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Two Essays on Dynamic Analysis of Imperfectly Competitive Markets

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Resumo

O objetivo desta dissertação é contribuir para a literatura que utiliza modelos de oligopólio dinâmicos, baseados no artigo seminal de Ericsson e Pakes (1995), para analisar questões relevantes de organização industrial. A relevância desses modelos está na capacidade de gerar resultados mais próximos dos dados observados na realidade, com firmas se tornando heterogêneas endogenamente, como resultado das ações tomadas por elas em resposta a choques idiossincráticos e comuns à indústria. A dissertação primeiramente expõe o framework criado por Ericsson e Pakes (1995) e algoritmos desenvolvidos pela literatura para encontrar um equilíbrio de modelos dinâmicos baseados nesse framework. Em seguida, modifica-se o modelo dinâmico de Chen (2009) para possibilitar entrada e saída de firmas. Chen analisa o impacto de fusões com produtos quase homogêneos, mas não considera entrada e saída; ele também encontra sinais de corridas de capacidade, que levam a indústria para estados assimétricos. Ao possibilitar entrada e saída, encontro resultados distintos. No capítulo seguinte, desenvolvo um modelo dinâmico capaz de analisar integração vertical, pois modela explicitamente os segmentos upstream e downstream de uma indústria. Então simulo a evolução de uma indústria com parâmetros inspirados na literatura do mercado de gasolina, onde a integração entre refino e revenda foi bastante analisada, e obtenho resultados compatíveis com a literatura empírica. Em particular, identifico que integração vertical pode, em determinadas circunstâncias, dificultar entradas, ao impedir entradas não integradas. Por fim, identifico possíveis extensões para os modelos analisados na dissertação.

Palavras-chave: oligopólio dinâmico, entrada, saída, integração vertical, gasolina.

Abstract

The goal of this dissertation is to contribute to the literature that uses dynamic oligopoly models, based on the seminal paper by Ericsson and Pakes (1995), to analyze relevant questions in industrial organization. The relevance of these models is in the capacity to generate results closer to data observed in reality, with firms becoming heterogeneous endogenously, as a result of actions taken by them in response to idiosyncratic and common industry shocks. The dissertation first exposes the framework created by Ericsson and Pakes (1995) and algorithms developed by the literature to find an equilibrium of dynamic models based on this framework. Following, we modify the dynamic model of Chen (2009) to allow for entry and exit of firms. Chen analyzes the impact of mergers with near-homogeneous products, but does not consider entry and exit; he also finds signs of preemption races in capacity, which take the industry to asymmetric states. When we allow for entry and exit, we find different results. In the next chapter, we develop a dynamic model capable of analyzing vertical integration, because it explicitly models the upstream and downstream segments of an industry. Then we simulate the evolution of an industry with parameters inspired in the literature on the gasoline market, in which vertical integration between refine and retail was often analyzed, and obtain results compatible with the empirical literature. In particular, we find that vertical integration might, under some circumstances, make entry more difficult, by preventing nonintegrated entries. In the end, we identify possible extensions to the models analyzed in the dissertation.

Keywords: dynamic oligopoly, entry, exit, vertical integration, gasoline.

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Chapter 1

Introduction

These essays relate to the literature started by Ericsson and Pakes[10] on applied dynamic analysis of imperfectly competitive markets. The authors created a general framework for a dynamic stochastic game, with a discrete state space. The framework was able to simulate industry evolution paths closer to what was observed in real world data. Firms faced both idiosyncratic and common industry shocks, leading to reversals of fortunes and simultaneous entry and exit, both phenomena observed in real data but, until that date, rarely replicated by theoretical models. Simultaneously, Pakes and McGuire[24] created an algorithm to solve for an equilibrium of this game. These two developments led to several papers that, building their models on this framework, were able to bring insights to important questions on the dynamics of imperfectly competitive markets, and often showed results that were much more compatible with patterns observed in real world data than the results obtained from other more elegant theoretical models.[1]

For instance, Besanko and Doraszelski[3] used the framework to analyze the issue of how industries have persistent asymmetric structures over time. Doraszelski and Markovich[7] analyzed the effects of advertising on the evolution of the industry structure. Gowrisankaran[16] analyzes an industry in which mergers may appear endogenously.

[1]Furthermore, although they are not the focus of these essays, it is worth noting that this framework was used to develop methods to estimate the industry parameters needed for simulations from real world data. In Chapter 5 we provide a brief overview of a possible estimation strategy and how it could be used in the models object of these essays. For a more detailed survey, see Ackerberg et al.[11].

Benkard[2] analyzes competition in the wide bodied commercial aircraft industry and includes learning-by-doing as a dynamic feature. Besanko et al.[12] then shows that when organizational forgetting is added to a learning-by-doing model, results might be different. Finally, Fershtman and Pakes[14] develop a model of collusion. And these are only a few examples, but that already show the potential of the dynamic framework.

The purpose of the essays here is to contribute to this literature. First, in Chapter 3, we have a simple goal. Chen[4] analyzed the dynamic effects of mergers in triopolies with near-homogeneous products and found results opposite to those of often used static models. In sum, price increases after the merger, and keep increasing with time, as the industry adjusts to the new structure and reduces installed capacity. In addition, Chen found evidence of preemption races, in that firms invest in capacity over time to become the industry leader, until one of them pulls ahead; then the others stop investing and become smaller firms, leading to industries with asymmetric structures. But the model of Chen does not consider entry and exit. In fact, previous models which analyzed the issue of preemption races and persistent asymmetric industry structures also did not consider entry and exit (Besanko and Doraszelski[3] and Besanko et al.[13]).

In view of that, we simulate a model similar to Chen's, but including the possibility of entry and exit - with random, but exogenous, entry costs and scrap values. We find that the possibility of entry and exit attenuates the effects of mergers with time, bringing prices back to the long-run level eventually (the length of time taken depending on the range of entry costs), and that preemption races are less likely, and the industry is more often symmetric, contrary to what Chen had found.

These results are important for two reasons. First, they show that we might mimic real world entry patterns and their effect on mergers, since our model brings prices back to the long run level, but not immediately, and the time taken depends on the primitives of the model. Second they demand caution when interpreting the asymmetric structures observed in the real world as the result of the dynamic competitive interaction of firms and preemption races, as the previous literature suggested. Once entry and exit are allowed, as we did here, industries appear more likely to be symmetric. Thus real world asymmetries, absent any other factors that facilitate preemption races (e.g., learning-by-doing), may be a sign of high barriers to entry and/or exit.

In Chapter 4, we have two goals. First we develop a dynamic model that is capable of analyzing vertically integrated industries. To the best of our knowledge, no one adapted the Ericsson and Pakes framework to explicitly model two segments of an industry: upstream and downstream. And several important issues appear in this context, such as vertical mergers, exclusive contracts between suppliers and clients, refusals to deal, among other business practices often adopted between manufacturers and retailers. Furthermore, these issues often have an important dynamic side. For instance, as indicated by the Federal Trade Commission of the United States in 2005[23], in a report on the gasoline industry, a possible harm of vertical integration is that it might increase barriers to entry in an industry by forcing potential entrants to enter both upstream and downstream segments. Since entry is an inherently dynamic issue, it cannot be assessed with a static model.

Since the issue of vertical integration was an important issue in the gasoline industry, our second goal is to simulate the model for parameters plausible to this industry and compare the results with the empirical literature on this market. This literature (see references in Chapter 4) found increases in wholesale (upstream) prices of gasoline to independent retailers following vertical mergers, and could link those to an incentive to raise rivals' costs (exclusionary incentives) on the part of integrated firms. But, nonetheless, the empirical literature could not establish increases in retail prices (and thus harm to consumers) as a result of these exclusionary incentives and wholesale price increases. Moreover, although the increase in entry barriers was an issue explicitly acknowledged, the empirical literature did not assess. We find, in our simulations, that there are instances in which our model predicts that retail prices would increase as a result of exclusionary incentives and, also, that these incentives may prevent non integrated entry. But, in the long-run, eventually integrated firms enter the industry and bring prices down.

This dissertation is divided as follows: (i) in Chapter 2 we provide a description of the basic Ericsson and Pakes framework and of two algorithms to compute its solution; (ii) in Chapter 3 we analyze a modified version of the model used in Chen[4], with the possibility of entry and exit; (iii) in Chapter 4 we develop a dynamic model capable of assisting in the analysis of vertically integrated industries and test it on the gasoline industry; and finally (iv) in Chapter 5 we indicate possible extensions.

Chapter 2

The Framework for Applied Dynamic Analysis in Industrial Organization

This chapter will explain the basic framework created in Ericsson and Pakes[10] and revised by Doraszelski and Satterthwaite[9], and then describe two algorithms used for computing an equilibrium of the dynamic game. The models analyzed in Chapters 3 and 4 were based on the framework exposed in this chapter and follow its structure closely.

2.1 The model

This section is mainly based on Doraszelski and Pakes[8], instead of the seminal article by Ericsson and Pakes[10]. This is because the original model could not guarantee the existence of an equilibrium in pure strategies (i.e., a computable equilibrium). Later Doraszelski and Satterthwaite [9] proved that by inserting random entry costs and random scrap values it was possible to guarantee the existence of an equilibrium in pure strategies.[1]

[1] Although this equilibrium cannot be calculated exactly, but only approximated, it may still be important to know that the solution we are trying to compute is an approximation to something that actually exists. For instance, as indicated by Doraszelski and Satterthwaite, if the model is correctly specified, but we still cannot obtain convergence, we will know there is a problem with the algorithm, not existence of equilibrium.

2.1.1 Overview

The players in the game are the incumbent firms in an industry and potential entrants. All players seek to maximize the expected sum of profits in all periods. These profits are obtained through static competition in "spot markets" every period. The outcome of this competition in each period depends on the states of the firms in that period, which can be affected by the firms' dynamic decisions (e.g., investments) in past periods.

The sequence of moves in the game is as follows. At the beginning of each period: (i) incumbents evaluate whether it is better to stay in the industry or exit, based on the exogenous scrap value they could obtain, and, if they stay, decide how much investment will maximize the present value of the sum of their expected future profits; (ii) potential entrants compare the exogenous costs of entry with the present value of the expected profits and decide whether or not to enter. These decisions are all simultaneous.

Entry and exit take one period to be completed: that is, firms leaving the industry still gain profits in the period they decide to exit, and entrants will only gain profits in the next period. After these decisions are made, spot market competition happens, idiosyncratic and common industry shocks to the firms' states are realized and the states of the firms evolve.

2.1.2 State space and evolution of the industry

Firms are described by their states. The possible states are integer values which are bounded above and below if some conditions on the primitives of the model are satisfied.[2] Therefore, the possible states of each firm can be represented by $\Omega = \{1, 2, ..., \overline{\omega}\}$. The states can be interpreted as indexes of productive efficiency, quality, capacity of production, among others (formally the state of a firm includes also the "states" of its competitors; but until the last paragraph of this section, we refer to the firm's state as solely its index).

The evolution of the states depends on stochastic processes that determine the

[2] They are listed below in the section 2.2, about the characterization of the equilibrium.

idiosyncratic and industry shocks firms suffer each period. By the end of a period, the state of each incumbent firm is defined by $\omega_i' = \omega_i + v_i - \zeta$, in which v takes the values $\{0, 1\}$ and represents a shock idiosyncratic to firm i, and $\zeta \in \{0, 1\}$ represents a shock affecting the whole industry. The presence of both shocks is an important feature. If all shocks were common to the industry, we would not see firms with different fortunes under the same economic environment, and if all shocks were idiosyncratic, we would not see positive correlation between the performances of the firms in the same industry.

The distribution of the idiosyncratic shocks depends on the amount of investment done by the firm. In most models used in the literature, and in the models of Chapters 3 and 4 as well, the distribution below is adopted:

$$\begin{aligned} p(v = 1|x) &= \frac{\alpha x}{1 + \alpha x} \\ p(v = 0|x) &= \frac{1}{1 + \alpha x}, \end{aligned} \tag{2.1}$$

in which x is the amount invested and α is a common investment multiplier representing its effectiveness. This probability is stochastically increasing in x.

The common shock affecting the whole industry is usually given by an exogenous probability:

$$\begin{aligned} p(\zeta = 1) &= \delta \\ p(\zeta = 0) &= 1 - \delta. \end{aligned} \tag{2.2}$$

It can be interpreted, for instance, as developments in the products not analyzed in this industry, but that compete, to some extent, with those analyzed.

The state of an entrant by the end of the period it decided to enter is usually similarly defined: $\omega^{e\prime} = \omega^e + v^e - \zeta$, with v^e and ζ following the same distributions above, and ω^e an exogenously determined initial state.

The exit and investment decisions of all incumbents, and entry and investment decisions by the entrant determine the probabilities of transiting from an industry structure to another.

As mentioned above, the state of a firm includes both its index and the indexes of its competitors. In this paragraph we define a firm's state in this way. The states of the incumbent firms completely characterize the industry structure in any period. There are several ways to represent the state of a firm. For instance, because Doraszelski and Pakes assume symmetric and anonymous primitives and focus on symmetric and anonymous equilibrium[3], they can represent the state of a firm i as a tuple (ω_i, s), $s = (s_1, ..., s_{\overline{\omega}})$, in which s_j is the number of firms in state j, and ω_i is the state of firm i. The industry state space is then (in which $\overline{n}$ is the maximum of firms allowed in the industry):

$$\overline{S} = \Big\{(\omega_i, s) : \omega_i \in \Omega, s = (s_1, ..., s_{\overline{\omega}}), s_\omega \in Z^+, \sum_{\omega \in \Omega} s_\omega \leq \overline{n}\Big\}. \tag{2.3}$$

2.1.3 Spot market competition

The model we will use here treats the outcome of spot market competition, the per period profit function $\pi(\omega_i, \omega_{-i})$, as a primitive. That is because, once a demand system, a cost function and an equilibrium assumption are assumed for the static per period competition, the profits of the firms in each period are a function of their states. The profits resulting from every state can then be calculated beforehand and stored to be used in calculating expected value functions and optimal policies in the dynamic game.[4]

2.1.4 Incumbent firms

At the beginning of each period, each incumbent draws a scrap value ϕ from a random distribution $F(.)$. The scrap values are independently and identically distributed across firms and periods - usually an uniform distribution on a bounded interval is used. In view of that, in principle, we should consider the scrap value as an additional state variable when writing the problem of an incumbent. However, to make computation easier, the model considers the problem before the scrap value is realized, using an expected value

[3]A set of functions $\{f_i(.)\}_{i=1}^n$ is symmetric if $f_i(\omega_i, \omega_{-i}) = f_j(\omega_i, \omega_{-i}), \forall i, j$; and a function $f(.)$ is anonymous if $f(\omega_i, \omega_{-i}) = f(\omega_i, \omega_{\pi(-i)})$ for all permutations $\pi(-i)$. Although I also make these assumptions in the models analyzed in chapters 3 and 4, I use a different representation of the state space, as explained in the Appendix.

[4]But that does not need to be the case. It is possible to include decisions with dynamic impact in the per period competition, which would prevent calculating profits beforehand and using them as a primitive. For instance, Besanko et al.[12] does this to analyze learning-by-doing.

function:

$$V(\omega_i, \omega_{-i}) = \pi(\omega_i, \omega_{-i}) + r(\omega_i, \omega_{-i})\tilde{\phi} \\ + (1 - r(\omega_i, \omega_{-i}))\big\{ \max_{x_i} (-x_i + \beta E[V(\omega_i', \omega_{-i}')|\omega_i, \omega_{-i}])\big\}, \quad (2.4)$$

in which $r(.)$ is the probability that the firm exits (i.e., that it draws a scrap value above its continuation value), $\tilde{\phi}$ is the expected scrap value, conditional on the firm exiting, and E is the expectation operator over the probability distribution of possible states next period.

A useful notation for the continuation value is used by Doraszelski and Pakes:

$$\begin{aligned} E[V(\omega_i', \omega_{-i}')|\omega_i, \omega_{-i}] &= \sum_v W(v|\omega_i, \omega_{-i})p(v|x_i) \\ W(v|\omega_i, \omega_{-i}) &= \sum_{\omega_{-i}', \zeta} V(\omega_i + v - \zeta, \omega_{-i}')q(\omega_{-i}'|\omega_i, \omega_{-i}, \zeta)p(\zeta), \end{aligned} \quad (2.5)$$

in which $W(.|.)$ represents a firm's expected continuation value, conditional on its investment resulting in outcome v, and using $q(.)$ as the perception about the probability of next period's state of its competitors, ω_{-i}'. In equilibrium, this perception matches the actual probabilities governing the evolution of the industry.

2.1.5 Entrants

At the beginning of each period, each potential entrant draws an entry cost ϕ^e from a random distribution $F^e(.)$ - again, usually an uniform distribution on a bounded interval. Entrants cannot delay entry: if an entrant does not enter, it disappears, and new potential entrants appear next period. For simplicity, here and in Chapters 3 and 4, we will assume that only one entry can happen each period (as in Ericsson and Pakes and most of literature). The entry cost is independently and identically distributed across periods.

The entrant may also invest some amount to improve the distribution of the states in which it enters. Similarly to the incumbents, we consider the expected value function of an entrant to reduce the state space.

$$V^e(\omega) = r^e(\omega)\big\{ \max_{x_i^e} (-\tilde{\phi}^e - x_i^e + \beta \sum_v W^e(v|\omega)p(v|x_i^e))\big\}, \quad (2.6)$$

in which W^e is defined analogously to the W defined above for the incumbents, $\tilde{\phi}^e$ is the expected cost of entry, conditional on having entered, and $r^e()$ is the probability of entering (i.e., of drawing an entry cost lower than the continuation value).

2.2 Equilibrium characterization and existence

Doraszelski and Pakes, as do most of the literature, work with a Markov perfect equilibrium as the solution to the game.[5] In this equilibrium, for every state $\omega \in \overline{S}$, each incumbent and the potential entrant: (i) chooses optimal investments, and entry and exit decisions based only on the current state of the industry and on their perceptions about the distribution of the future states, and (ii) these perceptions are consistent with the objective evolution of the industry resulting from their actions. In order to check that an equilibrium was found, one can check whether the value and policy functions found satisfy the Bellman equations described above.

In order to guarantee the existence of an equilibrium in pure strategies to the game described here, in addition to random entry costs and scrap values (which are required to obtain continuous best-reply correspondences, standard in existence proofs in game theory), finite players and a finite state space are also required.[6] These conditions will be observed in Chapters 3 and 4. Moreover, they make the state space finite, which allows us to compute value and policy functions. Also, because of the finite state space,

[5]This concept was developed by the theoretical industrial organization literature, namely the seminal articles by Maskin and Tirole [22], and later used by the applied literature. As explained by Doraszelski and Pakes, the equilibrium is Markov because value and policy functions are functions only of the current state, and is perfect because it is subgame perfect: since any influence of past behavior, regardless of what it was, is already captured by the current state, each state is a subgame and vice-versa, and the equilibrium definition ensures optimal behavior at each state, thus at each subgame.

[6]If certain conditions, provided in Ericsson and Pakes[10], are satisfied, then, in equilibrium, there is a maximum number of firms that the industry will ever support, and a maximum state above which no firm will ever go. The main condition is that the profit function is bounded from above and tends to zero as the number of firms increase. This makes the return from investment become smaller than the additional profit at some point, therefore imposing a maximum state above which firms will not go, and also makes the profits go below the positive scrap value for a finite large enough number of firms, therefore imposing a maximum number of firms.

the evolution of the industry is a finite state Markov chain, which guarantees that there exists at least one recurrent class - that is, a group of states such that, once the industry enters this group, it will remain there with probability one. This will be important for the stochastic algorithm described in the next section.

2.3 Algorithms to Solve for an Equilibrium

2.3.1 Deterministic

A deterministic algorithm was developed by Pakes and McGuire[24] to calculate a Markov perfect equilibrium[7]. The algorithm tries to find a fixed point for the Bellman equations 2.4 and 2.6 above. It starts with an initial guess for the value function for each state and for the entry, exit, and investment policies. Then, having these values, the algorithm updates them by the following procedure.

For each state, we treat the values from the previous iteration as the true expected values and policies as optimal policies. These are then used to calculate the conditional continuation values $\{W(.|.)\}$ according to the formula given above in equation 2.5.

As suggested by Doraszelski and Pakes, one can first calculate the probabilities of each future state of each agent, conditional on the current states of other agents, the information in memory and the industry shock. The probabilities of a future state ω_i' for a firm i are given by:

$$p^{l-1}(\omega_i'|\omega_i,\omega_{-i},\zeta)=\begin{cases} r^{l-1}(\omega_i,\omega_{-i}), & \text{if } \omega_i'=\text{exit} \\ (1-r^{l-1}(\omega_i,\omega_{-i}))p(v_i=1|x^{l-1}(\omega_i,\omega_{-i})), & \text{if } \omega_i'=\omega_i+1-\zeta \\ (1-r^{l-1}(\omega_i,\omega_{-i}))(1-p(v_i=1|x^{l-1}(\omega_i,\omega_{-i}))), & \text{if } \omega_i'=\omega_i-\zeta \end{cases} \tag{2.7}$$

in which $l-1$ indicates the iteration, and $r(.)$ is the probability of exiting the industry.

[7] As indicated by Doraszelski and Pakes, we notice that there may be multiple equilibria, and often there are. How to deal with this issue is still an open question in the literature.

Having calculated these probabilities for each firm, and also the probability that a new firm enters the industry ($r^{e,l-1}(\omega)$), for each state, it is possible to calculate, again for each firm and each state, the perceived probability of the future states of the competitors of the firm. Based on information from the last iteration, $l-1$, the current iteration's expression for the perception of a firm i about the probability of the future states of its competitors being ω'_{-i} (with an entry occurring) is:

$$q^l(\omega'_{-i}|\omega_i, \omega_{-i}, \zeta) = \prod_{j \neq i} p^{l-1}(\omega'_j|\omega_j, \omega_{-j}, \zeta) r^{e,l-1}(\omega). \tag{2.8}$$

Then one can substitute this perception in the expression for $W(.|.)$ in equation 2.5, and use the expected values in memory from the previous iteration and the probability of the industry shock to calculate $\{W(.|.)\}$ for each possible investment outcome of each firm.

Because the $\{W(.|.)\}$ are the expected future values conditional on the outcome of the agents' investment, it is possible then, for each agent, to obtain its optimal investment decision as the result of a single agent optimization process. That is, the investment decision determines the probability that the agent obtains each outcome (therefore, each $\{W(.|.)\}$), and the firm can calculate the expected value of its investment decisions based on these probabilities - the problem is then to choose investment so as to maximize this expected value.

Given the optimal investment, we can use the expression on the right hand side of the value function and the distribution of scrap values to determine the optimal exit policy. That is, the probability that the firm will draw a scrap value above the continuation value and then exit the industry. Note that exit policies are probabilities because we are using expected value functions.

Having obtained the new value function and policies, we begin the process again (the process for entry values and policies is similar, but using the entry expressions). This iteration will continue until the distance between two consecutive iterations is below a chosen limit.

2.3.2 Stochastic

The burden of computing equilibrium using the deterministic algorithm above is very large. That is because the algorithm goes through all the states in each iteration (it is synchronous), and the number of states grows polynomially in the maximum number of firms allowed and in the bounds on the possible states[8]. Moreover, when there is more than one state per firm, the state space grows exponentially in the number of states per firm. Also, at each state, the algorithm has to calculate continuation values for each firm, which involves summing over all possible future states of the industry for each firm. The number of possible future states grows exponentially in the number of firms and states per firm.

To address this issue, Pakes and McGuire[25] developed a stochastic algorithm. This algorithm is asynchronous, in that it does not update all the states in each iteration. Instead it acts as if it were an agent learning optimal values and policies.

It works as follows: given an initial state and an initial guess for expected values and policies, we can calculate new optimal policies and values for this initial state, following a single agent optimization procedure similar to the one used by the deterministic algorithm; then we take a draw from the transition distribution generated by these policies, in order to simulate what would be the next state, and use this simulated next state to calculate the continuation value, based on the expected value functions held in memory. After a significant number of iterations, we will have passed through each state several times and obtained different continuation values, over which we take an average, resulting in accurate value and policy functions (the more often a state was visited, the more accurate the value and policies for that state will be). Therefore, the algorithm simulates a learning procedure and eliminates the burden of calculating continuation values by summing over all possible future states, as it takes draws of the future state each iteration and calculates the continuation value only for that draw, averaging over all draws in the end. This eliminates the curse of dimensionality affecting this part of the deterministic algorithm.

[8]In fact, it grows polynomially because of the symmetry and anonymity assumptions. Otherwise it would grow exponentially.

Also, the algorithm only attempts to calculate accurate policies and value functions on a recurrent class. That is because after a finite number of iterations, the algorithm will enter a recurrent class and stay there forever. Thus, because the number of states in a recurrent class may be much smaller than the state space and does not need to grow in any way when the state space grows, the algorithm may eliminate the burden of solving large state space problems entirely.

A downside of the algorithm is that it increases the number of iterations needed. Because the algorithm works as if an agent is learning its value function and optimal policies through time, it takes a lot of iterations to obtain accurate policies and values for every state in the recurrent class, specially those visited less often. Thus the algorithm is more efficient only for large scale problems, with more than one state variable per firm and/or a large number of firms, to which a deterministic algorithm could not be applied.

Pakes and McGuire stopped every million iterations and calculated the value functions exactly over the states visited in these latter million iterations. Then they compared these value functions with the ones obtained through the learning algorithm. If their correlation (weighted by the number of times each state was visited) is above 0.995 and the difference between their weighted means is less than one percent, then the algorithm stopped.

The testing part is still subject to the curse of dimensionality, as they have to sum over all possible future states, for each state in the recurrent class, in order to obtain exact continuation values. But because the recurrent class may be significantly smaller than the state space, this burden may be smaller as well.[9]

[9]At last, it is worth mentioning that, more recently, Fershtman and Pakes[15] developed a test by simulation that is not subject to the curse of dimensionality and can be applied to the stochastic algorithm.

Chapter 3

Entry and Preemption Races

3.1 Introduction

Chen[4] analyzed the dynamic effects of a merger in an industry with three firms, and in which firms produce almost homogeneous goods, but have different capacity constraints. The objective of the author was to compare the effects of a merger in this scenario to the predicted effects that would be obtained if one fitted the data generated by such an industry to an asymmetric costs static competition model - as antitrust agencies often do.

The result was that the asymmetric static model underestimates the price increase and reduction in welfare created by the merger, while the dynamic model shows that prices get higher and the welfare losses get worse over time. That is because it takes some time for the industry to adjust its capacity to the new scenario. But the static model did not capture that.

Moreover, Chen finds that the industry dynamics is characterized by a preemption race. Firms race each other to become the market leader, investing to increase their capacity, until one of them reaches a capacity of 25. Then the others retreat and stop investing, resulting in their capacity shrinking to 5 (the lowest level admitted in the model) eventually, due to stochastic depreciation. This state (one dominant and two small firms) was the most likely state in the industry.

But the model used by Chen does not allow for entry and exit. In view of that, the goal of this chapter is to test if these results are affected by the possibility of entry and exit in the industry. We will modify his model to allow for entry and exit, and simulate the evolution of the industry using the same parameters. As we will see, we find that the results with entry and exit do differ from the results obtained by Chen.

The next sections describe the model analyzed in this chapter and report the results of the simulations.

3.2 The spot market

Chen used a model of price competition with symmetrically differentiated products and a demand derived from a representative consumer with quadratic utility function. The demand and inverse demand systems are:

$$\begin{aligned} p_j(q_j, q_{-j}) &= a - bq_j - \theta \sum q_{-j},\ \ j = 1, ..., n. \\ q_j(p_j, p_{-j}) &= \frac{1}{b(1+\theta+2\theta^2)}[a(1-\theta) - (1+\theta)p_j + \theta \sum p_{-j}],\ \ j = 1, ..., n, \end{aligned} \tag{3.1}$$

in which a and b are parameters, θ indicates the degree of product differentiation (with 0 being the highest degree, and 1 being totally homogeneous), q is quantity and p is price.

The firms have a cost function with soft capacity constraints, in that they can produce any quantity, but, depending on the parameter η, costs can get prohibitively high if the firm is producing above its capacity k.

$$C(q|k) = \frac{1}{1+\eta}\left(\frac{q}{k}\right)^{\eta} q. \tag{3.2}$$

Also, notice that firms have the same cost functions, differing only in their production capacities. That is, they are considered ex-ante identical, since the capacity is endogenously determined by the investment decisions in the model.

Firms compete each period by setting price and producing to demand. They set price to maximize profit given the prices of the other firms and the demand function. These prices solve the following system of first order conditions, which we solve numerically:

$$\frac{-(1+\theta)}{b(1+\theta+2\theta^2)} * p_j + q_j - (\frac{q_j}{k_j})^{\eta} * \frac{-(1+\theta)}{b(1+\theta+2\theta^2)} = 0, \ j = 1, ..., n. \tag{3.3}$$

3.3 The Dynamic Game

Differently from Chen's original model, an incumbent here must choose whether to exit or continue in the market, by comparing the scrap value ϕ with the expected present value of future profits. Also, as seen in Chapter 2, to guarantee that the model has an equilibrium in pure strategies, we will consider random entry costs and scrap values, and will also consider expected value functions. Therefore, entry and exit policies will not be zero/one functions, but rather probabilities.

In order to obtain the continuation value and compare to the scrap value, the incumbent must choose an optimal level of investment because it influences the level of profits in the future. The incumbent then solves the following Bellman equation:

$$V(k_j, k_{-j}) = \pi(k_j, k_{-j}) + r(k_j, k_{-j})\tilde{\phi} + (1 - r(k_j, k_{-j}))\Big\{ \max_{x_j \geq 0} \big(- x_j + \beta E[V(k'_j, k'_{-j})]\big) \Big\}, \tag{3.4}$$

in which $r(.)$ is the probability that the firm exits, $\tilde{\phi}$ is the expected scrap value conditional on the firm exiting the industry, k is capacity, and x is investment in capacity. Firms do not exit immediately. If a firm decides to exit on period t it will still obtain the spot market profits in that period, completing its exit only on period $t+1$.

Similarly, entry takes one period to occur and entry decisions are simultaneous to exit decisions, occurring at the beginning of a period. Entrants must compare the costs of entry, ϕ^e, with the expected value of future profits. Entrants also cannot invest for the first period and, thus, enter with a pre-determined level of capacity - k^e. But the common industry shock of the period they enter (see below) affects their capacity.

Again, we consider random entry costs and expected values. The entrants solve:

$$V(k^e, k) = r^e(k)\Big\{ - \tilde{\phi}^e + \beta E[V(k'^e, k')]\Big\}, \tag{3.5}$$

in which $r^e(.)$ is the probability of entry and $\tilde{\phi}^e$ the entry cost conditional on entering.

Following Doraszelski and Pakes[8], we can represent the continuation value as follows:

$$\begin{aligned} E[V(k'_j, k'_{-j})] &= \sum_{v} W(v|k)p(v|x_j) \\ W(v|k) &= \sum_{k'_{-j},\zeta} V(k_j + v - \zeta, k'_{-j}|W)q(k'_{-j}, k, \zeta)p(\zeta), \end{aligned} \tag{3.6}$$

in which v is the idiosyncratic shock affecting the firm and $p(.|x)$ is its probability conditional on the investment of the firm (see more below). Thus $W(.)$ is the expected value of future profits for the firm, conditional on the investment result being v and current industry structure being k. ζ is an industry-wide shock affecting all firms and $p(.)$ its probability. Finally $q(.)$ is the perception of the firm about the distribution of the future states of its competitors.

Both idiosyncratic and common shocks take only one or zero values. The probability that an investment will result in a one-unit increase in capacity is given by the same formula used in Chen and in most of the literature:

$$\begin{aligned} p(v = 1|x) &= \frac{\alpha x}{1 + \alpha x} \\ p(v = 0|x) &= \frac{1}{1 + \alpha x} \end{aligned} \tag{3.7}$$

in which α is an investment multiplier.

The common shock affecting the capacities in the industry is given by an exogenous probability:

$$\begin{aligned} p(\zeta = 1) &= \delta \\ p(\zeta = 0) &= 1 - \delta. \end{aligned} \tag{3.8}$$

Below we analyze simulations based on a Markov perfect equilibrium for this game, which was obtained using the deterministic algorithm described in Chapter 2.[1]

[1]We used a modified version of the code made available by Ariel Pakes in his website. The original code did not have random exit and the profit function we use here, and did not compute some of the statistics we analyze, and was modified accordingly.

3.4 Results

As indicated in the introduction, Chen analyzes a model similar to the one described in the previous sections, but without entry and exit. His results were obtained with the following parameters: $a = 4, b = 0.1, \alpha = 0.0625, \beta = 1/1.05, \eta = 10, \theta = 0.9$, and 9 possible states, ranging from 5 of capacity (state 1) to 45 of capacity (state 9). We will use these same parameters here since our goal is to assess the impacts of entry and exit on his results.

As explained, entry costs are treated as random variables. Each period that entry is feasible, the potential entrant draws the entry cost from a uniform distribution between 20 and 100 and can enter the industry with capacity 5 (state 1). The range for entry costs was based on the fact that the mean expected value function for the smallest firm is 34.9, with a standard deviation of 27.8. Therefore, entry costs can be considered relatively high compared to the expected value an ordinary entrant would obtain after entering. Entry takes one period to be completed, as explained above. Also, we inserted a range of scrap values from 8 to 15, which a firm draws each period to evaluate whether or not it exits the industry. Thus, we might consider the costs of entering this industry partially sunk.

Simulating such an industry, we find that although price does increase immediately after the merger, it starts decreasing after that due to entry. But, with these parameters, the decrease is not fast. In fact, it takes more than 25 periods before the industry returns to its old price level. And since the discount rate is based on an annual interest rate of approximately 5%, each period can be compared to a year, resulting in a very long period with higher prices. Figure 3.1 below shows this result. The industry structure in period 0 is (1,1,1) and a merger between the two smallest firms occurs in period 100.[2]

[2]Consumer and total surplus were calculated using the same expressions used by Chen[4].

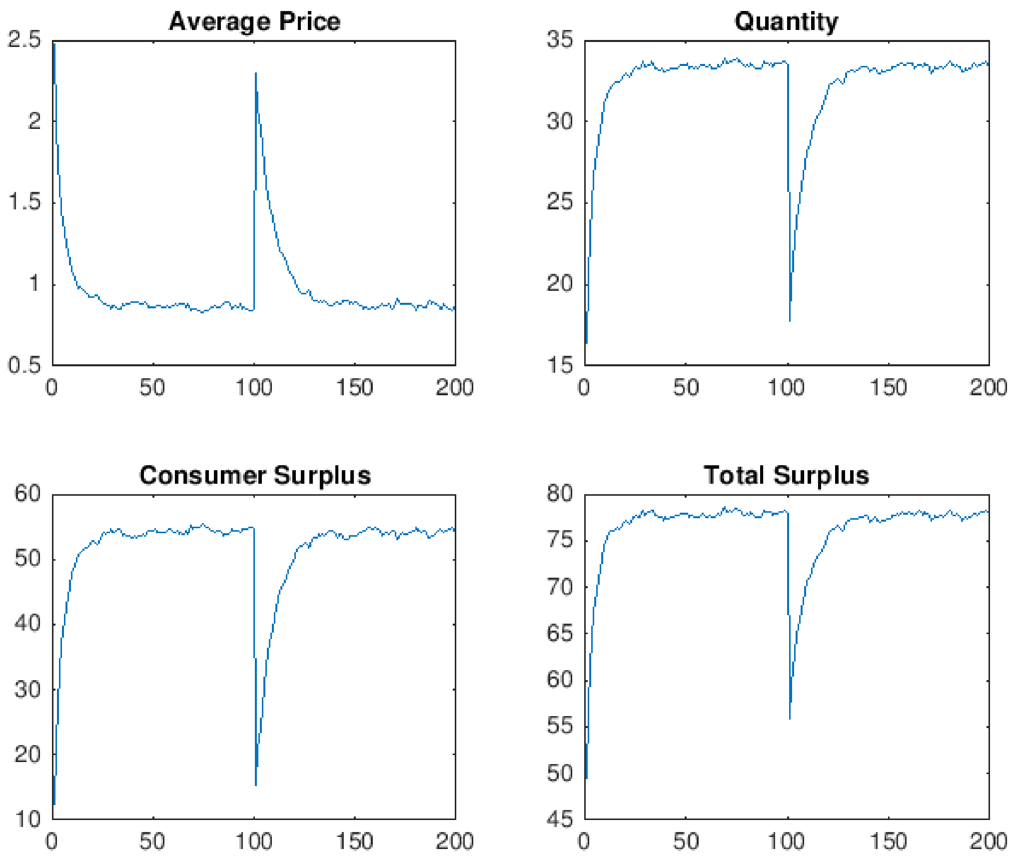

Figure 3.1: $\theta = 0.9$ - Entry costs: 20-100 - Scrap value: 8-15

The lag between the merger and the date when the level of prices returns to the pre-merger level is due to the fact that entry is uncertain. In some of the simulations it occurred right after the merger, and in others it may have taken longer. This uncertainty captured by working with random entry costs could then plausibly match entry patterns in real world industries through the choice of an appropriate range of entry costs. For instance, by reducing the upper bound of the range of entry costs, the industry reaches the original level of prices faster. This can be seen in Figure 3.2 below.

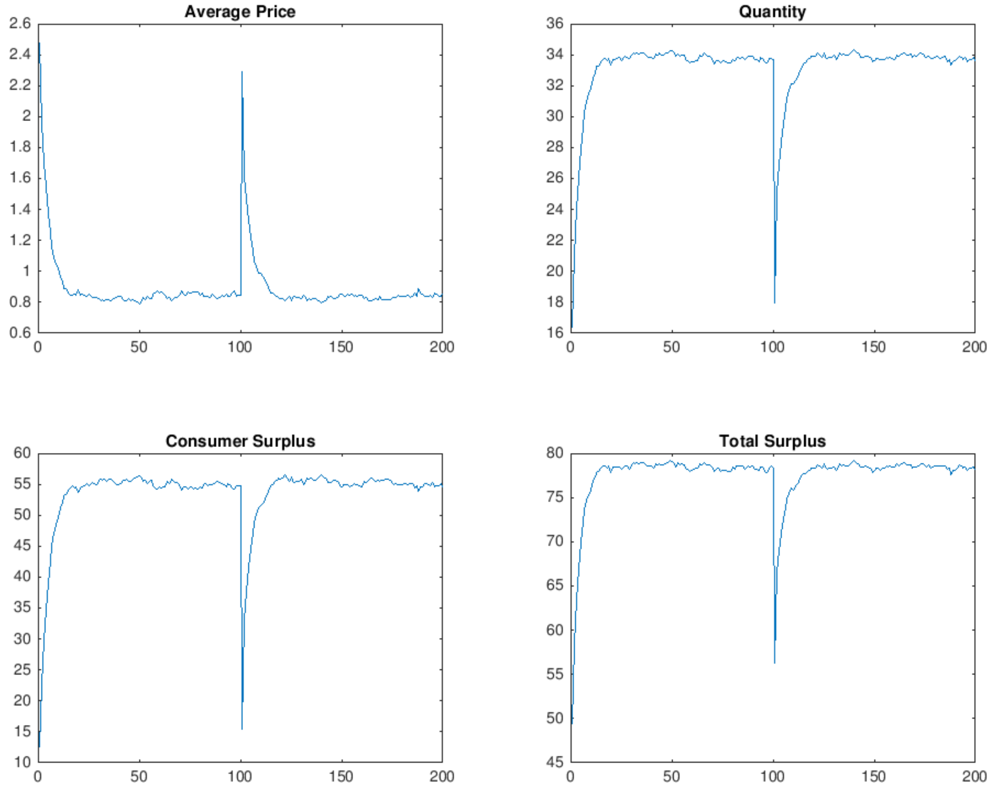

Figure 3.2: $\theta = 0.9$ - Entry costs: 20-35 - Scrap value: 8-15

Also, it is interesting that with more differentiated products ($\theta = 0.5$) - and a range of entry costs comparable with the first specification simulated above (Figure 3.1), that is, relatively high (between 100 and 400[3]) - prices appear to decrease somewhat faster to their level before the merger - see Figure 3.3. This might be explained by the fact that, all else equal, firms earn higher profits with differentiated products and, thus, entrants expect higher profits, accepting higher entry costs. In fact, the mean of the entry probabilities (considering only non-zero probability scenarios) in the specifications below with $\theta = 0.1$ and 0.5 is 25%, whereas with $\theta = 0.9$ this mean is less than 10%.

[3]They are in line with the mean (188.5) and standard deviation (115.8) of the value function of the smallest firm.

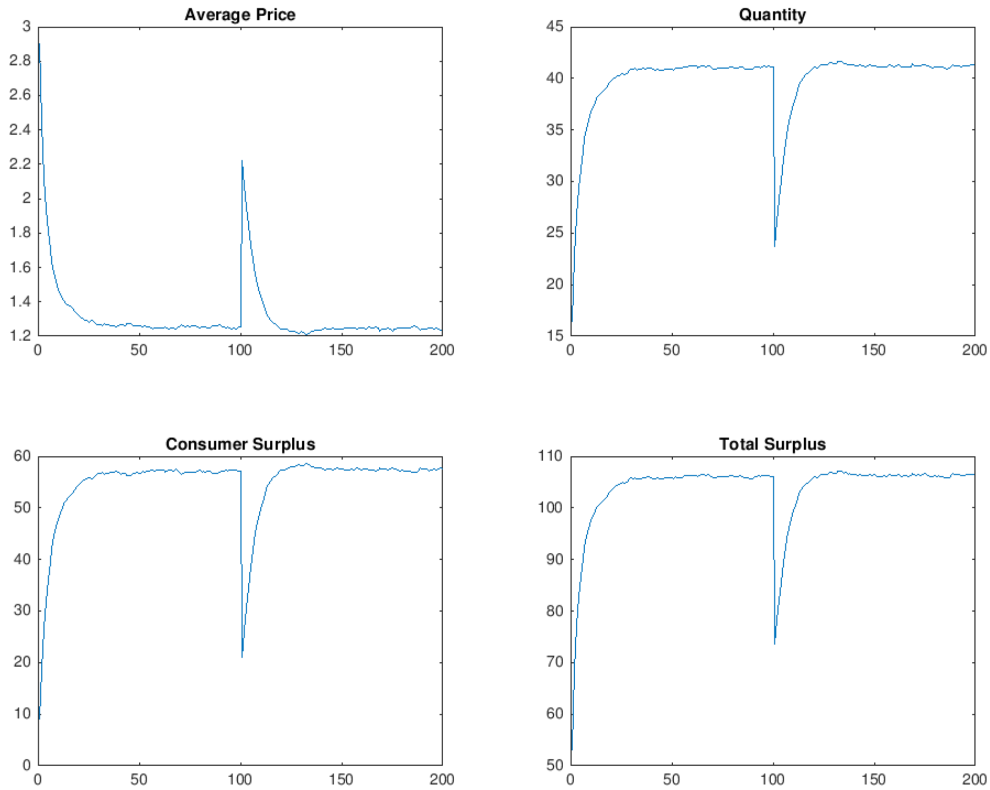

Figure 3.3: $\theta = 0.5$ - Entry costs: 100-400 - Scrap value: 8-15

A similar picture appears with even more differentiated products: $\theta = 0.1$ and entry costs in the range between 300 and 600, again to keep entry costs relatively high, comparable to the first scenario with $\theta = 0.9$. See Figure 3.4 below.

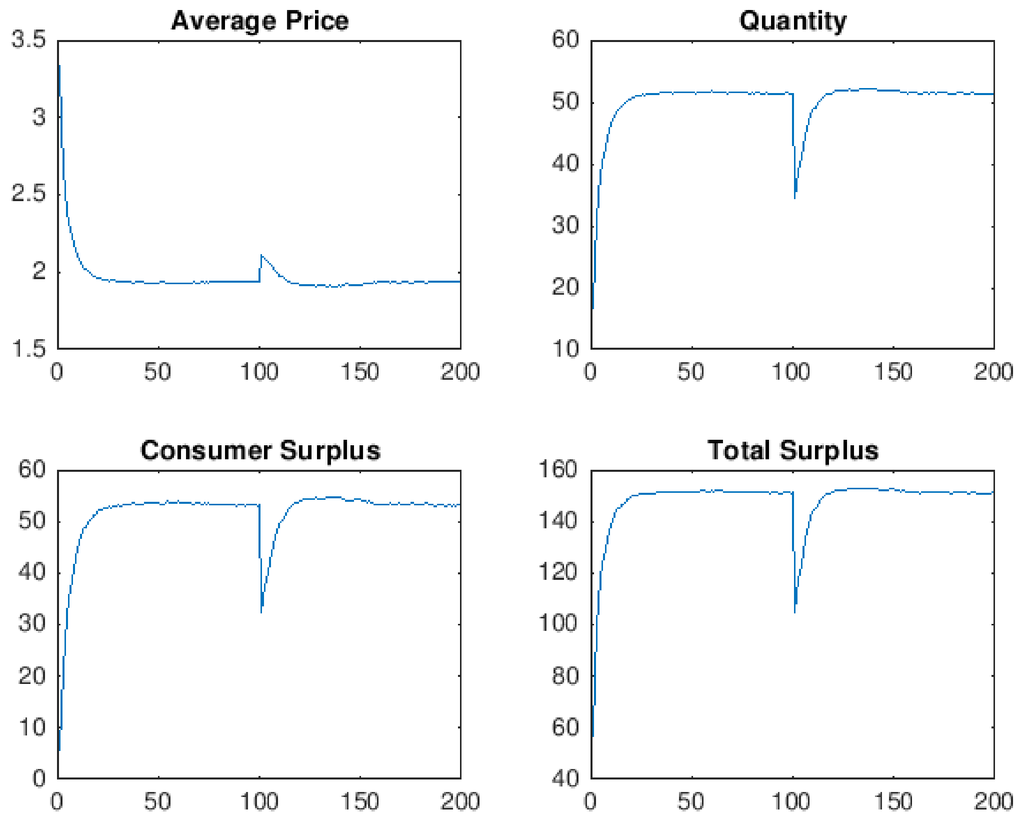

Figure 3.4: $\theta = 0.1$ - Entry costs: 300-600 - Scrap value: 8-15

Furthermore, regarding the preemption race that Chen found with the parameters described, when entry and exit were inserted in the model (with the same parameters as in Figure 3.1) there was no clear sign of it. As can be seen in Figure 3.5 below, a firm invests a positive amount only until it reaches capacity 15 (state 3), no matter the capacity of its rivals. If its rivals have a lower capacity, then the firm might invest more. But it does not stop investing, for instance, when one of its rivals reaches a capacity of 15. Moreover, the most likely state, as measured by its frequency, was (3,3,3), a symmetric state.

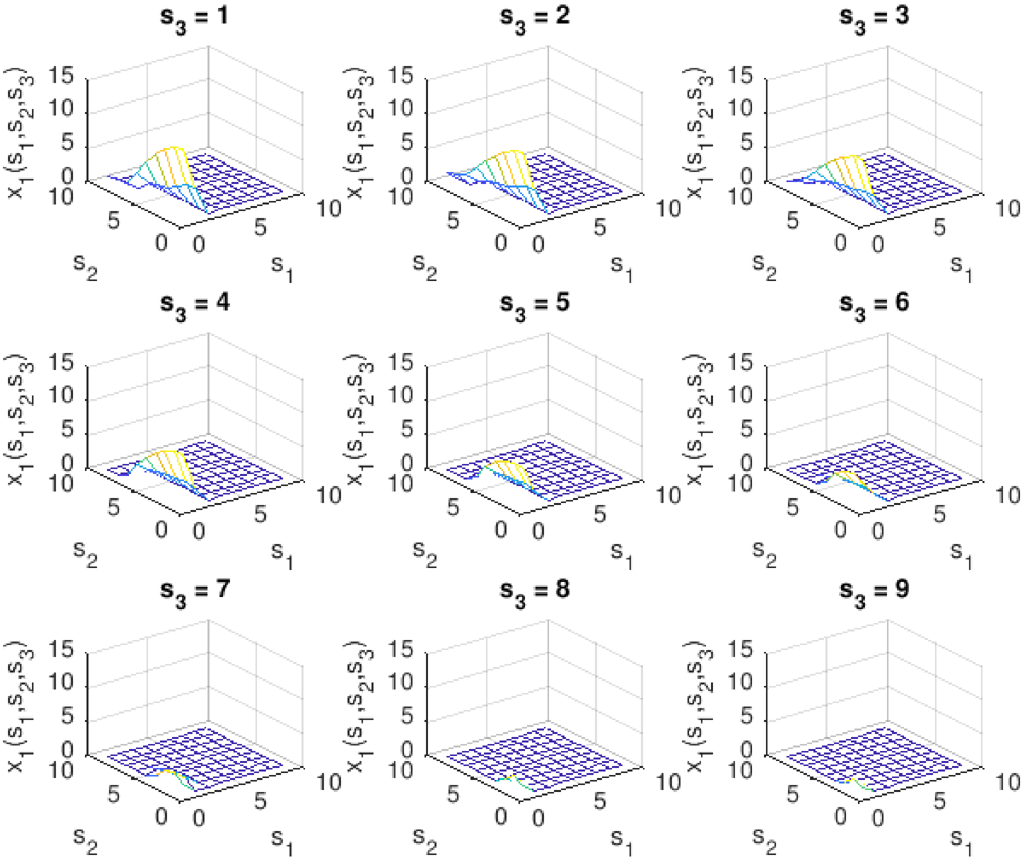

Figure 3.5: Investment function (s_i is the state of the firm i). $\theta = 0.9$ - Entry costs: 20-100 - Scrap value: 8-15

This result suggests that entry and exit might make a preemption race less likely - at least using the model and parameters of Chen. A possible explanation is that no firm is guaranteed to stay at the state 1, with zero investments. If a firm reaches 1 and stops investing, it will exit the industry due to stochastic depreciation. Therefore, it has to invest some amount to stay alive and, with this investment, it has a positive chance of increasing capacity above the state 1. When reaching a higher state, it might be better to continue investing rather than stop investing (since this again would increase the risk of depreciation and future exit). Therefore, contrary to the model of Chen, with the possibility of exit, a firm might find it profitable to invest and eventually reach the same capacity of the others. This is corroborated by what we see in the value function. Figure 3.6 below shows that the value function of a firm is increasing in its investment at lower states, even when competitors already have a somewhat high capacity.

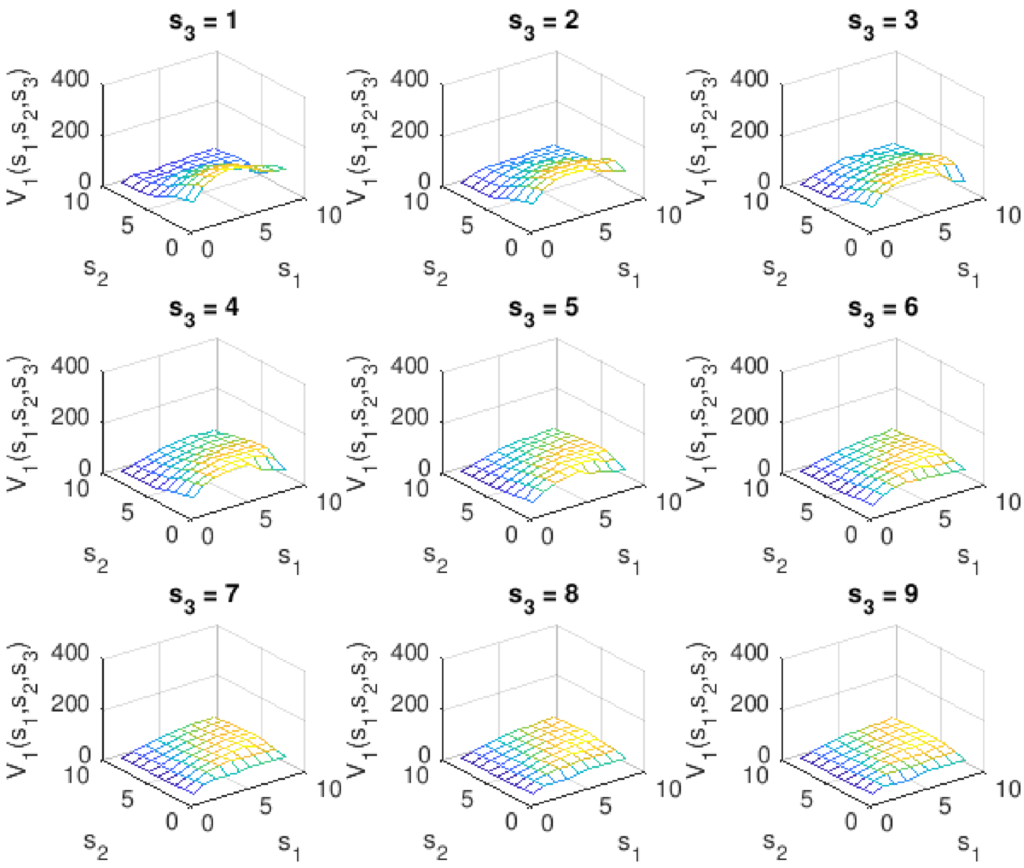

Figure 3.6: Value function. $\theta = 0.9$ - Entry costs: 20-100 - Scrap value: 8-15

But that does not mean that preemption races are impossible with entry and exit. In fact, Besanko et al.[13] relate preemption races to product differentiation. If products are sufficiently homogeneous so that firms do not expect the industry to support more than one firm, they might race to become the leader. Otherwise, they might avoid the excess capacity built in such a race and the outcome would be a symmetric industry.

For instance, in the model simulated here, if we increase the θ from 0.9 to 0.99, that is, decrease product differentiation, we obtain signs of a preemption race. As can be seen in Figure 3.7, investment is higher in the regions of the graphs in which the firms have similar capacities.

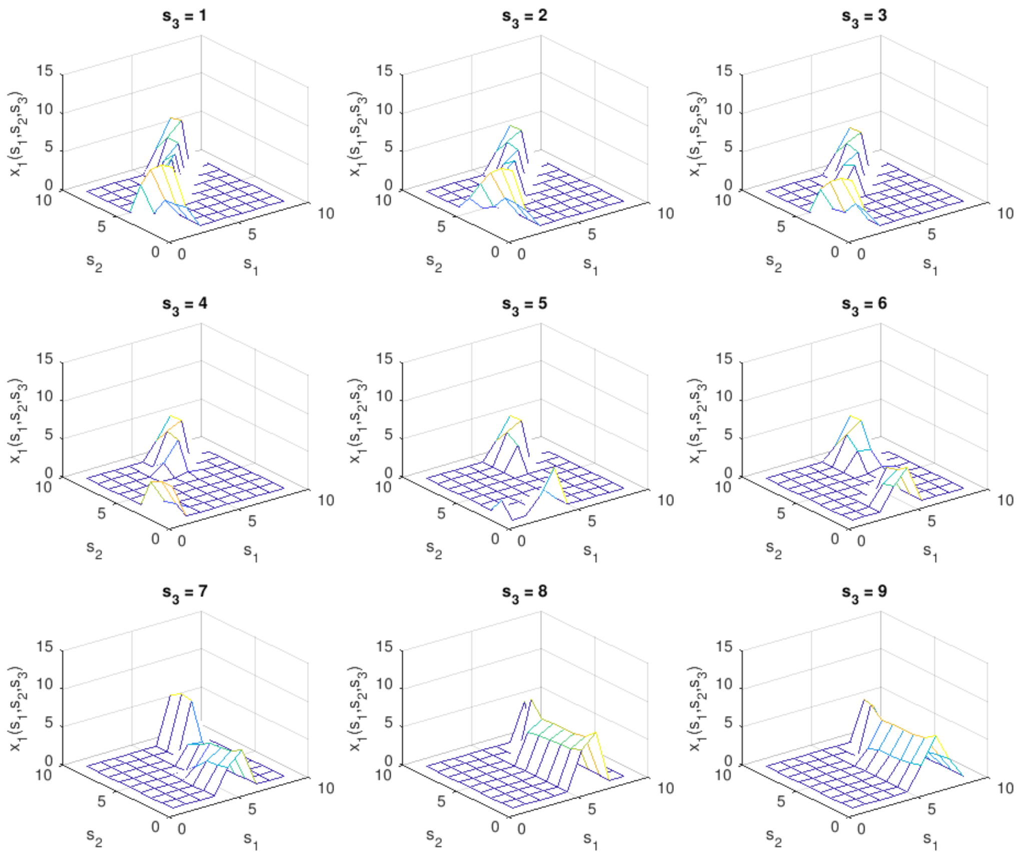

Figure 3.7: Investment. $\theta = 0.99$ - Entry costs: 20-100 - Scrap value: 8-15

Also, Figure 3.8 below, depicting value functions for this specification, shows that when the asymmetry between a firm and its competitors is large enough, its incentives to invest diminish. The value function becomes flatter in these regions, showing that the benefit from increasing capacity is smaller than for the specification shown in Figure 3.6.

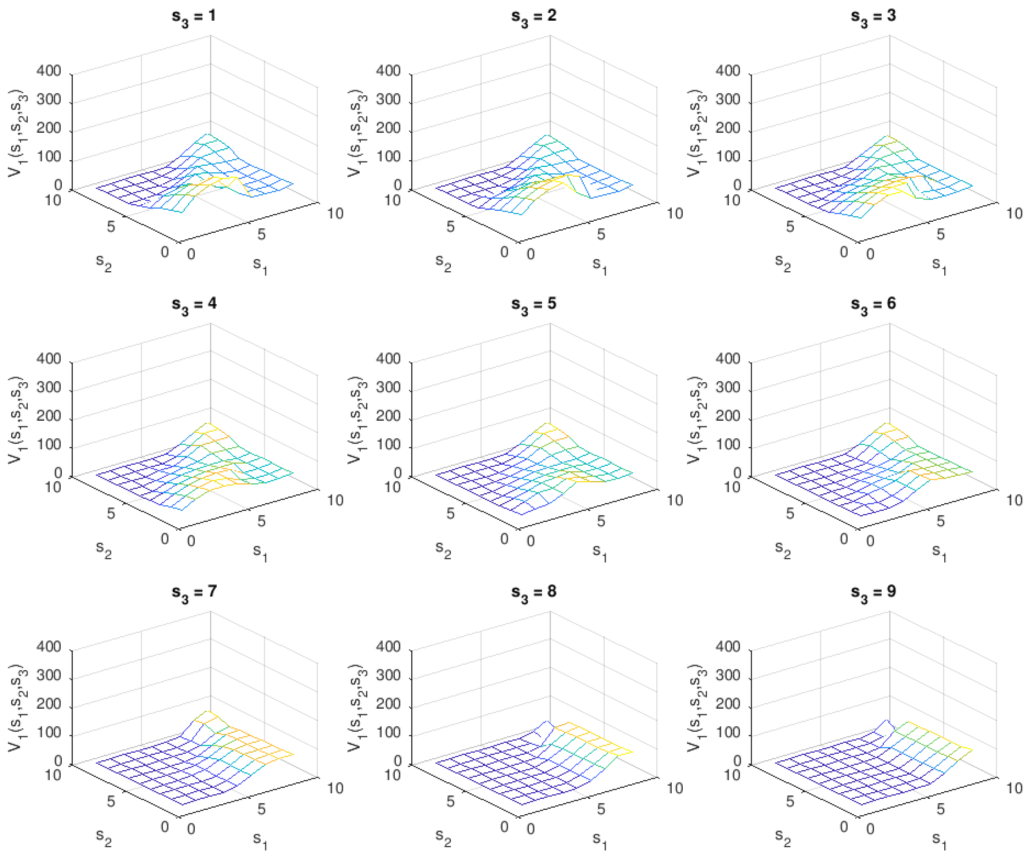

Figure 3.8: Value function. $\theta = 0.99$ - Entry costs: 20-100 - Scrap value: 8-15

3.5 Conclusion and possible extensions

The results obtained above indicate that, by choosing appropriate ranges for entry costs, the model could plausibly replicate the uncertainty in entry in the real world. That is, when we chose a relatively high upper bound on the entry range, the averaging over various simulations produced a pattern in which prices increase immediately after a merger occurs but may take a long time to reduce after that. In real markets this can happen because entry often takes a long time to be completed, with the length being uncertain, and entrants take some time to establish their reputation and capture consumers from competitors, among other factors. Similarly, by choosing a lower upper bound, entry was able to bring prices down faster. Therefore, it should be possible to calibrate this range to match entry patterns found in the real world.

Another interesting feature is that, in this model, only actual entry, and not threat of entry, can constraint prices after the merger. This result is partly due to the fact that we used a static profit function every period. This does not allow incumbent firms to take into account future states of the industry when pricing. In other words, they cannot choose lower prices in order to discourage entry. On the other hand, lower prices alone would not be able to prevent entry because the entrant is forward-looking and takes into account not the price today, but the prices that will realize when it enters the industry. Thus, for an incumbent to be able to price strategically in order to prevent entry, it would have to price below the optimal price level for an industry structure in which the entrant is already in. And this non-optimal price might lead to profits below what would be obtained with optimal pricing with an entrant. To analyze what would actually happen under some specific parameters, we would have to adapt the model in order to make the pricing decision dynamic.[4] Furthermore, we would have model the expectations of entrants and how they are affected by incumbents' behavior.

Also, we saw that, all else equal, entry appears to occur more often with more product differentiation, which might be explained by the industry being more profitable. But it would be interesting to build a model with two state variables for each firm: capacity and quality, for instance. The latter would be an index indicating in which part of the quality spectrum a firm's product are. If two firms produce in closer parts of the spectrum, they are stronger constraints on each other. A potential entrant, then, could choose to enter in any part of the spectrum, and the costs of entry and expected revenues would differ according to that.

Regarding preemption races, the results show that entry and exit might make it less likely to occur. Using the same parameters as Chen, but with the addition of entry and exit, we were not able to obtain signs of a preemption race. But by making products even more homogeneous, we could obtain some signs.

[4]For instance, Besanko, Doraszelski, and Kryukov[6] analyze predatory pricing by making pricing a dynamic decision. They also insert learning-by-doing, which can turn even below-cost prices into an optimal strategy, for a firm to achieve a better position on its cost curve in the future and earn more profits in the long-run.

Chapter 4

A Dynamic Model to Assess Vertical Integration

4.1 Introduction

The goals of this chapter are two. First, we will suggest a dynamic model, based on the Ericsson and Pakes[10] framework, capable of analyzing vertically integrated industries by explicitly modelling the upstream and downstream segments. It may assist researchers in analyzing important issues such as vertical integration, exclusive contracts, and other common business practices between manufacturers and retailers. The construction of such a model becomes even more important when one notices that these issues often have important dynamic consequences, which so far have not been analyzed through a structural dynamic model such as the one built here.

A particular example of an industry in which vertical integration is a relevant issue, but in which, to our knowledge, this issue was not analyzed dynamically so far, is the gasoline industry. The second goal is then to test the model by simulating the evolution of an industry with parameters similar to the characteristics of real world gasoline markets.

The market for gasoline was the focus of several studies and debates on the effects of vertical integration, more specifically the vertical relations between the segments of refining (wholesale) and retail of gasoline. The main concern was that vertically integrated firms would be able to raise the wholesale price of gasoline sold to independent retailers, thereby raising their costs and increasing the retail price to the final consumer. Also, these exclusionary incentives might raise barriers to non integrated players, making entry more difficult, as a potential entrant would have to enter both segments - an example of a dynamic issue.

Hastings and Gilbert[18], for instance, empirically analyzes whether vertical integration between refiners and retailers of gasoline increases wholesale prices due to an incentive to raise rivals' costs. The authors mention three instances that, according to the theoretical literature, should increase incentives to raise rivals' costs: integrated and independent gas stations are close to each other, refiners sell a large portion of their production to integrated retailers, and refiners have significant market power. They use wholesale price data on the West Coast of the United States from before and after the acquisition of Unocal's refining and marketing assets by Tosco Corporation. This was a transaction that increased the level of vertical integration of the latter firm in some locations. Controlling for confounding factors, the authors found that the degree of competition with independent retailers has a significant and positive effect on wholesale prices, which consistent with the theoretical incentives to raise rivals' costs.

However, Taylor and Hosken[29] criticizes these results, pointing that the authors should have looked at retail prices in order to assess whether there was impact on welfare. They analyzed the effects of a joint venture between two integrated players in the gasoline industry, Marathon and Ashland, and also found evidence of increase in wholesale prices. In fact, in some locations, independent retailers faced increases, while integrated ones faced lower wholesale prices. Nonetheless, the authors could not find evidence of these wholesale price increases being passed on to retail prices, and, therefore, there was no consumer harm - at least in the short run, since the authors themselves pointed that the margin squeeze found, if persistent, could result in the exit of independent retailers.

Hastings[17] did analyze and found evidence of retail price increase following the conversion of several independent gas stations to stations with the ARCO brand, a firm vertically integrated in refining and retailing in the West Coast of the United States. But, although there was a change in the level of vertical integration following the merger, the author concludes that the cause was likely the fact that independent gas stations compete more on price and, thus, their elimination would decrease price pressure on the remaining competitors. This would also be evidence in favor of a brand-loyalty model of competition in the gasoline retail. Nonetheless, these results were questioned later by Taylor, Kreisle, and Zimmerman[5], who could not replicate the results, and, in fact, although using a different a data set, found results opposing the conclusions of Hastings.

Also, a report by the Federal Trade Commission of the United States in 2005[23], after analyzing several papers on the issue of vertical integration, concluded that retail prices of companies active in both refining and retailing tend to be lower. Nonetheless, they acknowledge that vertical integration may be harmful, for instance, due to the incentive to raise rivals' costs and because they may make entry more difficult by forcing potential entrants to enter both segments (refining and retailing), and mention the results of Hastings[17] and Hastings and Gilbert [18]. And they point that vertical integration between refining and retailing may have been decreasing in the United States as a whole (although it was still high in the West Coast, object of the studies of Hastings and Hastings and Gilbert), and non integrated retailers have entered in some locations.

In sum, some studies found that vertical integration may cause high wholesale prices, but no connection with higher retail prices could be made. In fact, several papers found connection between vertical integration and lower retail gasoline prices. Moreover, although this was suggested by Taylor and Hosken[29], no study assessed the impact of vertical integration on entry and exit of independent competitors and a possible long-run reduction in competition.

To analyze these issues, we propose a dynamic model based on the Ericsson and Pakes framework. The static "per-period" competition follows the model proposed in Hendricks and McAfee [19]. This is an homogeneous product model, plausible for the gasoline market. But one could experiment with different models, such as bargaining models, for instance, as we intend to do in the future, in the case of differentiated products. On the dynamic side, firms can invest in capacity upstream and/or downstream every period, in order to improve their outcomes from the competition in future periods. Also firms can exit and enter the industry. In order to be able assess the impacts of vertical integration on entry, we allow entry to happen either as an integrated firm, or only upstream/downstream, with different costs for each option.

This analysis is similar to the one carried by Carranza, Clark, and Houde[20], who analyze the impacts of price floors in the retail of gasoline through a dynamic model and conclude that it may prevent efficient firms from entering, although competition between the less efficient firms may become more intensive. And the final result on welfare can vary. But the authors consider only the retailing segment. Here we explicitly model both refining and retailing segments and obtain results consistent with the empirical literature mentioned above: we see evidence of incentives to raise rivals' costs and, depending on the market structure, we see this resulting in higher retail prices. But, on average, prices tend to be lower with integration, due to the reduction of the double margin problem. Furthermore, we found evidence of vertical integration preventing non integrated entry in some instances, resulting in lower accumulated welfare over time - but not much lower because integrated entry eventually happened in the long-run. This novel feature was only hinted at by the empirical literature, but was not assessed yet. The next sections describe our model in more detail and the results from the simulations.

4.2 Spot market competition

We chose a simpler static model for the per-period market competition, involving an homogeneous goods industry, developed by Hendricks and McAfee [19]. Nonetheless, this choice is reasonable for the assessment of the gasoline market, an almost homogeneous product.[1] The model is a bilateral oligopoly, in that the upstream firms have some market power and also the downstream firms have some buyer power. Hendricks and McAfee develop a game in which suppliers of an intermediate homogeneous good submit their cost functions and buyers (sellers of the final good) submit value functions, and a market mechanism selects prices that equate supply and demand. In the model, sellers can exercise market power by exaggerating their costs and buyers by understating their valuation. Hendricks and McAfee developed several variations of this basic framework. In this essay we use the "wholesale market" model. The buyers are retailers that sell the intermediate good in an imperfectly competitive downstream market, incurring some cost

[1] As mentioned in the introduction, the conclusions of Hastings[17] that the gasoline retail market was characterized by brand loyalty were questioned later by Taylor, Kreisle, and Zimmerman[5].

(e.g., costs of distribution).

The model is as follows. In the upstream market, each firm i produces and sells an intermediate output, with constant returns to scale and fixed capacity, having cost functions in the form below:

$$C(x_i) = \gamma_i c(\frac{x_i}{\gamma_i}), \tag{4.1}$$

in which γ_i is capacity, x_i is the quantity produced and $c(.)$ is a convex and strictly increasing function.[2]

The general form of the valuation function for the buyer is as follows:

$$V(q_j) = k_j v(\frac{q_j}{k_j}), \tag{4.2}$$

in which V is homogeneous of degree one, $v(.)$ is concave and strictly increasing[3], q_j is the volume of intermediate output consumed by the buyer, and k_j is its capacity for processing the good.

For this essay, because the buyers are retailers, the valuation function V takes a more specific form:

$$V(q_j) = r(Q)q_j - k_j w(\frac{q_j}{k_j}), \tag{4.3}$$

in which $w(.)$ represents selling costs, Q is the total quantity of good sold downstream, and $r(Q)$ is the downstream inverse demand curve (i.e., the price of the good sold downstream).

In view of this, the profit function for a vertically integrated firms is:

$$\pi_i(\gamma, k) = r(Q)q_i - k_i w(\frac{q_i}{k_i}) - \gamma_i c(\frac{x_i}{\gamma_i}) - p(q_i - x_i), \tag{4.4}$$

in which p is the price of the intermediate good. The profit functions of non-integrated firms are analogous.

In the game, firms report their cost and valuation functions, i.e., their capacities (since the functions must follow the form specified above, differing only in the capacity

[2]Hendricks and McAfee also assume that $c'(z) \to \infty$ as $z \to \infty$.

[3]Hendricks and McAfee also assume that $v'(z) \to 0$ as $z \to \infty$.

parameter). The reported production capacity is $\hat{\gamma}$ and the reported retailing capacity is $\hat{k}$, which may or may not be their true capacities. Then the market mechanism chooses the prices to equate reported supply and reported demand - a Nash equilibrium to this game.[4] As indicated by Hendricks and McAfee, this equilibrium is characterized by the balance condition:

$$Q = \sum_{i=1}^{n} q_i = \sum_{i=1}^{n} x_i = X, \tag{4.5}$$

and the marginal conditions:

$$v'\big(\frac{q_j}{\hat{k}_j}\big) = p = c'\big(\frac{x_i}{\hat{\gamma}_i}\big),\ i, j = 1, ...n. \tag{4.6}$$

In words, supply of the intermediate output equals its demand and the marginal valuation and marginal costs of production of the intermediate output (based on the reported, not true, capacities) equal the price of this good. Because larger firms upstream, for instance, may be able to report higher costs (less capacity) than they have, without the other firms being able to fully supply the demand left unattended, they can inflate the price p, obtaining positive profits; and similarly for buyers that can report a valuation lower than the true one. Thus, as pointed by the authors, if the agents do not misreport their capacities, the outcome can be efficient, but also inefficiencies may arise from the "rational exercise of market power by firms with significant market presence". The structure of the market, not the model, is responsible for the outcome in each case.

In order to make the equilibrium amenable to computation, Hendricks and McAfee adopt functional forms with constant elasticities. The upstream cost function is:

$$C(x_i) = \gamma_i\big(\frac{\eta}{\eta+1}\big)\big(\frac{x_i}{\gamma_i}\big)^{(\eta+1)/\eta}, \tag{4.7}$$

in which η is the elasticity of supply upstream and the valuation function downstream is:

$$V(q_i) = k_j\big(\frac{\beta}{\beta+1}\big)\big(\frac{q_j}{k_i}\big)^{(\beta+1)/\beta}, \tag{4.8}$$

[4] But notice that Hendricks and McAfee state that "The firms' actual types are common knowledge to the firms. Thus, in choosing their reports, firms know the true types of other firms." In other words, it is a complete information game.

in which β is the elasticity of supply downstream.

Finally, Hendricks and McAfee define an inverse demand function downstream with constant elasticity of demand α:

$$r = Q^{-1/\alpha}. \tag{4.9}$$

Given the elasticities and true capacities (i.e., the structure of the market), an equilibrium to the game comprises reports of capacities by each firm, their market shares, and prices and quantities for both segments (which enables us to calculate profits for each firm). Hendricks and McAfee then developed an algorithm that is able to take vectors of true capacities of firms, in both segments, and outputting the equilibrium solution. This is the algorithm used in this essay.[5]

4.3 The dynamic model

The dynamic decisions in this model are entry, exit and investment in productive capacity (both upstream and downstream).

First, each period, an incumbent firm must determine whether it continues active in the industry, or exits. If it exits, it receives a one-time payment of ϕ, the scrap value of its assets. And, as in the framework exposed in Chapter 2, we use random scrap values distributed uniformly on a bounded interval, and identically and independently over firms and time, and expected value functions. If the firm continues, it will receive the profits of the current and future periods in which it is still active.

In order to make its decision, the firm must calculate the expected present value of the future profits and compare this to the scrap value drawn. And, as seen above, spot market profits are a function of capacities of the firms upstream, γ, and downstream, k: $\pi(\gamma_i, k_i, \gamma_{-i}, k_{-i})$. Therefore, each incumbent firm must determine each period an amount of investment that maximizes its future profits and then compare this continuation value

[5]The algorithm can be obtained in the website of McAfee: http://vita.mcafee.cc/Bin/Vertical/.

to the scrap value.

$$V(\gamma_i, k_i, \gamma_{-i}, k_{-i}) = \pi(\gamma_i, k_i, \gamma_{-i}, k_{-i}) + r(\gamma_i, k_i, \gamma_{-i}, k_{-i})\tilde{\phi}$$
$$+ (1 - r(\gamma_i, k_i, \gamma_{-i}, k_{-i}))\Big\{ \max_{x_i^u, x_i^d} \big(- x_i^u - x_i^d + \beta E[V(\gamma_i', k_i', \gamma_{-i}', k_{-i}')]\big) \Big\}, \quad (4.10)$$

in which $r(.)$ is the probability of the firm exiting, $\tilde{\phi}$ is the expected scrap value conditional on the firm exiting, x_i^u is investment upstream and x_i^d investment downstream. It can be seen in the expression that a firm cannot exit the industry immediately. If a firm decides to exit in period t, it will still receive the profits of that period and will cease to be active only in period $t + 1$.

Similarly, it takes one period to establish a new firm. A potential entrant that decides to enter on period t must compare the entry costs, ϕ^e (also a random draw distributed uniformly in a limited range, and identically and independently over time), to the expected present value of future profits, considering that it will start receiving profits only from period $t + 1$ on. Also, an entrant will enter with fixed capacities upstream γ^e and downstream k^e, in which he cannot invest for $t + 1$, but which can be degraded due to the common industry shock at the end of t (see below for explanations on the shocks). The entrant's problem is then:

$$V^e(\gamma^e, k^e, \gamma, k) = r^e(\gamma, k)\Big\{ - \tilde{\phi}^e + \beta E[V(\gamma'^e, k'^e, \gamma', k')]\Big\}, \quad (4.11)$$

in which $r^e(.)$ is the probability of entering the industry and $\tilde{\phi}^e$ is the expected entry cost conditional on the entrant actually entering.

Specifically to this model, we also inserted the possibility that the entrant enters as an integrated firm, or only one of the segments. The choice is made by comparing the expected value of each option before starting the efforts to enter. The entrant then chooses the type of entry with the highest expected value and takes the draw of entry costs to see if it is worth entering. We choose this sequence of decisions, to mimic uncertainties in the real world, in which the agents often have to decide on a course of action based only on expectations about what may happen as a result of each action.

More importantly for the purposes of this essay, we also allowed the possibility of integrated entry being disproportionally more expensive than the entry in any one segment alone. For instance, if the range of entry costs in a single segment is between 1 and 3, we multiply these bounds by 4 to obtain the range of costs of integrated entry. This could reflect additional costs of coordinating the entry efforts in both segments of the industry, for instance.

As seen in the expressions above, the continuation value depends on the future capacities of the industry, which in turn are stochasticaly determined by the investments of the firms. Therefore, following Doraszelski and Pakes[8], we can write the continuation value as follows:

$$E[V(\gamma_i', k_i', \gamma_{-i}', k_{-i}')] = \sum_{v^u, v^d} W(v^u, v^d | \gamma, k) p(v^u, v^d | x_i^u, x_i^d), \tag{4.12}$$

in which

$$W(v^u, v^d | \gamma, k) = \sum_{\gamma_{-i}', k_{-i}', \zeta^u, \zeta^d} V(\gamma_i' + v^u - \zeta^u, k_i' + v^d - \zeta^d, \gamma_{-i}', k_{-i}' | W) q(\gamma_{-i}', k_{-i}', \zeta^u, \zeta^d) p(\zeta^u, \zeta^d). \tag{4.13}$$

In words, $W(.|.)$ is the continuation value conditional on the result of the firm's investment being v^u, v^d, and $p(.|.)$ is the probability of obtaining this result, conditional on the investment done. Notice that $W(.|.)$ is already integrated over the probabilities of all possible future states of the competitors and of the common industry shocks ζ^u, ζ^d. Below we explain the distribution of the idiosyncratic and industry shocks.

As in most of the literature, the probability that a given investment will result in an increase of 1 unit of capacity is given by the expressions below, in which a^u and a^d are investment multipliers upstream and downstream respectively:

$$\begin{aligned}
p(v^u = 1, v^d = 1 | x^u, x^d) &= (\frac{a^u x^u}{1 + a^u x^u}) * (\frac{a^d x^d}{1 + a^d x^d}) \\
p(v^u = 0, v^d = 0 | x^u, x^d) &= (\frac{1}{1 + a^u x^u}) * (\frac{1}{1 + a^d x^d}) \\
p(v^u = 1, v^d = 0 | x^u, x^d) &= (\frac{a^u x^u}{1 + a^u x^u}) * (\frac{1}{1 + a^d x^d}) \\
p(v^u = 0, v^d = 1 | x^u, x^d) &= (\frac{1}{1 + a^u x^u}) * (\frac{a^d x^d}{1 + a^d x^d})
\end{aligned} \tag{4.14}$$

Also, the probability that an industry will suffer an adverse shock affecting equally all firms is given exogenously, according to the expressions below:

$$\begin{aligned} p(\zeta^u = 1, \zeta^d = 1) &= \delta * \delta \\ p(\zeta^u = 0, \zeta^d = 0) &= (1-\delta) * (1-\delta) \\ p(\zeta^u = 1, \zeta^d = 0) &= \delta * (1-\delta) \\ p(\zeta^u = 0, \zeta^d = 1) &= (1-\delta) * \delta \end{aligned} \tag{4.15}$$

As seen in the expressions above, both the idiosyncratic and common shocks upstream are assumed independent of the shocks downstream, and independent over time as well.

For more than two firms, this model is very burdensome to solve using the deterministic algorithm described in Chapter 2 and used in Chapter 3, because it has two state variables for each firm (upstream and downstream capacity), and also there is no closed form solution to the maximization problem solved by the firm to determine its investment decisions upstream and downstream. Therefore, in order to be able to analyze the dynamics of an industry with up to 4 firms, a code that applies the stochastic algorithm, also described in Chapter 2, is used (some details of the code particular to this application are explained in the Appendix).

4.4 Results

In this section we report results[6] from a model with up to four firms, using the stochastic algorithm. For all the results below, the spot market static profit function carried the following specifications. The cost elasticity downstream was $\beta = 1$ and the cost elasticity upstream was $\eta = 1/10$, to reflect the fact that refining capacity (upstream) is more difficult to expand relative to retailing capacity, due to being more technology intensive, and thus requires a lower, inelastic, supply elasticity. Also, the supply elasticity in retailing was not chosen high, to reflect possible regulatory restrictions on where to build and

[6] We iterated the stochastic algorithm until the weighted (by the number of visits to each state) correlation between the exact continuation values and the continuation values obtained through the learning process of the algorithm was above 0.995 and the difference between the weighted means was below 1%.

expand gas stations. The demand elasticity downstream was chosen at a relatively low value, $\alpha = 2$, to reflect the known fact that demand for gasoline is not very elastic - but was nonetheless chosen above unitary elasticity, because we will test monopoly industry structures, which cannot theoretically happen with inelastic demand. The states range from 1 to 5 upstream, corresponding to the same levels of capacity, and 1 to 5 downstream, corresponding to $s * 3$ of capacity (s is the state). These values, as well as the maximum number of firms, were chosen due to computational constraints. Also, the depreciation rate δ is 0.01, since refining and retailing assets in the gasoline market last decades, and the discount rate is 0.925, reflecting an annual interest rate of approx. 8% .

The range of entry costs was between 1 and 3 (and these bounds being multiplied by 4 for the case of integrated entry to reflect the additional cost of coordinating entries up and downstream), which were relatively high entry costs. A firm enters in state 2 on the segments it chose to enter. The range of scrap values was between 0.1 and 0.5, to reflect relatively high sunk costs as, in the model, firms do not exit the industry by selling assets to another entrant and, thus, their assets would not have the same value for other uses. Also, the investment multiplier was 0.0625 upstream and downstream, to reflect regulatory difficulties in increasing capacity through investment.

Figure 4.1 below shows the evolution of retail prices, quantity, and accumulated consumer and total welfare[7] of such an industry for a 100 periods (again, averages over 1,000 runs), starting with two non-integrated firms in states 2, one upstream and the other downstream. The first thing we notice is that the price falls fast in the beginning, and that is due to entry. In this specification, we had approx. 3,000 integrated entries, 500 only upstream and 500 only downstream, over all simulations.

[7] The consumer surplus was calculated by integrating the demand function downstream presented in the section 3.1 and subtracting the revenues of the firms. Total surplus was obtained by summing profits to consumer surplus.

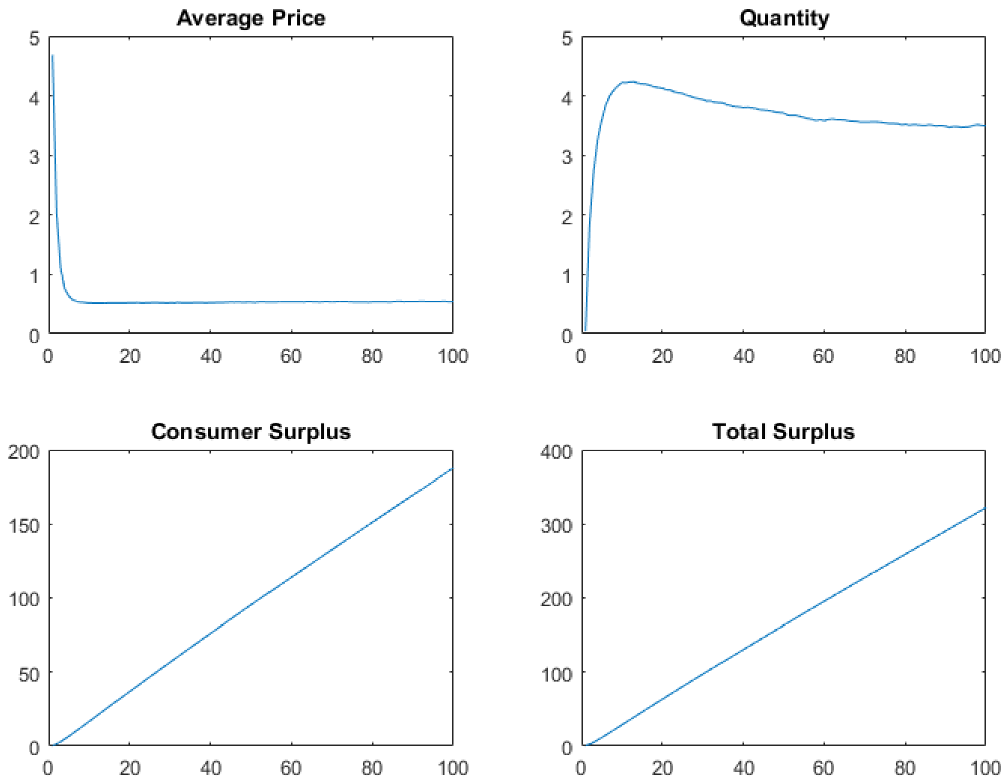

Figure 4.1: Initial state: non-integrated firms [(2,0,0,0),(0,0,0,2)] - Higher integrated entry costs (multiplier 4) and investment multipliers (0.0625)

When we start with an integrated industry, with a firm in state 2 in both segments, as shown in Figure 4.2, we again reach a similar level of prices in the long-run as in Figure 4.1, but prices begin already much lower, which corroborates the results in the literature that integrated firms have lower retail prices, due to the reduction of the double margin problem. But the welfare accumulated over time is almost the same in the two initial states: integrated and non-integrated industry. That appears to be because entry occurs in both scenarios and prevents the double margin problem from persisting when the industry starts with a non-integrated structure. But, somewhat unexpectedly, the number of entries of each type was much more similar starting with an integrated firm: approx. 2,000 integrated, 1,500 upstream, and 1,500 downstream; when intuition would suggest that non integrated entries should have been much less frequent than when we started with a non integrated industry, reported above.

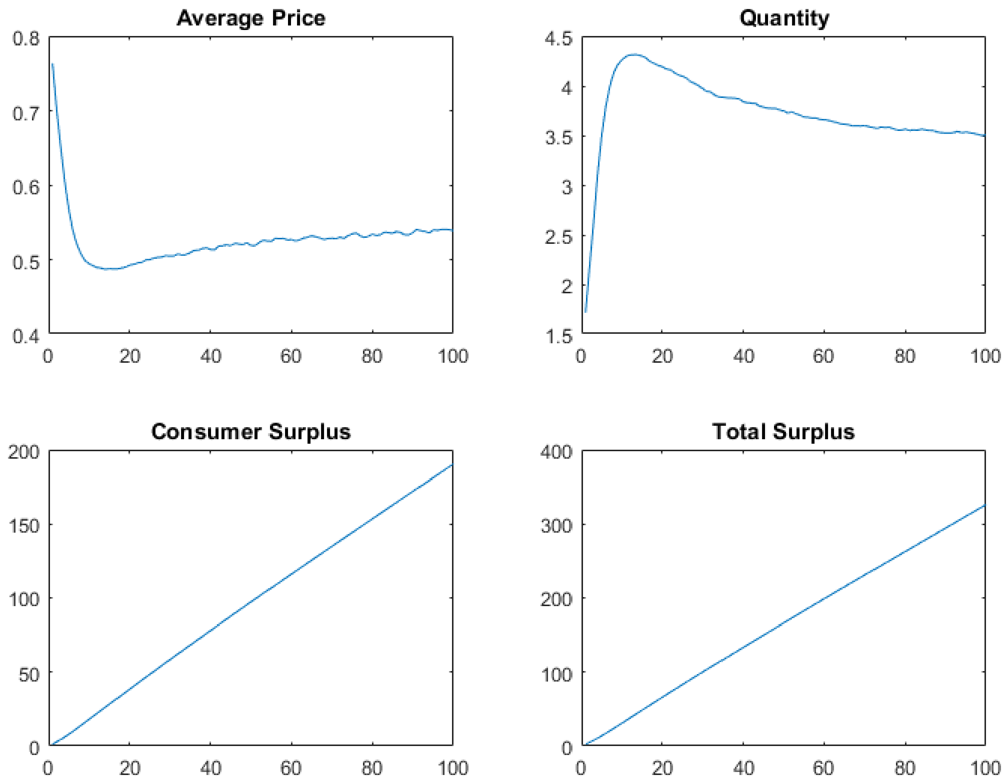

Figure 4.2: Initial state: integrated firm [(2,0,0,0),(2,0,0,0)] - Higher integrated entry costs (multiplier 4) and investment multipliers (0.0625)

But this unexpected result might be due to a pattern that often emerged in the simulations with integrated initial state. First we would have an integrated entry and then two consecutive non-integrated entries, one upstream and one downstream, and the industry would remain in this configuration, with two integrated firms and two non-integrated firms, for long periods. Thus, non integrated entries often happened once there was already two integrated firms competing in the market, therefore reducing the market power of each, a requisite the theoretical literature had identified for the incentives to raise rivals' costs to appear.

Also, very often we had industry configurations with integrated and non-integrated firms. And these often appeared with the non integrated initial state as well. One possible explanation for the appearance of non integrated entries and the often found industry structure with integrated and non integrated firms is that low investment multipliers make it more difficult for integrated firms to expand their own capacity, and thus they find it profitable to use the capacity of a non integrated firm.

Indeed, when we run the simulation starting from an integrated monopoly with state 2 upstream and 1 downstream, we found some runs in which the retail price increased initially, after a non integrated entry, indicating exclusionary incentives. Those were the runs in which capacity upstream was depreciated to 1, and there was entry downstream with capacity 2, resulting in a capacity of 1 upstream and 3 downstream (1 of the integrated firm and 2 of the non integrated entrant). Because the integrated firm had much lower capacity upstream than the available downstream capacity (recall that downstream capacity is multiplied by 3), it may have been able to sell it all through its own capacity downstream, but since markets had to clear, in order for the incumbent to sell to the independent downstream firm, price had to increase. But this was a rare event, due to the low depreciation rate, $\delta = 0.01$, and thus was not sufficient to prevent entry in the first place.

But when we started with a monopolist with state 1 both upstream and downstream (thus lower capacity upstream relative to the downstream capacity, that is multiplied by 3), we do not see non integrated entry at period two, and, most of the time, we do not see entry at all, due to the higher integrated entry costs. Thus, despite the high number of non integrated entries found above in the scenario of Figure 4.2, this might be due to the higher capacity upstream of the integrated firm in that scenario and low investment multiplier, creating a need to use the downstream capacity of independent firms in the market. When we reduced the capacity upstream of the integrated incumbent, the exclusionary incentives appeared and they lowered incentives to non integrated entry, preventing them altogether and, thus, making entry overall more difficult, due to the higher costs of integrated entry. In the end, though, as can be seen in Figures 4.3 and 4.4 below, we observed levels of accumulated welfare smaller, but not much different, because eventually integrated entry would happen and lead the industry to states in which exclusionary incentives were not present and non-integrated entry could happen. Again, often leading the industry to states with integrated and non-integrated firms.

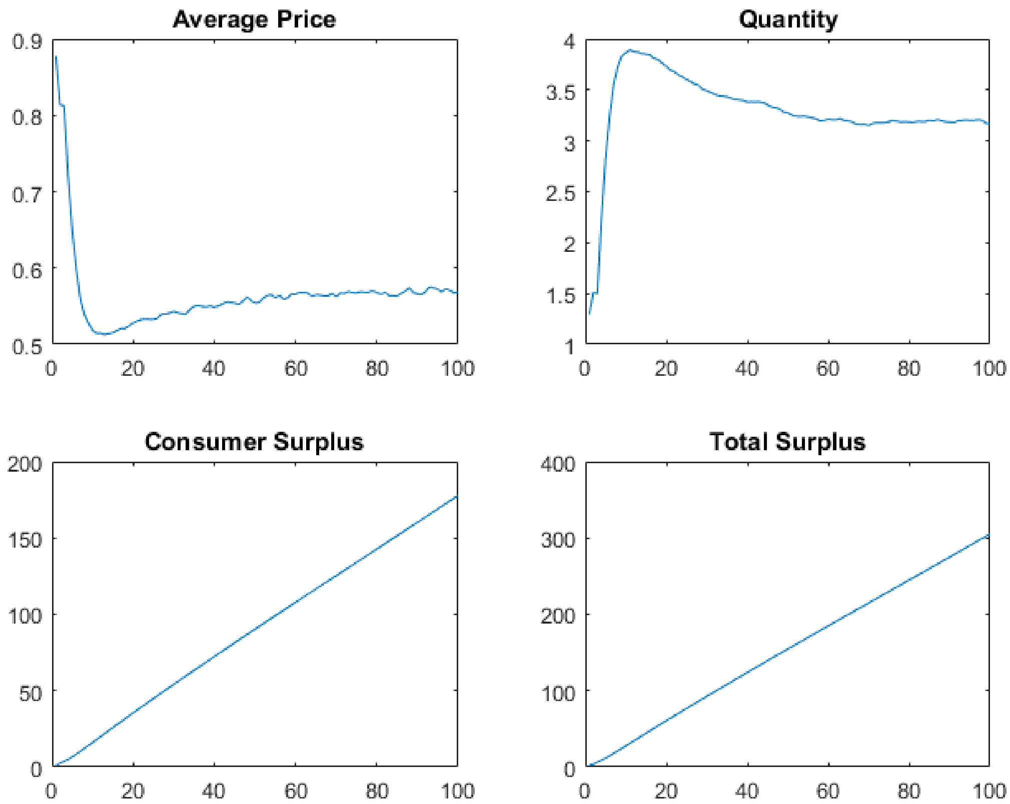

Figure 4.3: Initial state: integrated firm [(2,0,0,0),(1,0,0,0)] - Higher integrated entry costs (multiplier 4) and investment multipliers (0.0625)

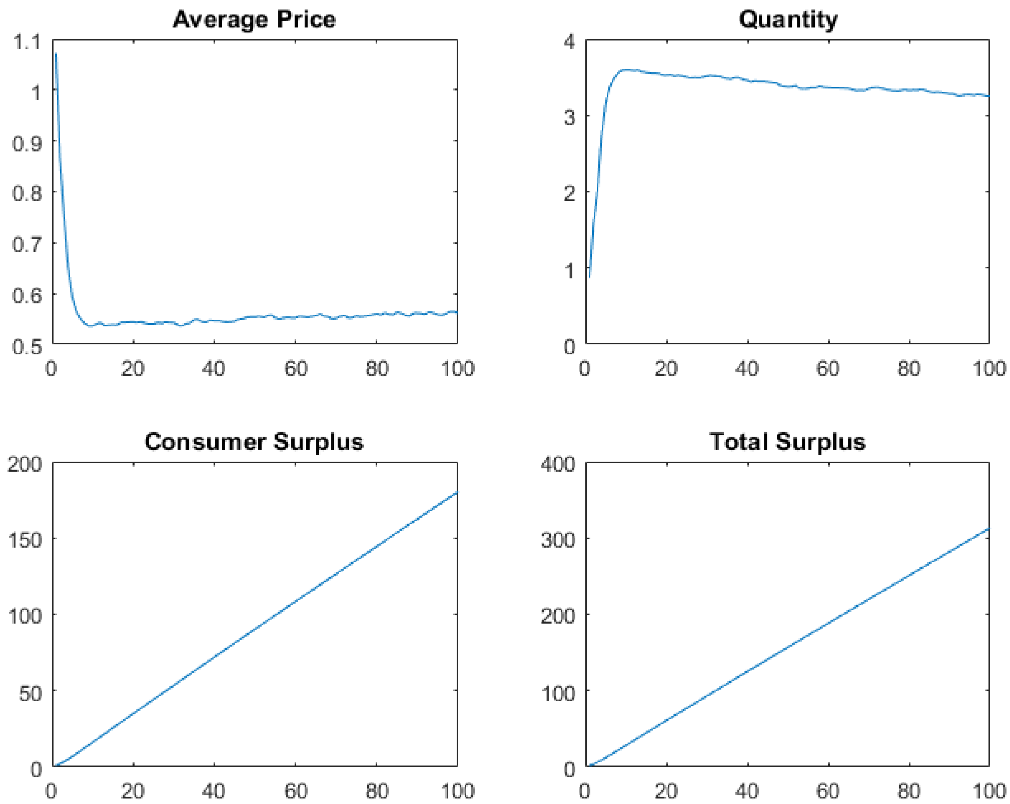

Figure 4.4: Initial state: integrated firm [(1,0,0,0),(1,0,0,0)] - Higher integrated entry costs (multiplier 4) and investment multipliers (0.0625)

4.4.1 Conclusion and Extensions

Through the model constructed in this section, we were able to simulate the evolution of the industry under specifications inspired in the gasoline industry, namely refining and retailing, starting both from integrated and non-integrated industry structures. The motivating issue, as indicated in the introduction, was to assess whether the dynamic model developed here showed signs of exclusionary incentives as reported by the empirical literature reviewed in the introduction and, more importantly, whether this could increase barriers to entry, reducing welfare in the long run, a dynamic issue that was not analyzed by the empirical literature yet.

First, our results showed that the level of prices may be lower with integrated firms, as claimed in part of the empirical literature - as seen in the scenario in which we started with an integrated firm; but entry rapidly brought prices down and accumulated welfare did not differ much in the long run between the two initial states. Nonetheless, we also found that there were indeed incentives to raise rivals' costs evidenced by instances in which retail prices increased right after entry of a non integrated firm. But these appeared only when the capacities of the integrated firms were low and asymmetric, and therefore they did not need the capacity of independent firms. In general, though, the industry tended to configurations with both integrated and non integrated firms in the long run - as we indeed see in the real world gasoline markets.

More importantly, we found evidence that, in the instances in which capacity of the integrated firm was low and asymmetric, the exclusionary incentives did indeed prevent non integrated entry, therefore making entry less likely, due to the higher costs of integrated entry. Moreover, we obtained lower accumulated welfare over time when we started with integrated monopolies with lower and asymmetric capacities upstream and downstream, but not much lower, because in the long run, integrated entry would eventually happen and bring the industry to states in which exclusionary incentives were no longer present, and there were both integrated and non integrated firms present.

These results, however, may be related to the particular model we chose for the per period competition. Continuing this line of research, it might be interesting to experiment with other models of competition, such as bargaining models, often used in static analysis of vertical integration. For instance, Sheu and Taragin[28] develop an interesting and simple model, in which firms compete downstream according to a logit model and prices for the intermediate input are the result of a bargaining game between suppliers upstream and buyers downstream, in which each player's payoff function depends on its level of vertical integration and market position in both segments. This model could handle differentiated products as well.

Chapter 5

Future research

5.1 Asymmetric information

The essays here analyzed dynamic issues under two different industry configurations, but always using a Markov perfect equilibrium as the solution to a game without asymmetric information. Although practical, the assumption that firms observe the actions and states of each other perfectly might be unrealistic.

To address this issue, Fershtman and Pakes[14] extends the framework developed in Ericsson and Pakes [10] to allow for asymmetric information in dynamic oligopoly games and also develop a new concept of equilibrium that can be computed for these games, the Experience-Based Equilibrium (EBE). A future line of research, thus, would be to follow their approach and adjust the complete information model described above in Chapter 4 so that firms have access only to their own state and decisions and the public information available to all players (which would be only the total number of firms active in the market). Below we describe some of the adjustments that we might be needed to construct this model.

5.1.1 Overview

Again, it is assumed that there is a maximum number of firms every period, and the possible values of the state variables are also finite. To accommodate asymmetric information, the authors distinguish between observed and non-observed variables and, within the observed variables, between payoff relevant variables and informationally relevant variables. They define payoff relevant variables as variables that are not controls and affect current period profits of at least one firm, and informationally relevant variables as those that may improve the outcomes of the agent's actions by conditioning on its value or there is at least one player whose strategy depends on it. Each agent has a different information set, called J, which contains his observed payoff and informationally relevant variables, and the player conditions its actions on this information set. The state of the game is the set of all information sets of the players. The assumption of finite values for the state variables requires some restrictions on informationally relevant variables[1] to avoid actions depending on the whole history of the game. In the game we propose, the only informationally relevant variable is the number of firms in the market, which is assumed limited as well.

In our version of the game, firms have two controls each period: m^u, m^d, which represent discrete reports of capacity (upstream and downstream) by the firm for the current period market competition (which is also based on the Hendricks and McAfee[19] model described above), which can take any of the possible values in the discrete spaces of capacities, $\mathcal{M}^u, \mathcal{M}^d$; and x^u, x^d, which are investments to improve capacity for the future and can take any non-negative real value.[2] None of these controls are observed by competitors. The strategy of a player is a function of its current period information set, and includes decisions on reports of capacities and investments.

The problem of an incumbent firm can then be described similarly to the com-

[1] As indicated by the authors, the assumption on payoff relevant variables can be derived from the primitives of the game.

[2] Although the framework presented in Fershtman and Pakes[14] has only discrete controls, they mention the possibility of continuous controls, which were in fact used in the original version of their article.

plete information model, by the following equation:

$$\max_{m^u, m^d, x^u, x^d} \Big[\sum_{\eta} W(m^u, m^d, \eta | J_i) p_\eta(\eta | x^u, x^d) \Big]$$

$$W(m^u, m^d, \eta | J) = \pi^E(m^u(J), m^d(J) | J) + r(J)\tilde{\phi} + \\ (1 - r(J))\beta \sum_{J'} \Big\{ \sum_{\eta} W(\eta', m^{u*}(J'), m^{d*}(J') | J') p'_\eta(\eta' | x^{u*}(J'), x^{d*}(J')) \Big\} p(J' | J, \eta) \tag{5.1}$$

in which $\pi^E(.)$ is the expected profit, as a function of the information set, given that the firm does not know the capacities of the competitors and thus cannot know the exact profits ex-ante; $W(|)$ is the value function of the agent, and is a function of the reports of capacity and η, which is the result of the investment decision, stochastically increasing on the investment amount, and the variables with $*$ represent optimal decisions, that is, decisions that maximize the value function. In words, then, the agent chooses its control to maximize its value function, which itself takes a recursive form, such that its value today from adopting certain decisions is the sum of current expected profits and the expected value from future periods, assuming optimal decisions are taken in the future.

Moreover, as indicated by the exit probability $r(J)$, we also consider the possibility of the agent exiting the industry, and exit takes one period. Similarly to the complete information model, firms compare the expected continuation value (i.e., the value function above excluding the profits of the period) to the scrap value, and decide to exit if the scrap value is greater. The scrap value is a random variable uniformly distributed on a bounded interval and we consider expected value functions, based on the probability of the firm drawing a scrap value greater than the expected continuation value and on the expected scrap value, conditional on exiting, $\tilde{\phi}$.

The problem for the entrant would be defined analogously. There can be only one entrant per period, and it takes one period to enter. The potential entrant compares the expected continuation value to the entry cost, which is also random, and enters if the former is greater. Also, as indicated in the complete information model, the potential entrant compares the expected values of entering as an integrated firm, only upstream or only downstream, and chooses the one with the highest expected value.

For this problem, we would use the EBE concept. As defined by Fershtman and Pakes[14], an EBE includes (i) a subset $\mathcal{R} \subset \mathcal{S}$ of the state space; (ii) strategy functions, $m^*(J)$ and $x^*(J)$, indicating control decisions for every information set (and every player)[3]; and (iii) expected discounted value functions for every decision on controls and information sets, $W(m|J)$; such that:

1. ($\mathcal{R}$ is a recurrent class). Starting from any $s_0 \in \mathcal{R}$ the Markov process generated by the strategy functions stays within $\mathcal{R}$ with probability 1.

2. (Optimality of strategies on $\mathcal{R}$). Given $W(|)$, for every J component of an s in $\mathcal{R}$, the decisions given by the stragey functions maximize $W(|)$.

3. (Consistency of values on $\mathcal{R}$). For every J component of an s in $\mathcal{R}$, and for the decisions given by $m^*(J)$ and $x^*(J)$, $W(|)$ satisfies the following recursive equation:

$$W(m^{u*}(J), m^{d*}(J), \eta|J) = \pi^E(m^u(J), m^d(J)|J) + r(J)\tilde{\phi}+$$

$$(1 - r(J))\beta \sum_{J'} \Big\{ \sum_{\eta} W(\eta', m^{u*}(J'), m^{d*}(J')|J') p'_{\eta}(\eta'|x^{u*}(J'), x^{d*}(J')) \Big\} p(J'|J, \eta) \tag{5.2}$$

As noted by the authors, the consistency requirement applies only to optimal actions within the recurrent class $\mathcal{R}$, thus the name "experience-based", as the firms "learn" accurate values solely for states they visit repeatedly.[4]

Finally, the authors use a reinforcement learning algorithm to compute the EBE, based on the Stochastic algorithm describe above in Chapter 2. In summary, the algorithm mimics the learning process of agents. It starts with an initial state and initial guesses for π^E and $W(|)$. Then each agent chooses optimal actions based on these initial guesses, which allows us to calculate the probabilities of investment success and of transiting to

[3]Note that the function is the same, and players can take different actions only if they have different information sets

[4]Fershtman and Pakes[14] also develop a more restrictive version, called Restricted Experience-Based Equilibrium, which imposes consistency for all feasible actions (instead of only optimal actions) from states in the recurrent class. But the restriction may also require adjustments to be able to access the information needed to compute this new equilibrium (which restricts actions outside the equilibrium path).

any other state. Then we take a draw from these probabilities and obtain the state. Also, we update the initial guess of π^E by taking an average of the actual profit obtained after the uncertainties were solved with the value in memory, and similarly update $W(|)$ by averaging with the expected value held in memory for the state that was drawn from the probabilities derived from the actions of the agents. Having updated the values, we begin the process again, from the drawn state. After a significant number of iterations the agents will have passed through the states in a recurrent class several times and will eventually learn the correct values on the recurrent class and for optimal actions within these states. And since the values are functions solely of the players own information sets, this would be an EBE for an asymmetric information game.

Below we describe some further details on how profits and values would be calculated and updated.

5.1.2 Spot market competition

As in the complete information model, firms also compete every period by submitting supply and value functions for an homogeneous intermediate input, following the static model of bilateral oligopoly competition by Hendricks and McAfee[19]. But now we cannot compute the equilibrium of this game off-line and use it as an input to the dynamic game, because we cannot assume that firms know each others types (capacities), as does the solution of Hendricks and McAfee[19].

Instead, in computing the solution through the algorithm described above, we start with an initial guess for the profit that would be obtained for every possible capacity report. As suggested by Fershtman and Pakes[14], we use the value of profits a firm would obtain had it submitted each capacity and its competitors submitted nothing. Then, once the firms take their actions based on these initial values, we can calculate the actual profits obtained by each, and use this value to update the profit function for the action chosen by each firm. We can calculate both the initial and actual profit values using the marginal conditions given in Hendricks and McAfee[19].

5.1.3 Public information and update of $W(|)$

In order to update $W(|)$, we would need the evaluation of the agent for the state drawn from the transition probabilities. To do that, we search in memory to check if we have the value $W(|)$ for this new state and, if not, we initiate its value by using a discounted infinite sum of the initial profit for this state. But notice that the value of $W(|)$ held in memory contains the value of the profit function, as indicated by equation 5.1, which is being updated as well. Therefore, in addition to knowing how many firms actually remained in the industry after the uncertainties were realized, the agent also uses a expected value function based, to some extent, on real obtained profits in the past to learn the correct values of $W(|)$. Therefore, although our game has very few public information, the agent may be able to learn its values and optimal decisions through actual profits obtained for each line of action.

In sum, by making these adjustments and using an algorithm very similar to the one already used in Chapter 4, we might be able to assess the dynamic impacts of vertical integration without assuming complete information, which might provide us with more relevant results to some industries.

5.2 Estimation

As indicated in Ackerberg et al.[11], there is a growing literature using the dynamic framework described in Chapter 2 to estimate the structural parameters of an industry and, with those, be able to analyze more accurately dynamic issues, using the simulation techniques used here in Chapters 3 and 4. In view of that, a natural extension to the work done here would be to take the models to the data. For instance, with respect to the model of Chapter 4, if we were able to estimate the parameters of the model for a particular industry, we might be able to quantitatively compare the short-run efficiencies from a vertical merger (e.g., the elimination of the double-margin) with possible dynamic impacts on entry costs (e.g., a vertically integrated industry might require integrated entry, which could be more difficult).

A possible strategy for doing that would be similar to what Ryan[27] did for the cement industry. Ryan estimated the structural parameters of a dynamic model for the cement industry based on the framework described in Chapter 2. He estimated the parameters both before and after a change to the environmental legislation affected the costs of the companies (e.g., sunk entry costs were affected). With both estimates, he then simulated the evolution of the industry with and without the change in legislation, being able to compute the difference in welfare brought by the change. He concluded that sunk entry costs had increased, and this could not be captured by the static models used by the authorities to assess the impacts of the new regulation.

In order to estimate the parameters of the dynamic model, Ryan adopts a two-step strategy, developed by Bajari, Benkard and Levin[26]. In the first step, he obtains estimates of the parameters of the static profit function and empirical probabilities of investment[5], entry and exit. Then, in the second step he recovers the value function and estimates the distribution of entry probabilities.

Finally, in connection with the model and issues analyzed in Chapter 3, it is worth mentioning that there is already work being done to estimate the relevant parameters and analyze the dynamic effects of a merger, using the same strategy described above. Benkard, Bodoh-Creed, and Lazarev[21] use the Bajari, Benkard, and Levin method to estimate policy functions from pre-merger data in the airline industry, and then run simulations to assess the medium and long-run effects of a merger. Based on that they are able to identify important factors that influence the ability of potential entrants to constraint prices of the merged firm. They apply the method to three airline mergers recently analyzed by United States antitrust authorities. Their analysis show how dynamic models based on the Ericsson and Pakes[10] framework can be brought to data (with some modifications) and provide relevant complementary information to the static analysis already being done by antitrust authorities. In fact, only with dynamic models one could obtain structural estimates of entry probabilities after the merger, as they do.

[5]Notice that the model in Chapter 4 has continuous investment, and Ryan's model uses lumpy investment. This would have to be adjusted. For instance Ryan states that continuous investment is a limit case of lumpy investment as the boundaries of the (S, s) strategy shrink.

Appendix A

Algorithm used in Chapter 4

In order to carry the analysis in Chapter 4, we used the stochastic algorithm to solve for the equilibrium of the model and simulated the evolution of the industry. In this Appendix we explain how the code that uses the stochastic algorithm developed in Pakes and McGuire[25], described in Chapter 2, was constructed.

A.1 State space

First, in order to go through the states in the learning procedure, we need a method to access each state's information in memory, similar to a code for each state that indicates to the computer from which state it must get information or save information. For this essay, we used a method based on the one developed in Pakes, Gowrisankaran, and McGuire[1]. In that paper, the authors represented the industry structure as a descending tuple, with the firms having the highest indexes first, and an indication of the position that the firms hold in the tuple. For instance, $[(4, 2, 2, 1, 0), 2]$ would represent the state of a firm with index 2 in an industry with three competitors, having indexes 4,2 and 1. Then, the authors construct a bijective function that takes a descending tuple and converts it into an integer, having an inverse function that does the opposite. This allows them to represent the entire state space with a matrix, in which each line is a possible industry configuration, and each column the information for the firm holding the position equivalent to that column's number.

Similarly, we also use this bijection to code upstream and downstream industry structures, separately, into integers. But, because we can have different ownership relations between upstream and downstream positions (for instance, given two tuples, up and downstream, the first firm upstream can own any of the other firms downstream), we need another index for this ownership relation. The total number of possible ownership relations for any two tuples up and downstream vary depending on the tuples, but an upper bound is the number of possible permutations of the vector going from 1 to the maximum number of firms allowed. Therefore, we number each such permutation and consider an industry structure to be represented by the two tuples and this number indicating the actual ownership relation in place. But this will result in the same industry structure being represented by different codes (e.g., [(2,2,1,1),(3,2,2,1),(1,2,3,4)] is the same as [(2,2,1,1),(3,2,2,1),(2,1,4,3)], as we have firms with capacity up and downstream, respectively, (2,3), (2,2), (1,2), (1,1) in both). Therefore, we also reorganize the vector indicating the ownership relation in several relevant moments within the code, so that it is also ascending, thereby preventing duplication of states in the calculations. Finally, the positions of the firms are indicated by two more dimensions, containing their position upstream and downstream respectively.

In sum, the state of any firm is a five-dimension vector, such as (1,2,3,1,1), in which the first element is the code for the industry structure upstream, the second for the structure downstream, the third for the ownership relation, and the fourth and fifth for the position of the firm up and downstream.

A.2 The main code

After constructing the bijection mentioned above and functions to code and decode states, the main part of the code takes the relevant parameters, an initial state and initial guesses for the conditional continuation value ($W(.|.)$) and policies, and start the learning procedure by updating values and policies and transiting to the next states through simulation. To do that it calls three types of subroutines: optimize (which updates values and policies and take the draw of the next state), calcprofit (which calculates the static profits in the spot market for a given state), and initW (which calculates initial values for the conditional continuation values based on the static profit function).

The calcprofit subroutine is based on the algorithm developed by Hendricks and McAfee[1] and is called mainly the first time each state is visited, in order to calculate the profits for each firm in that state and store those in memory. Similarly, initW uses the profits to generate initial values for the $W(.|.)$ - the infinite discounted sum of the profits for the state. The most important subroutine is optimize. It takes a given state, updates the conditional continuation values $W(.|.)$ and value functions for all players for that state and returns another state (the next state in the iteration).

The first step done in the subroutine is taking two draws from a uniform distribution between zero and one to assess whether the common industry shocks upstream and downstream were zero or one, and save these values. Then we check if the state given to the optimize function has been visited before. If yes, we retrieve the conditional continuation values $W(.|.)$ and use them to obtain optimal policies for each incumbent by solving numerically the single agent optimization problem, calculate the value functions and update these values in memory. If the state has never been visited, we first call the initW routines to generate initial values for the conditional continuation values and then similarly obtain optimal policies.

Given the policies of each incumbent, we can calculate the probability that its investment in capacity will be successful. We then use these probabilities and generate numbers from an uniform distribution between zero and one, for each agent separately, to check whether they were successful or not in increasing capacity.

Similarly, for entrants, we retrieve the continuation values from memory, or generate initial ones using initW, and calculate the expected value of entering upstream, downstream and integrated. The entry type chosen is the one with the maximum expected value. We then take a draw from an uniform distribution between zero and one again and, based on the entry probability for the chosen type of entry, check whether entry occurred or not.

Based on the draws taken before for the outcomes of investments, exit and entry decisions, which were taken from the distribution of possible next states given the actions

[1] As already indicated above, it can be found in the website: vita.mcafee.cc/Bin/Vertical/.

of the players, we can construct the next state, which will be one output of the function.

Then the next step is to update the values of $W(.|.)$ for the current state. This is done by calculating value functions for the state drawn before (from the distribution of possible next states) through numerically solving the single agent optimization problem, and then taking a simple average between the old value of $W(.|.)$ and the value function obtained.

Then, after calling optimize a sufficient number of times [2], the main code calculates exact continuation values using the method of the deterministic algorithm for the states visited in a certain large number of the latest iterations and value functions from this continuation value. Then, it calculates a weighted correlation between the value functions obtained through the exact continuation values and the ones in memory from the learning part of the algorithm and the difference between their weighted means; if these are above 0.995 and below 1%, respectively, the code stops. The weights are the number of times each state was visited.

[2]Including an initial averaging procedure, by which old estimates of the value and policies are eliminated and later estimates given more weight - see Pakes and McGuire[25].

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