.0113	0.0373	0.0084	0.0327	0.0065	0.0297	0.0052
0.15	0.3342	0.0694	0.2085	0.0390	0.1623	0.0274	0.1420	0.0216

The results are similar, though one can notice the variance values for both output and inflation vary more in this case. Additionally, note that the interval in which the fiscal authority improves its performance is between [0.01, 0.10], wider than in the behavioral case. This is interesting, since more product volatility does not necessarily translate into higher variances for inflation.

Above all, one can conclude that under monetary dominance the central bank should fiercely pursue its goals - while the monetary authority should carefully adjust not to cause higher volatility in the variances of product and inflation. This is a result in both specifications, considering behavioral and fully rational agents. One main difference is that variances are smoothed when behavioral agents are considered - which makes theoretical sense, once inattentive agents do not pay attention to all prices and hence soften their trajectories.

Next, the results from fiscal dominance situations are exposed. Let us first emphasize that some changes are made in the original model to make it fit for such conditions. First, it is assumed $\phi_{\bar{S}} = 0$, implying the primary surplus target is not relevant for the fiscal authority. That means the actual primary surplus is not influenced by a target anymore, depending only on its past values. This implies that fiscal policy does not place any weights on debt trajectory.

Note that the model does not allow for both monetary and fiscal policies to be dominant - it results in an explosive situation in which the Blanchard-Kahn conditions are not usually satisfied. Such a situation resembles a context similar to the one treated by Loyo [1999] for explaining hyperinflation in Brazil during the 1970s and 1980s. In addition, the central bank becomes responsible for debt trajectory and thus $\Gamma_b > 0$. Also, since monetary policy is not dominant, parameter Γ_Π is run for values less than 1 - meaning the central bank does not follow a Taylor-rule in these cases, it just adjusts for fiscal policy. In addition, simulations are also made for $\Gamma_\Pi \geq 1$, which often results in explosive situations - just as mentioned above. However, some simulations are made for hypothetical contexts in which the monetary authority wishes to fight inflation at the same time as it pursues debt stability.

Again, variance values for output and inflation are analyzed as proxies for social welfare. Simulations are run for different Γ_Π and Γ_b values. These values were chosen based on the fact that monetary policy is not dominant and on the determination region for the model. Note that Γ_b values are run for certain values close to ϕ_B, and that is done on purpose: the attention given to debt by the fiscal authority under monetary dominance could be close to the attention given to debt by the monetary authority under fiscal dominance. The parameter Γ_b is also tested with higher values. The table below summarizes the results for the behavioral case:

Table 11 – Behavioral Case ($M = 0.7580$, $M^f = 0.8649$) - Fiscal Dominance.

	Γ_Π							
	0.00		0.5		1.0		2.0	
Γ_b	y	π	y	π	y	π	y	π
0.01	0.1402	0.0723	-	-	-	-	-	-
0.05	0.1176	0.0563	-	-	-	-	-	-
0.5	0.0517	0.0148	0.0669	0.0241	0.1119	0.0549	-	-
1.0	0.0383	0.0088	0.0434	0.0114	0.0514	0.0156	0.1024	0.0481
1.5	0.0336	0.0070	0.0366	0.0083	0.0407	0.0102	0.0549	0.0174

In the table above, results with “-” (no numerical value) signifies the Blanchard-

Kahn conditions were not met and there is no stable equilibrium for those parameter values.

Let us say a few words about the results in the table. First, given a certain value of Γ_b, when the central bank responds with higher intensity to inflation, the outcome is increasing inflation and output variances. Take for instance $\Gamma_b = 1.0$. For higher values of Γ_Π one observes increasing variances for both output and inflation.

Additionally, for a given value of Γ_Π, the more the monetary authority responds to debt trajectory (the higher Γ_b), the lower the product and inflation variances become. This suggests the monetary authority should take into account public debt trajectory when faced with fiscal dominance contexts and not fight inflation fiercely. If it pursues the latter, it could cause price instability. In fact, what this model specification suggests is that the monetary authority ought to adjust for fiscal policy and accommodate for public debt. By doing this, it keeps inflation and product variances low.

This result is in line with the literature reviewed in chapter 2. Woodford [2001] and Blanchard [2005] argue a similar response for monetary authorities faced with fiscal dominance contexts: when there are no controls on government spending and the fiscal authority is free to sets its own policy, inflation may become a fiscal phenomena and there is not much monetary policy can achieve in terms of price stability.

Next, a similar simulation is conducted for the rational expectations case, in which $M = M^f = 1$. The table below summarizes the results:

Table 12 – Rational Expectations Case ($M = 1$, $M^f = 1$) - Fiscal Dominance.

	Γ_Π							
	0.00		0.5		1.0		2.0	
Γ_b	y	π	y	π	y	π	y	π
0.01	0.0433	0.0492	0.0426	0.1370	-	-	-	-
0.05	0.0430	0.0471	0.0427	0.1268	-	-	-	-
0.5	0.0403	0.0320	0.0428	0.0661	0.0382	0.2552	-	-
1.0	0.0390	0.0247	0.0427	0.0429	0.0451	0.1020	-	-
1.5	0.0389	0.0213	0.0430	0.0332	0.0478	0.0623	0.0262	1.635

Generally, the results are in line with the behavioral case seen above. For instance, for a given value of Γ_b, the more the central bank responds to inflation, the higher the variances for product and inflation becomes. Let us consider again $\Gamma_b = 1$. Then for higher values of Γ_Π, product and inflation variances increase. Additionally, for a given value of Γ_Π, higher responses of the monetary authority to public debt (a higher Γ_b) generate lower product and inflation variances.

This leads one to conclude the same as above: under fiscal dominance, the central bank should not fiercely pursue its inflation target. In particular, when central bank response to public debt and inflation is high enough, a strong instability can be generated - as is the case for $\Gamma_b = 1.5$ and $\Gamma_\Pi = 2$, which results in an inflation variance of 1.635.

It is also worth mentioning differences. Under fully rational agents (with $M = M^f = 1$), for instance, the product variance is relatively stable - more stable than in the behavioral case, for sure. This may suggest that the real economy is not much affected by monetary policy under a fiscal dominance context with fully rational agents. The results also carry greater inflation variances across most of their values, which indicates that fully attentive agents take into account full price variations. It also suggests that a fiscal dominance context does not differ much from a monetary dominance situation in terms of output variances. Empirical evidence generally goes against this, as Loyo [1999] has argued for Brazil in the 1970s and 1980s.

Comparing the behavioral and rational cases is interesting to check how the theoretical results differ. Since the modifications made to add the behavioral feature of inattention to firms and consumers build on the traditional rational expectations model, the results ought to differ though not considerably. Placing a cognitive discount factor for economic agents helps generate smoother paths for the resulting variables under normal conditions (e.g. monetary dominance). This has happened in the comparisons above. For instance, one arrives at similar conclusions for both the behavioral and rational cases under monetary dominance, though the amount of weight the fiscal authority places on debt trajectory should be taken into account with care - because the rational model suggests more weight than the behavioral. The resulting variances, and thus social welfare, can differ significantly due to higher or lower weights placed by the fiscal authority on public debt trajectory.

Under fiscal dominance, though the conclusions drawn from both cases are similar in terms of policy prescriptions, the variance results differ. The behavioral model, as seen above, can generate explosive situations when the monetary authority responds too fiercely to inflation and lightly to public debt trajectory. On the other hand, it can generate lower variances when the opposite is true, that is, when the central bank places lower weight on fighting inflation and responds highly to public debt trajectory. Taking this result to the limit, this means that under a fiscal dominance context the best the central can do about inflation is almost nothing, just accommodating for fiscal policy and adjusting for debt trajectory. For the rational expectations case, fiscal dominance is not so costly in terms of output. The generated product variance values, though a bit higher on average, are not much different from the rational monetary dominance simulations. This could suggest fully rational agents are able to understand inflation as a monetary phenomena, not having too much impact on real variables, as mentioned - while the opposite could be true for behavioral agents.

6 Conclusions

This dissertation began with a brief analysis of the existing literature on monetary and fiscal regimes. It discussed the theoretical foundations of dominant policies, their objectives and hurdles. Important economists have debated over these issues and their main insight is that monetary and fiscal policies should not be analyzed separately, under the penalty of not fully understanding their effects.

Some behavioral economics literature was also succinctly reviewed. The objective was to set the stage for introducing cognitive inattention of economic agents - that is, their limited cognitive capacity for processing information. The notion of inattentive agents was presented in light of Gabaix's recent developments. The author uses the rational expectations hypothesis to introduce a cognitive discount factor, which transforms fully rational agents into behavioral (inattentive) ones.

Based on this literature, a traditional DSGE model with Brazil's monetary and fiscal rules was derived. Subsequently, the model was transformed into behavioral, by incorporating the cognitive discount factors so that inattention to the future by economic agents, firms and consumers, was added to it.

After, a Bayesian estimation was conducted using Brazilian data, following the same data transformations used by Castro et al. [2011]. A few parameters were calibrated and others were estimated, taking the latter paper as a benchmark. A posteriori mean and confidence interval values were found and exposed. Additionally, a brief comparison with Gabaix's results [2018] was conducted for different sets of data under the same model specification proposed by the author in order to analyze the consistency of the cognitive discount factors' estimations. The main conclusion is that the estimation values do not differ significantly using different data transformation techniques and values are relatively close to Gabaix's results.

Monetary and fiscal dominance situations were then simulated with the original model, with fiscal policy, and the resulting variances of output and inflation analyzed. One main result is that the central bank should fiercely pursue its inflation target under a monetary dominance context, while the fiscal authority should carefully choose the amount of effort it places on controlling debt trajectory under the penalty of generating high output variance. The behavioral model shows a tenuous threshold for the fiscal authority, thus suggesting it should be vigilant.

Another solid result is that under fiscal dominance the monetary authority should only accommodate for fiscal policy, especially for public debt trajectory, under the penalty of generating higher inflation variance. This is the case in both the behavioral and rational models. The latter generated higher variance results for inflation while product variances were relatively stable as the central bank placed higher weights in fighting inflation and public debt trajectory. This may suggest fully rational agents cause less impact on the real economy (less product variance) while the opposite could be true with inattentive firms and consumers.

Additionally, it is worth mentioning the Bayesian estimation of the behavioral inattention parameters is the first one conducted for the Brazilian economy. The values, as discussed in chapter 4, carry theoretical sense. Also, they are relatively close to the estimates of Gabaix [2018] for the American economy using U.S. data - as mentioned. One may wish to conclude, from the comparison made, that Brazilian firms and consumers are less behavioral (i.e., more rational) than their American counterparts - once their cognitive discount factors have higher values. However, since the time span for data differed considerably for Brazil and the U.S.A., conclusions like that should be made with extreme care.

It is important to emphasize the behavioral DSGE model proposed here does not represent a full departure from the rational expectations hypothesis. In fact, as seen in the model derivation, the hypothesis itself is widely used to add inattentive agents to the model. As a consequence of such addition, some of the results given by the behavioral model indeed differ from the fully rational model - as discussed in the former chapter and in this conclusion.

Finally, further works can be conducted in the intersection of macroeconomics and behavioral economics. Placing more psychological realistic foundations in human micro behavior can make our analyzes and conclusions sharper, helping improve economic diagnostics, public policies and economic predictions.

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A Transformed Data

Figure 1 – Historical and Smoothed Variables

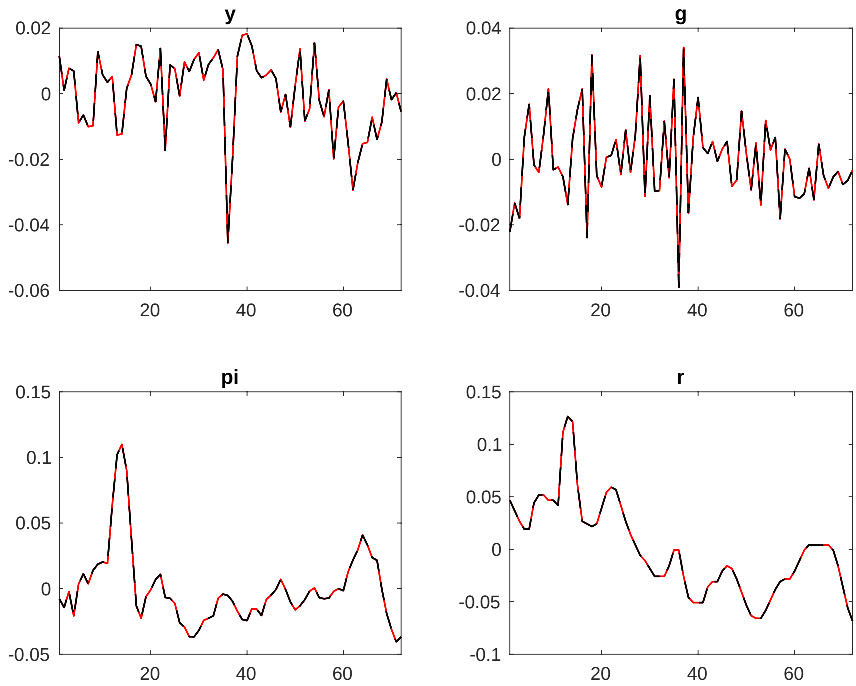
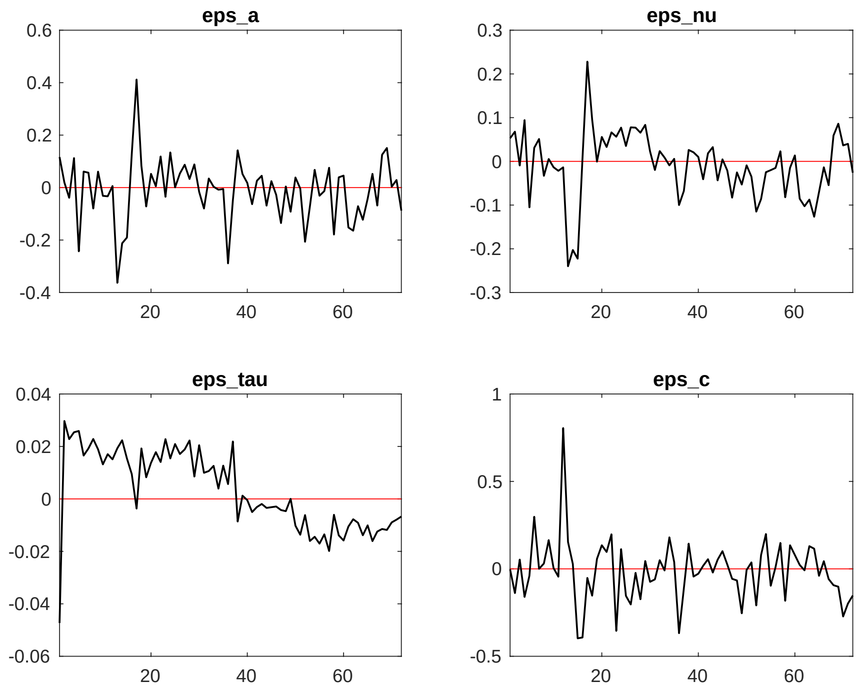


Figure 2 – Smoothed Shocks



B Impulse Response Functions

Behavioral Monetary Dominance Case: estimated parameters

Figure 3 – Impulse Response Function to a Monetary Policy Shock 1

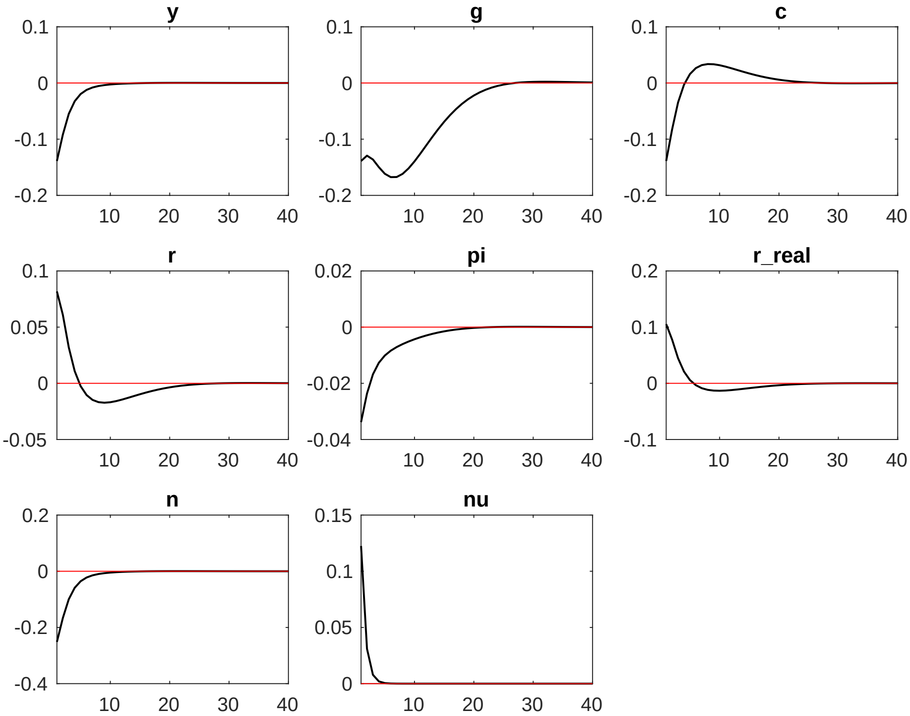


Figure 4 – Impulse Response Function to a Technology Shock 1

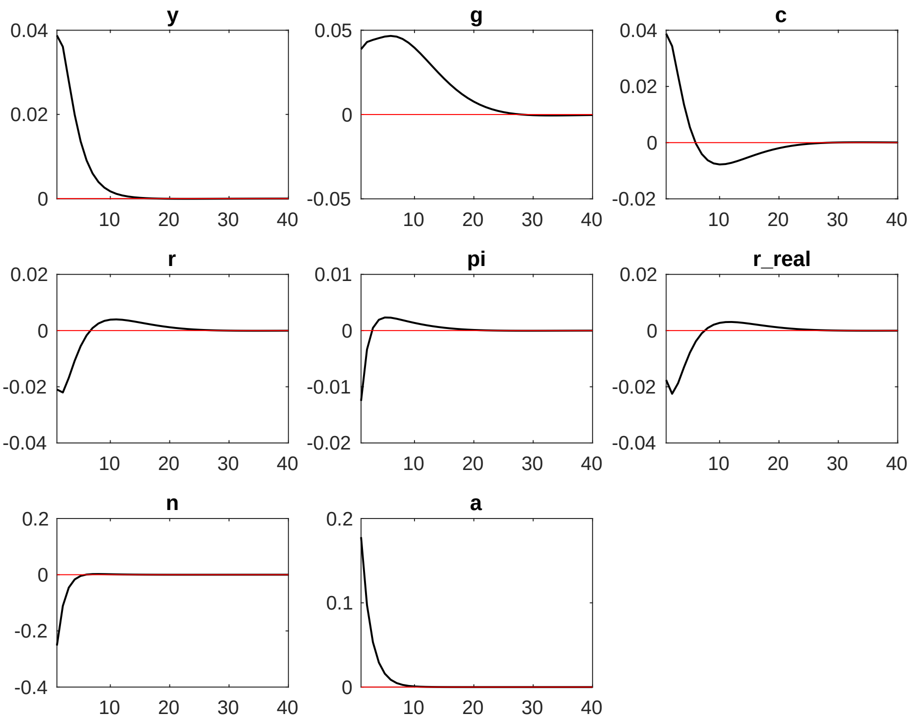


Figure 5 – Impulse Response Function to a Tax Rate Shock 1

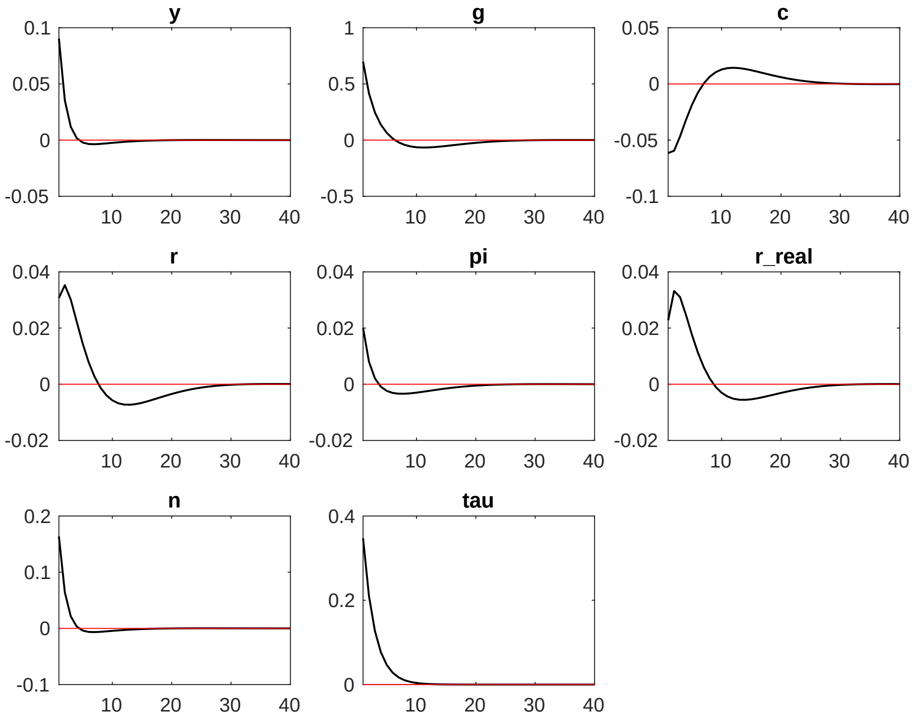
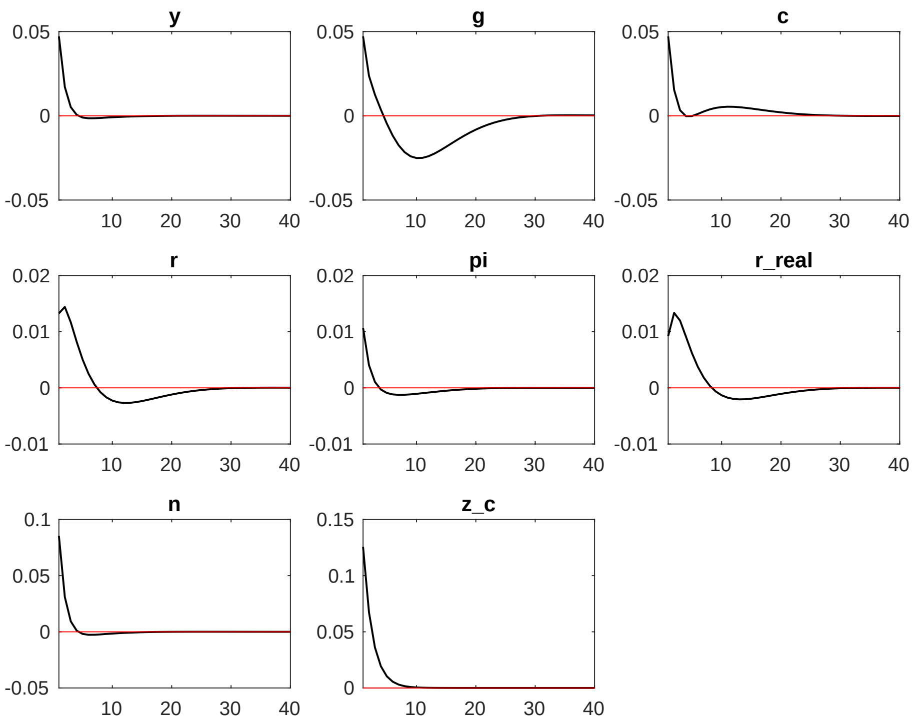


Figure 6 – Impulse Response Function to a Consumption Shock 1



Rational Monetary Dominance Case: estimated parameters

Figure 7 – Impulse Response Function to a Monetary Policy Shock 2

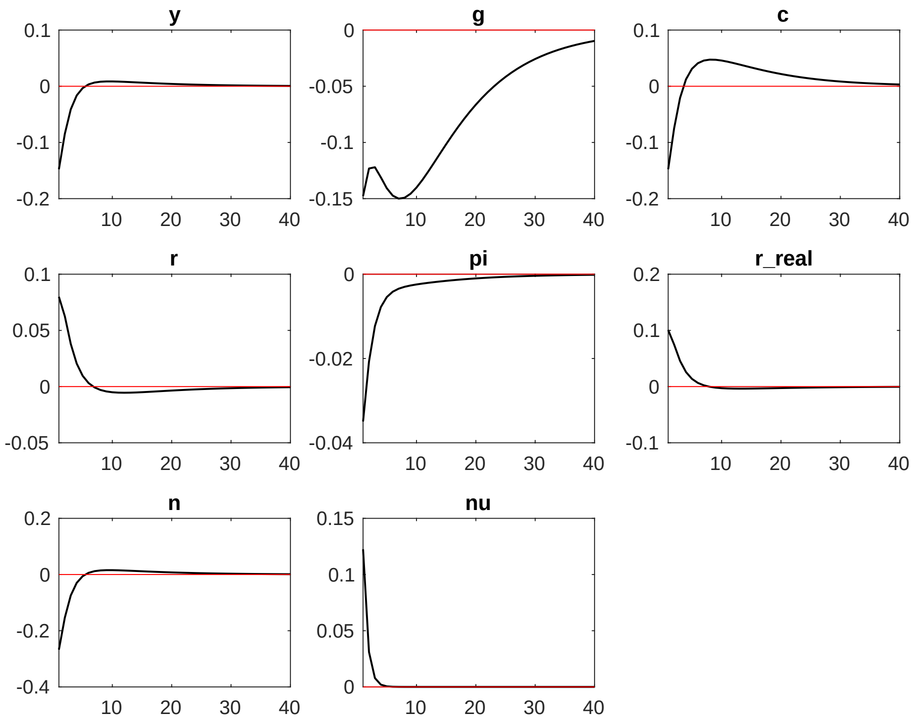


Figure 8 – Impulse Response Function to a Technology Shock 2

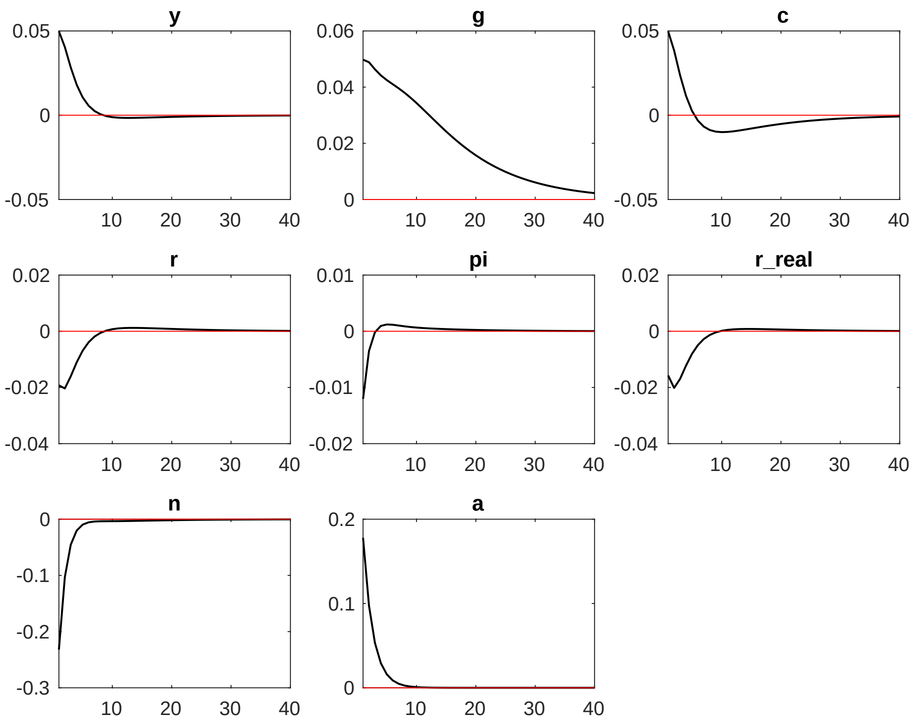


Figure 9 – Impulse Response Function to a Tax Rate Shock 2

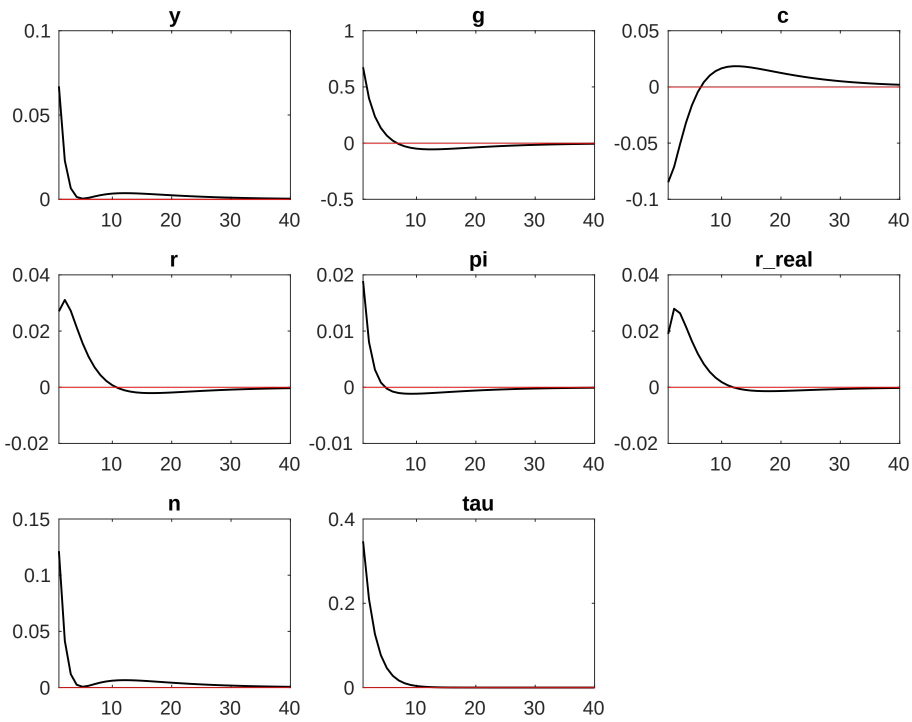
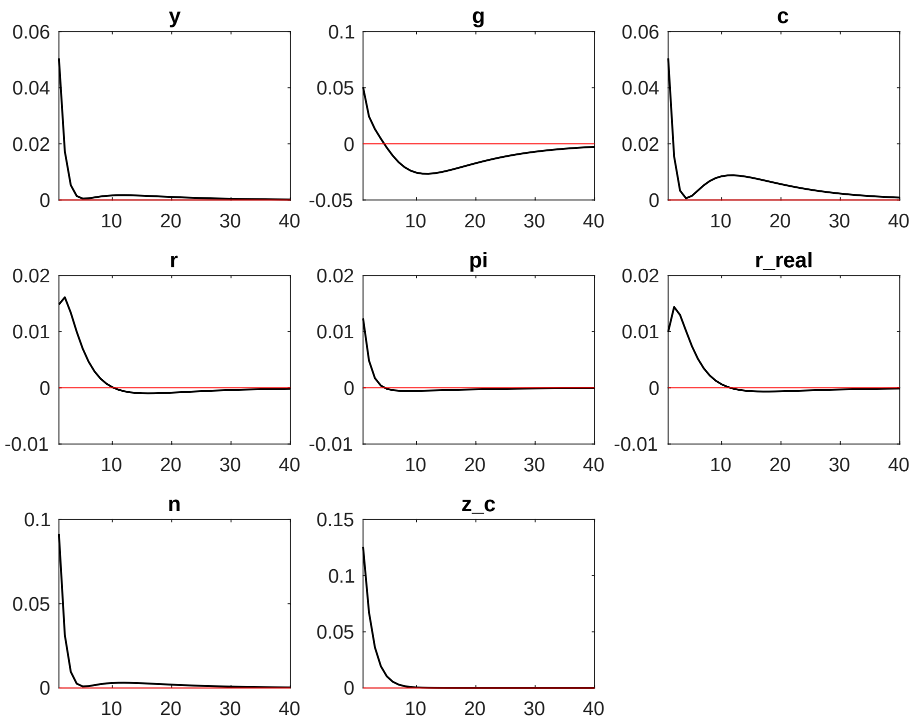


Figure 10 – Impulse Response Function to a Consumption Shock 2



Behavioral Fiscal Dominance Case: $\Gamma_{\Pi} = 1.5$ **and** $\Gamma_b = 1.0$

Figure 11 – Impulse Response Function to a Monetary Policy Shock 3

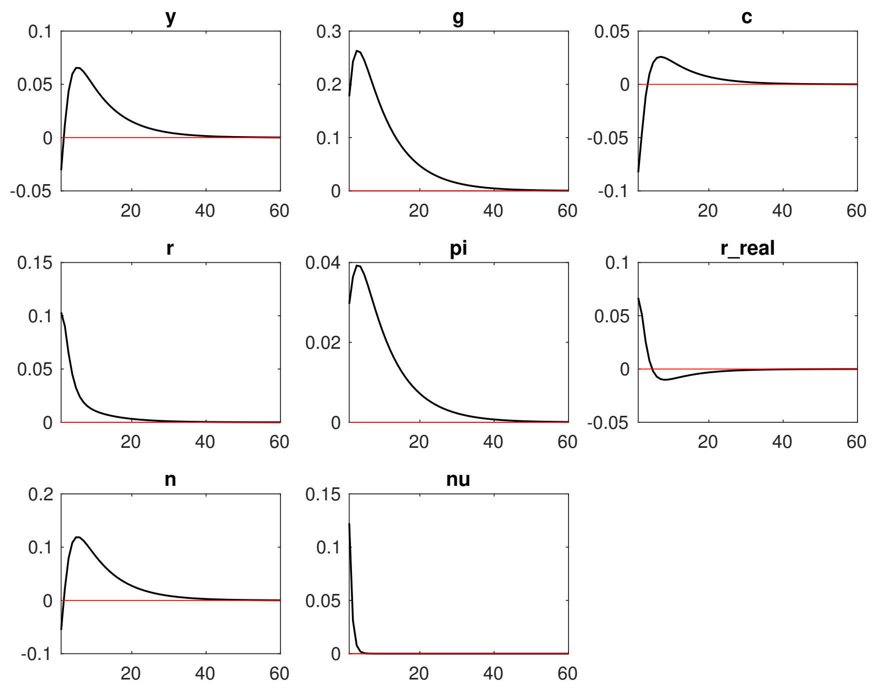


Figure 12 – Impulse Response Function to a Technology Shock 3

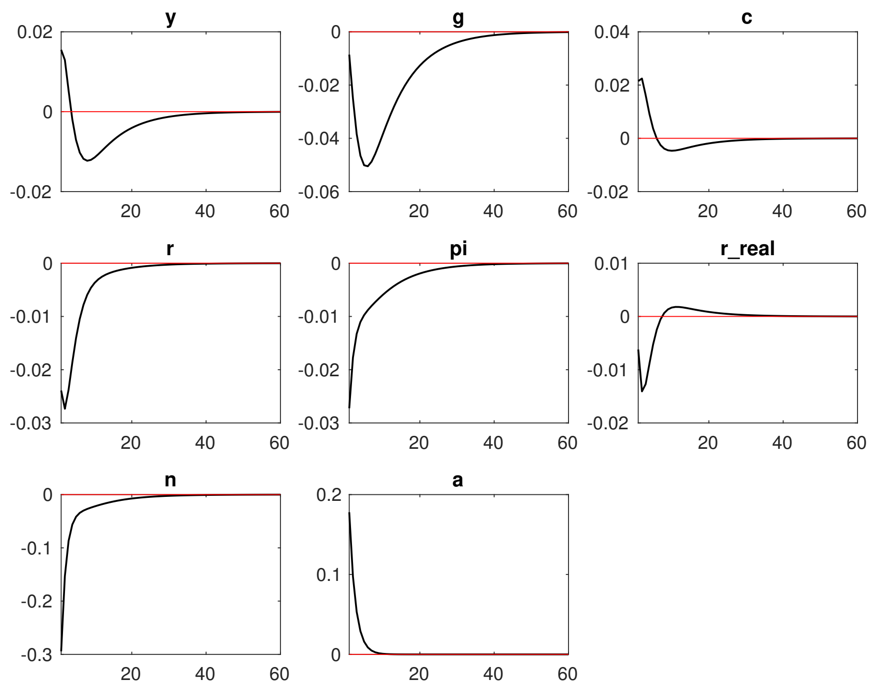


Figure 13 – Impulse Response Function to a Tax Rate Shock 3

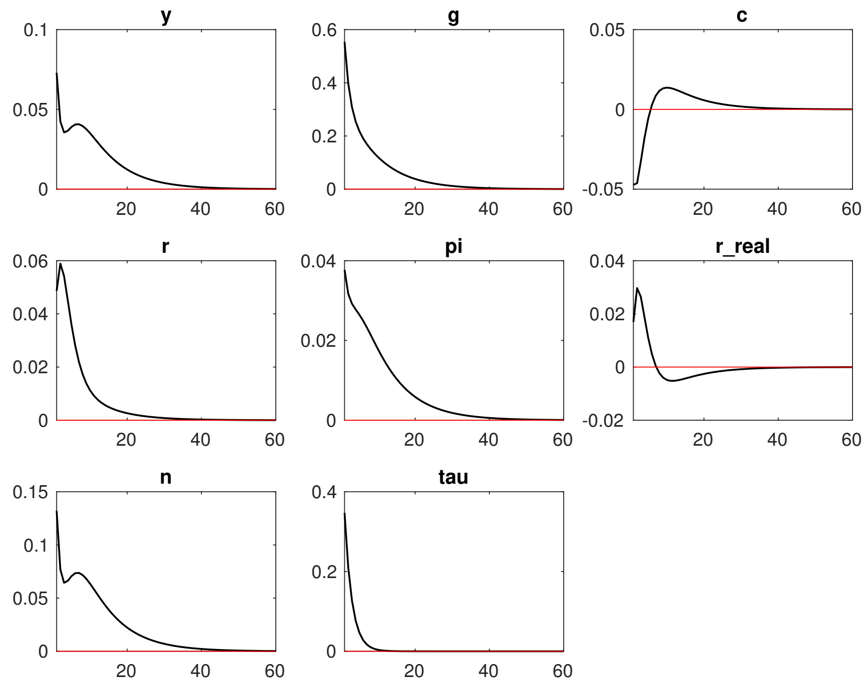
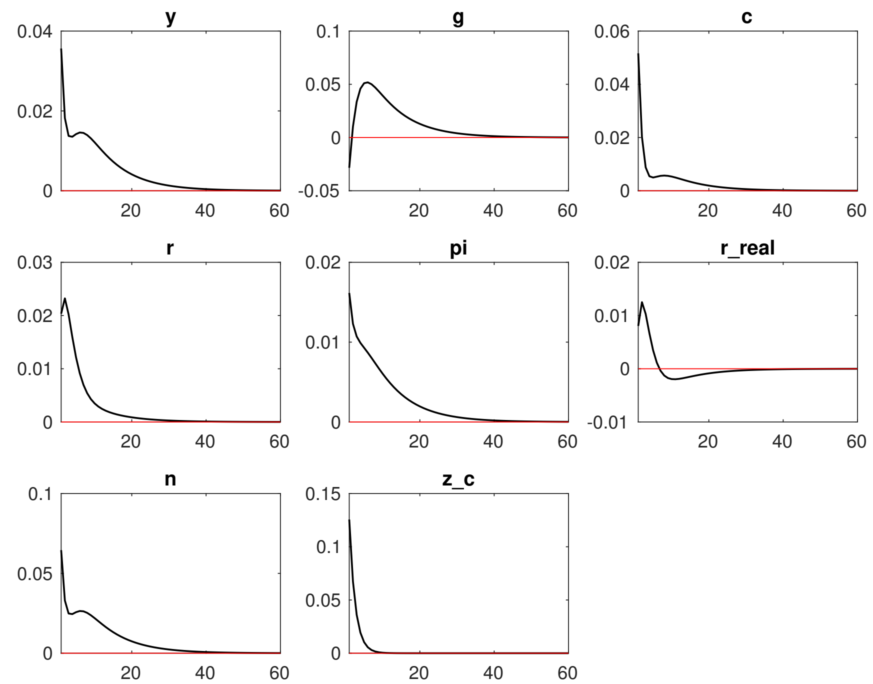


Figure 14 – Impulse Response Function to a Consumption Shock 3



Rational Fiscal Dominance Case: $\Gamma_{\Pi} = 1.5$ **and** $\Gamma_b = 1.0$

Figure 15 – Impulse Response Function to a Monetary Policy Shock 4

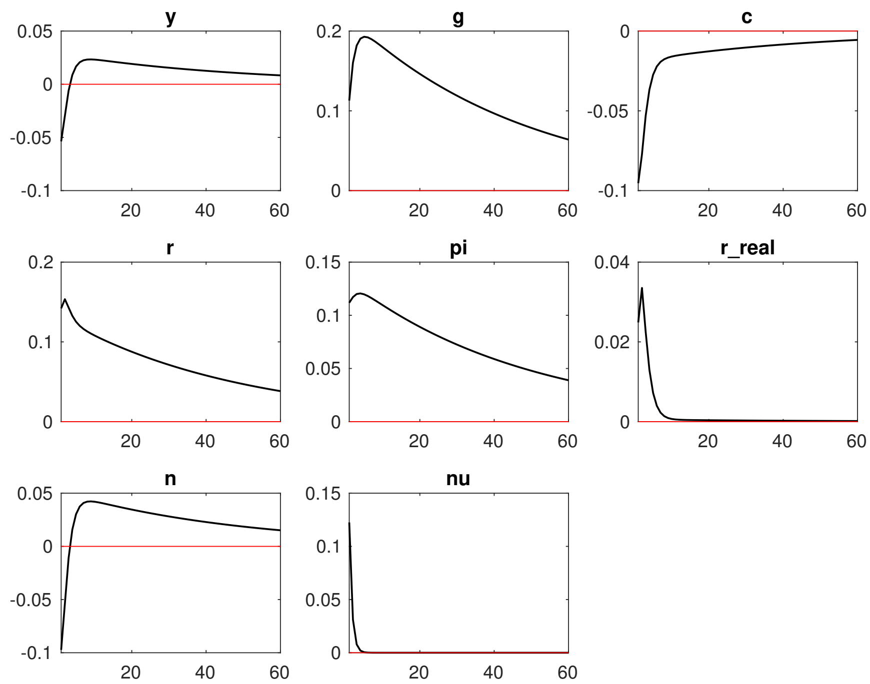


Figure 16 – Impulse Response Function to a Technology Shock 4

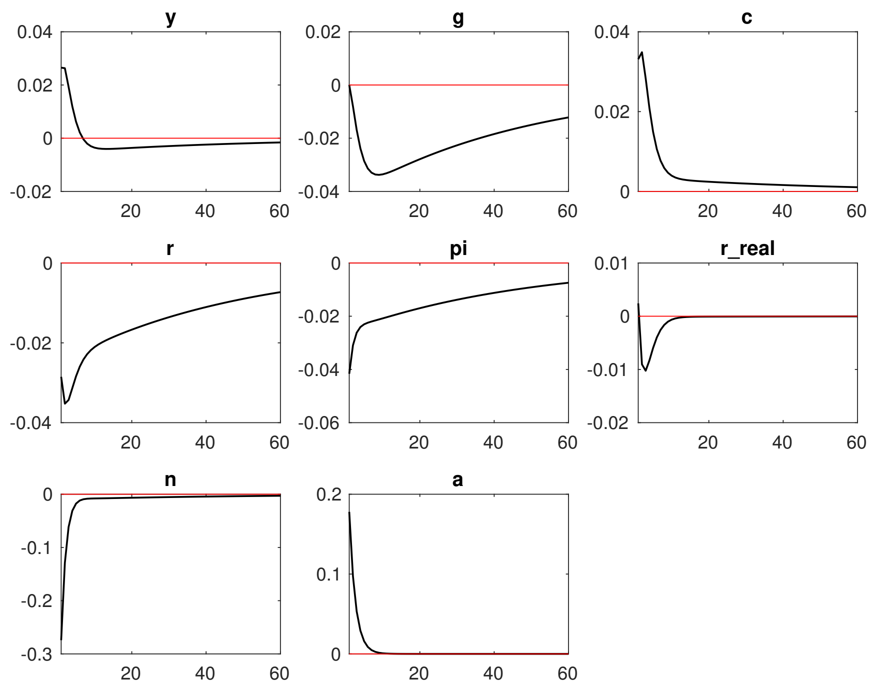


Figure 17 – Impulse Response Function to a Tax Rate Shock 4

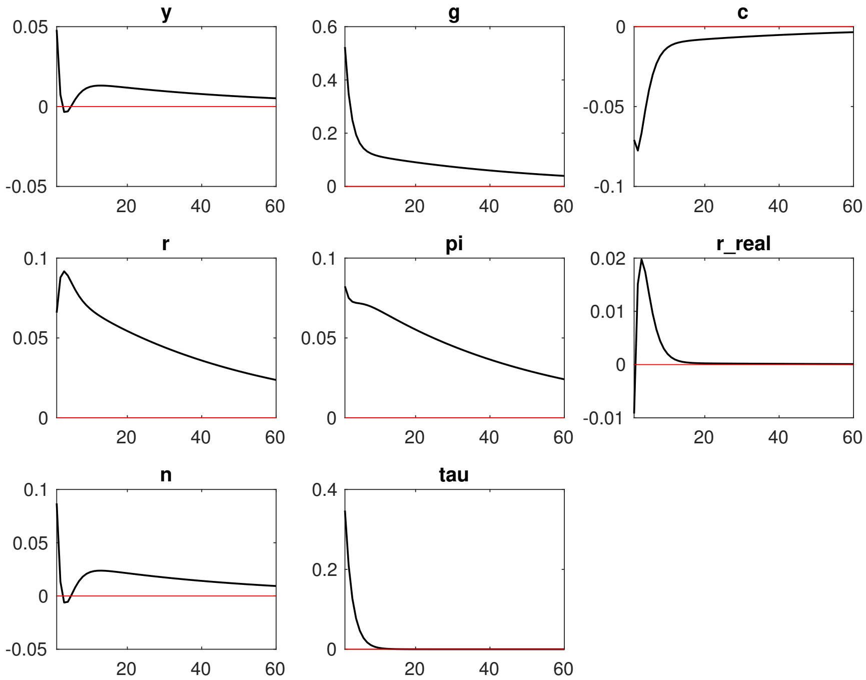
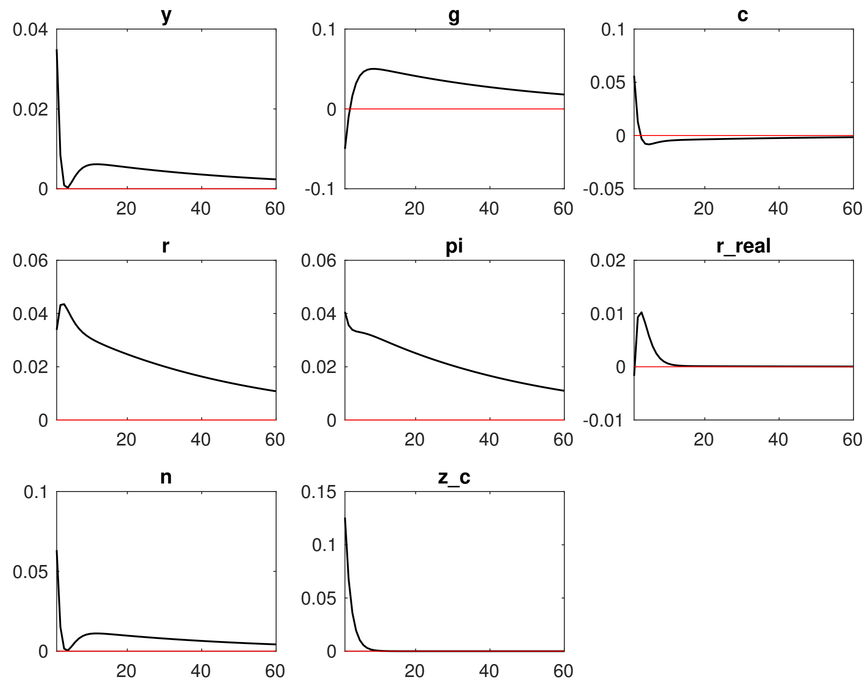


Figure 18 – Impulse Response Function to a Consumption Shock 4



C Model Code

Below the code used for the estimation of the full model, with fiscal policy, is presented in its complete form. The code is written in Dynare (`.mod` file), and it must be run in MATLAB. Data used for the estimation is described in section 4.1.

```
var
    pi ${\pi}$ (long_name='inflation')
    y_gap ${\tilde{y}}$ (long_name='output gap')
    y_nat ${y^{nat}}$ (long_name='natural output')
    y ${y}$ (long_name='output')
    r_real ${r^r}$ (long_name='real interest rate')
    r ${r}$ (long_name='nominal interest rate')
    n ${n}$ (long_name='hours worked')
    mc_hat ${\hat{mc}}$ (long_name='marginal cost deviation')
    c ${c}$ (long_name='consumption')
    b ${b}$ (long_name='public debt')
    s_y ${s_y}$ (long_name='primary surplus')
    s_y_bar ${\bar{s_y}}$ (long_name='primary surplus steady state deviation')
    g ${g}$ (long_name='government spending')
    nu ${\nu}$ (long_name='AR(1) monetary policy shock process')
    a  ${a}$ (long_name='AR(1) technology shock process')
    tau ${\tau}$ (long_name='AR(1) taxes process')
    z_c ${z_c}$ (long_name='consumption shock')
    ;

varexo
    eps_a ${\varepsilon_a}$ (long_name='technology shock')
    eps_nu ${\varepsilon_{\nu}}$ (long_name='monetary policy shock')
    eps_tau ${\varepsilon_{\tau}}$ (long_name='Government Spending/Taxes Shock')
    eps_c ${\varepsilon_c}$ (long_name='consumption shock')
    ;

parameters
    alppha ${\alpha}$ (long_name='capital share')
    betta ${\beta}$ (long_name='discount factor')
    rho_a ${\rho_a}$ (long_name='autocorrelation technology shock')
    rho_nu ${\rho_{\nu}}$ (long_name='autocorrelation monetary policy shock')
    siggma ${\sigma}$ (long_name='intertemporal elasticity of substitution')
    phi ${\phi}$ (long_name='unitary Frisch elasticity')
    Gamma_pip ${\Gamma_R}$ (long_name='past inflation weight on monetary rule')
    Gamma_pi ${\Gamma_{\Pi}}$ (long_name='inflation feedback on monetary rule')
    Gamma_b ${\Gamma_b}$ (long_name='public debt feedback on monetary policy')
    Gamma_y ${\Gamma_y}$ (long_name='output feedback on monetary policy')
    epsilon ${\epsilon}$ (long_name='demand elasticity')
    theta ${\theta}$ (long_name='Calvo parameter')
    Mf ${Mf}$ (long_name='Firms innatention')
    M ${M}$ (long_name='consumer innatention')
    phi_s ${\phi_s}$ (long_name='autocorrelation primary surplus')
    phi_s_bar ${\bar{\phi_s}}$ (long_name='autocorrelation steady state primary surplus')
    phi_b ${\rho_b}$ (long_name='steady state primary surplus response to debt')
    SG  ${SG}$ (long_name='Government Spending/GDP at steady state')
    BY ${BY}$ (long_name='Debt/GDP at steady state')
    SY ${BY}$ (long_name='Primary Surplus/GDP at steady state')
    rho_s_bar ${\bar{\rho_s}}$ (long_name='autocorrelation primary surplus target')
    tau_ss ${\tau_{ss}}$ (long_name='government taxes/GDP at steady state')
    rho_tau ${\rho_{\tau}$ (long_name='autocorrelation taxes')
```

```
    rho_c ${\rho_c}$ (long_name='autocorrelation consumption')
    ;

//CALIBRATED PARAMETERS//
betta = 0.989;
alppha = 0.448;
epsilon = 6;
phi = 1;
Gamma_b = 0;
SG = 0.2;
BY = 2;
SY = 0.022;
tau_ss = 0.35;
//PARAMETERS TO BE ESTIMATED//
siggma = 1.3;
theta = 0.65;
Mf = 0.8;
M = 0.8;
Gamma_pip = 0.75;
Gamma_pi = 2;
Gamma_y  = 0.25;
phi_s = 0.4;
phi_s_bar = 0.35;
rho_s_bar = 0.50;
phi_b = 0.02;
rho_nu  =0.5;
rho_a   = 0.5;
rho_tau = 0.5;
rho_c = 0.5;

model(linear);
//Composite parameters//
#Omega=(1-alppha)/(1-alppha+alppha*epsilon);
#lambda=(1-theta)*(1-betta*theta)/theta*Omega;
#kappa=lambda*(siggma+(phi+alppha)/(1-alppha));
#R = 1/betta;
#SC = 1-SG;
#psi_a = SC*(1+phi)/(siggma*(1-alppha)+SC*(1+phi)-SC*(1-alppha));
#psi_g = SG*siggma*(1-alppha)/(siggma*(1-alppha)+SC*(1+phi)-SC*(1-alppha));

//1. Behavioral New Keynesian Phillips Curve
pi=Mf*betta*pi(+1)+ lambda*mc_hat;
//2. Deviation of Marginal Cost
mc_hat = (siggma/SC + (1+phi)/(1-alppha) - 1)*y_gap + z_c;
//3. Behavioral Euler Equation
c = M*c(+1) - 1/siggma*(r-pi(+1)) + 1/siggma*(z_c - z_c(+1));
//4. Interest Rate Rule
r= Gamma_pip*r(-1) + (1-Gamma_pip)/betta*(Gamma_pi*pi + Gamma_y*y_gap - Gamma_b*b) + nu;
//5. Public Debt Law of Motion
b = r + R*(b(-1) - pi + y(-1) - y) - (R-1)*s_y;
//6. Primary Surplus Rule
s_y = phi_s*s_y(-1) + phi_s_bar*s_y_bar;
//7. Steady State Primary Surplus
s_y_bar = rho_s_bar*s_y_bar(-1) + phi_b*(R/(R-1))*b(-1);
//8. Government Spending
g = y + 1/SG*(tau_ss*tau - SY*s_y);
//9. Definition real interest rate (Fisher Rule)
r_real=r-pi(+1);
//10. Definition natural output
y_nat = psi_a*a - psi_g*g;
```

```
//11. Definition output gap
y_gap=y-y_nat;
//12. Market Clearing
y = SC*c + SG*g;
//13. Production function
y=a+(1-alppha)*n;
//14. Monetary policy shock
nu=rho_nu*nu(-1)+eps_nu;
//15. TFP shock
a=rho_a*a(-1)+eps_a;
//16. Government Spending/Taxes Shock
tau = rho_tau*tau(-1) + 1/tau_ss*eps_tau;
//17. Consumption Shock
z_c = rho_c*z_c(-1) + eps_c;
end;

resid(1);
steady;
steady_state_model;
    pi = 0;
    mc_hat = 0;
    c = 0;
    n = 0;
    r = 0;
    b = 0;
    s_y = 0;
    s_y_bar = 0;
    g = 0;
    r_real = 0;
    y_gap = 0;
    y_nat = 0;
    y = 0;
    nu = 0;
    a = 0;
    tau = 0;
    z_c = 0;
end;
check;

shocks;
    var eps_nu = 0.1^2;
    var eps_a = 0.1^2;
    var eps_tau = 0.1^2;
    var eps_c = 0.1^2;
end;

varobs y g pi r;

estimated_params;
siggma, normal_pdf,     1.3, 0.05;
theta, beta_pdf, 0.65, 0.1;
Mf, beta_pdf, 0.8,      0.05;
M, beta_pdf, 0.8, 0.05;
Gamma_pip, beta_pdf, 0.75, 0.05;
Gamma_pi, normal_pdf, 2.0, 0.35;
Gamma_y, gamma_pdf, 0.25, 0.1;
phi_s, beta_pdf, 0.4, 0.05;
phi_s_bar, normal_pdf, 0.35, 0.05;
rho_s_bar, beta_pdf, 0.5, 0.15;
phi_b,  inv_gamma_pdf, 0.05, 0.05;
```

```
rho_nu, beta_pdf, 0.5, 0.25;
rho_a,  beta_pdf, 0.5, 0.25;
rho_tau, beta_pdf, 0.5, 0.25;
rho_c, beta_pdf, 0.5, 0.25;
stderr eps_nu, inv_gamma_pdf, 1.0, inf;
stderr eps_a, inv_gamma_pdf, 1.0, inf;
stderr eps_tau, inv_gamma_pdf, 1.0, inf;
stderr eps_c, inv_gamma_pdf, 1.0, inf;
end;

estimation(datafile = data_final1, mh_replic = 100000, mode_compute = 6);
stoch_simul(order = 1,irf=60) y g c n r pi r_real b s_y nu a tau z_c;
write_latex_dynamic_model;
```