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### REFERÊNCIA

SILVA, R. C.; VIANA, L. S. Matlab programming for stress analysis in concrete gravity dams considering earthquake and silts. **Revista IBRACON de Estruturas e Materiais**, São Paulo, v. 4, n. 2, p. 213-246, jun. 2011. DOI: <http://dx.doi.org/10.1590/S1983-41952011000200005>. Disponível em: [https://www.scielo.br/scielo.php?script=sci\\_arttext&pid=S1983-41952011000200005&lng=pt&tln g=pt](https://www.scielo.br/scielo.php?script=sci_arttext&pid=S1983-41952011000200005&lng=pt&tln g=pt). Acesso em: 29 out. 2020.

# Matlab™ programming for stress analysis in concrete gravity dams considering earthquake and silts

## *Programação em Matlab para a análise de tensões em barragens gravidade de concreto com a consideração de sismos e sedimentos*

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### Abstract

Having in mind the social and economic importance that dams have in Brazil and the large investments made in relation to these structures, the objective of this work is to contribute to the study of these structures, through developing a program in MATLAB™ language for stress analysis in concrete gravity dams. The routines developed allow usual verifications in dam projects, considering the action of silts and earthquakes stands out and also the variation of the geometric parameters and physical properties of the dam. The earthquake analysis follows two methods the Pseudo-Static method (Westergaard Method) and the Pseudo-Dynamic method (Chopra Method). Before presenting the subroutines and the program's functions, these methods are briefly discussed and a validation of stress results are made using CADAM 2000. Additional analyses are presented in reference to program's potentialities and the conclusions.

**Keywords:** concrete gravity dams, pseudo-static method, pseudo-dynamic method, silts, Matlab™.

### Resumo

Tendo-se em vista a importância sócio-econômica que as barragens possuem no Brasil e os grandes investimentos realizados acerca destas estruturas, o objetivo deste trabalho é contribuir para o estudo destas estruturas, através do desenvolvimento de um programa em linguagem Matlab para a análise de tensões em barragens gravidade de concreto. As rotinas desenvolvidas permitem verificações comuns em projetos de barragens, dentre as quais se destaca a possibilidade de se considerar a ação de sedimentos e sismos, além da possibilidade de variar, em uma mesma análise, parâmetros geométricos e propriedades físicas dos materiais do perfil estudado. A análise dos sismos é abordada segundo dois métodos quais sejam: método Pseudo-Estático (Método de Westergaard) e Pseudo-Dinâmico (Método de Chopra).

Antes da apresentação das subrotinas desenvolvidas, os principais métodos de análise de tensões, ocasionadas pelas ações sísmicas, são brevemente discutidos e, após, são feitas diversas validações de resultados com o auxílio do programa CADAM 2000, desenvolvido pela Escola Politécnica de Montreal. Apresentam-se ainda análises adicionais referentes às potencialidades do programa, assim como as principais conclusões.

**Palavras-chave:** barragens de concreto gravidade, método pseudo-estático, método pseudo-dinâmico, sedimentos, Matlab.

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## 1. Introduction

According to data from (ANEEL, [1]), Brazil has a total of 1,706 electric energy generation projects in operation, producing 100.993.472 kW of power. Among the 1706 projects, 149 are Hydroelectric Plants (HEP), which represent 75.023.597 kW generated. Besides the HEP, Small Hydroelectric Plants (SHP) must be accounted for and they represent 2.004.617 kW of power, and Hydroelectrics, are responsible for 114.802 kW of power. Together the HEPs, SHPs and the LHs account for a total of 677 plants in operation in Brazil and they correspond to 70.7% of the Brazilian electric energy power framework.

Thus, the great importance of dams in the Brazilian energy sector becomes clear. In fact, Brazil is country that is privileged in terms of its hydro capacity and its terrain that are both favorable to constructing dams hence making these structures the most viable option for producing energy in the country. Corroborating this fact is that again according to data from ANEEL, there are 101 electric plants, among them HEP, SHP and Hydroelectrics under construction in the country.

Given the large number of existing dams and those under construction as well as their evident economic importance for the country, justifies a greater knowledge and study of these structures in order that projects be improved as well as to carry out more in-depth analyses.

In light of this scenario and considering the fact that electric energy generation in Brazil is predominantly grounded in water resources; that there is an eminent energy crisis in the country and in the world as well as considering the socio-economic impacts which failures in these structures may cause, the theme of this work becomes relevant. This is due to the fact that the main objective here is to develop a computational routine for analyzing the stress fields and stability of concrete gravity dams when subject to the action of extreme demands (earthquakes).

## 2. General features of concrete gravity dams and their loads

According to (Stewart and Floyd, [2]) concrete gravity dams are nothing but massive blocks of concrete that have their weight as a stabilizing factor and their length as the predominant dimension. A rupture in a dam may provoke a great loss of life, social damages and economic crises. For this reason, it is necessary that these structures are capable of resisting different types of loads that they may be subject to throughout their useful life as expected by the project. Hydrostatic weight and pressure, dead weight, uplift pressure, and weight and pressures due to silts may be cited as among the main loads acting on a dam. These loads are considered usual in dam structures.

Dead weight is one of the main loads active, not only due to its magnitude owing to the robustness of the concrete structure, but also because it is mainly responsible for the stability of these structures. Hydrostatic weight and pressure are loads that may or may not contribute to the stability of a dam. These loads are generated by water accumulated downstream (after the dam) and upstream (reservoir) of the structure.

Accumulated water, however, does not generate only the loads previously noted. Ground porosity permits that part of the accumulated water percolates into the interior, generating a type of load called uplift pressure. Its behavior occurs through vertical hydrostatic pressure at the foundation of the dam. This generates forces undesirable to the stability of the structure. This force tends to elevate the dam.

Silts are loads generated by the deposit and accumulation of material at the bottom of a dam. According to the (USBR, [3]), one very used hypothesis for determining pressures caused by silt is to consider that its horizontal component is equal to a fluid with the specific weight  $12\text{kN/m}^3$ . However, in the event that the designer

Figure 1 - Representation of usual demands acting on a typical cross section for a Concrete Gravity Dam

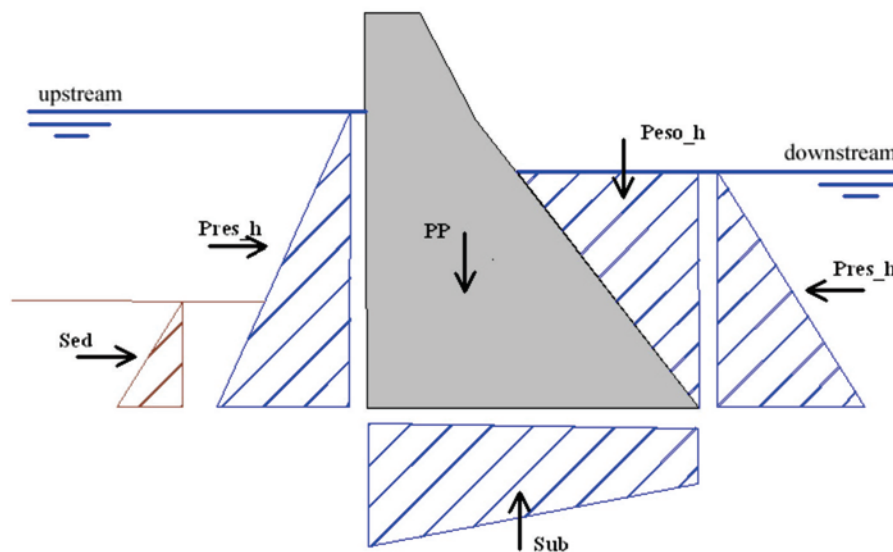
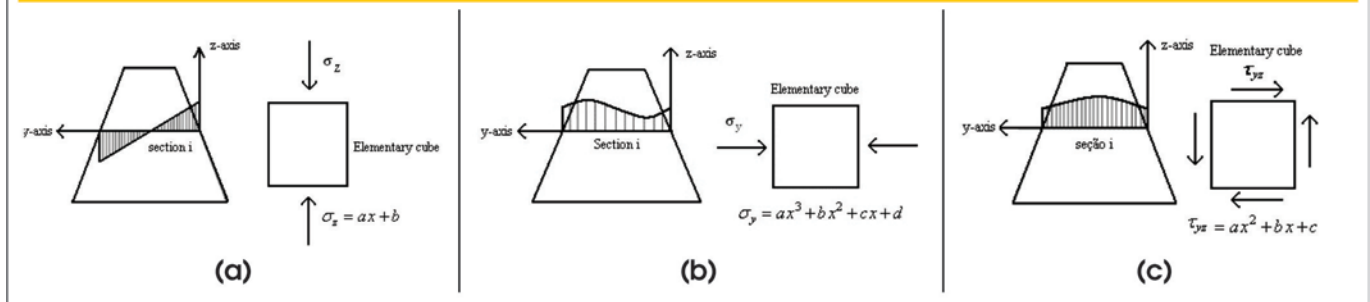


Figure 2 – Schematic representation of the distribution of normal and shear stresses in a given dam section “i”. (a): Variation of normal stress  $\sigma_z$ . (b): Variation of normal stress  $\sigma_y$ . (c): Variation of shear stress  $\tau_{yz}$



intends to give greater importance to these demands, this hypothesis may not be sufficient.

Combining hydrostatic pressure with the horizontal silt component through Rankine's formula, there is a good approximation of the effects of these two demands. These formulations consider three possible situations for silt behavior (Rankine, [13]). In this way, in addition to when silt behaves like a fluid, there is the active form which happens when the ground pushes the face of the dam; the passive form when the dam's face pushes the ground; and the static form in which both (silt/dam) are at rest.

Figure [1] shows a scheme with the distribution of usual active load pressures in a typical gravity dam. The results of these loads are denominated *Peso<sub>h</sub>* (*weight<sub>h</sub>*), hydrostatic weight in downstream face; *Pres<sub>h</sub>*, hydrostatic pressure in upstream and downstream; *Sub*, uplift pressure, *Sed*, pressure due to silt and *PP*, dead weight of the dam.

Besides the aforementioned usual static demands, dynamic loads arising from seismic activities should also be considered. The acceleration of the earth during an earthquake produces additional loads on the structure which must be computed into projects for new structures and in the analysis of already existing structures.

Among the methods for calculating usual and dynamic loads in this work, the Gravity Method for calculating stress in the static case and the Pseudo-static and Pseudo-Dynamic methods for calculating additional demands coming from seismic phenomena are presented here.

### 3. Gravity Method basic considerations

The Gravity Method for analyzing stress and stability is utilized for preliminary concrete gravity dam studies, depending on the phase of the project and on the information necessary. The Gravity Method is also used for final projects of straight gravity dams where the contraction joints are neither locked nor grouted (USBR, [3]).

The Gravity Method provides a means for the approximate determination of stress in cross sections of concrete gravity dams (USBR, [3]). According to this method, an analysis of the dam's body is made with representative sections of one meter in thickness, thus reducing the complexity of a 3D problem to a 2D one. Then, to establish the relationship among the demands acting on the body of the dam and the stresses developed in the dam a unitary width cross section is analyzed considering the Beam Theory.

According to (USBR, [3]) and (USACE, [4]), the structural model that assimilates itself most to the problem in question is that of the cantilever beam. As such, with the Gravity Method, the dam is considered to be like a beam perfectly embedded in the foundation. This approach favors safety, but there may be inaccuracies in the analysis of the regions near the foundation, as shown by (Ribeiro, [5]).

Stresses acting on a representative section of a dam cross section are, as cited in (USBR, [3]): vertical normal stress ( $\sigma_z$ ), that has linear distribution throughout the horizontal section of the cross section; shear stress ( $\tau_{yz}$ ), that has parabolic distribution and horizontal normal stress ( $\sigma_y$ ) which has a third degree polynomial distribution throughout the section. Figure [2] illustrates the development of these stresses in a given section of the dam's body.

#### 3.1 Stress Equations According to the Gravity Method

From the resulting forces and moments in the section,  $\sigma_z$ ,  $\sigma_y$  and  $\tau_{yz}$  stresses can be calculated. According to the (USBR, [3]), the equations for calculating these stresses are presented below:

$$\sigma_z(y) = \left( \frac{\sum W}{B} - \frac{6 \sum M}{B^2} \right) + \left( \frac{12 \sum M}{B^3} \right) y \quad (1)$$

$$\tau_{yz}(y) = \tau_{zYD} - \frac{1}{B} \left( \frac{6 \sum V}{B} + 2 \tau_{zYU} + 4 \tau_{zYD} \right) y + \frac{1}{B^2} \left( \frac{6 \sum V}{B} + 3 \tau_{zYD} + 3 \tau_{zYU} \right) y^2 \quad (2)$$

in which:

$$\tau_{zYD} = (\sigma_{zD} - p') \tan \phi_D \quad (3)$$

$$\tau_{ZYU} = -(\sigma_{ZU} - p)\tan\phi_U \quad (4)$$

in which,

$\sigma_Z$  - Vertical normal stress in an analyzed section;

$\sigma_{ZU}$  - Normal stress at dam's upstream face

$\sigma_{ZD}$  - Normal stress at dam's downstream face;

$\tau_{ZY}$  - Shear stress in an analyzed section;

$\tau_{ZYU}$  - Shear stress at dam's upstream face;

$\tau_{ZYD}$  - Shear stress at dam's downstream face;

$y$  - Point coordinates over an analyzed section;

$\phi_U$  - Angle of inclination of upstream face;

$\phi_D$  - Angle of inclination of downstream face;

$\sum V$  - Sum of all horizontal forces;

$\sum W$  - Sum of all vertical forces;

$\sum M$  - Sum of all moments;

$B$  - width of the section considered or joint studied;

$p$  - product between height of the column of water above the section analyzed, upstream, and the specific weight of the water;

$p'$  - product between height of the column of water above the section analyzed, downstream, and the specific weight of the water;

Normal stress to the vertical plane is given by:

$$\sigma_y = (a_1 \tan\phi_D + p') + \left( b_1 \tan\phi_D + \frac{\partial a_1}{\partial z} \right) y + \left( c_1 \tan\phi_D + \frac{1}{2} \frac{\partial b_1}{\partial z} \right) y^2 + \left( \frac{1}{3} \frac{\partial c_1}{\partial z} \right) y^3 \quad (5)$$

in which,

$$a_1 = \tau_{ZYD} \quad (6)$$

$$b_1 = -\frac{1}{B} \left( \frac{6\sum V}{B} + 2\tau_{ZYU} + 4\tau_{ZYD} \right) \quad (7)$$

$$c_1 = \frac{1}{B^2} \left( \frac{6\sum V}{B} + 3\tau_{ZYD} + 3\tau_{ZYU} \right) \quad (8)$$

The terms for each of these coefficients have already been defined. A more detailed analysis of the above equations may be verified in (USBR, [3]).

#### 4. The Pseudo-Static Method

The pseudo-static method basically consists of adding loads due to earthquakes. In this method, the dam is considered a rigid body accelerated to a certain fraction of gravity and water is considered incompressible. Calculating increases in hydrodynamic pressure was proposed in 1931 by (Westergaard, [6]). He deals with the fluid-structure interaction problem as a bi-dimensional interaction between the reservoir and the dam, submitted to horizontal movement of earth, where the foundation of the dam is considered non-deformable.

Seismic analysis begins with calculating the inertial forces (due to dead weight) induced by an earthquake and considered as static loads applied to the dam. The inertial forces are calculated using ground acceleration and are applied at the dam's center of gravity. When verifying the stability of a dam, the horizontal and vertical acceleration peaks are considered simultaneously.

Pseudo-static seismic analysis is completed when the hydrodynamic force of the reservoir and the silt, when considered fluid, are calculated. A peculiarity in this method is that this dynamic force of nature is thought of as static. Once calculated, it is considered that these forces act in a static manner along with other forces already determined. This means that to calculate the stresses using Equations (1), (2) and (5), it would be considered the sum of forces and moments.

Proceeding from numeric methods, the following expression for hydrodynamic pressure is proposed below:

$$p(h) = \frac{0.543}{0.583} \frac{7}{8} a_h \sqrt{Hh} \quad (9)$$

where:

$a_h$  = peak horizontal acceleration (m/s<sup>2</sup>);

$H$  = total height of dam (m);

$h$  = difference between water elevation and the analyzed section (m).

The hydrodynamic force is obtained through the integration of  $p(h)$  over  $h$ :

$$F_{Hd}(h) = 5.33 \frac{a_h}{g} \sqrt{Hh}^{1.5} \quad (10)$$

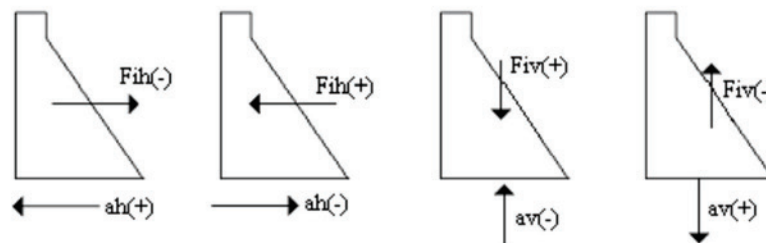
where  $g$  is the acceleration of gravity.

The hydrodynamic force is applied to 0,4h of the surface analyzed and its direction is opposite to ground acceleration.

To calculate inertial forces, it is assumed that accelerations, velocities and deformations are the same at all points of the dam at every instant. With that, the horizontal and vertical forces of inertia, per dam meter, are calculated as being the weight of the dam cross section times the earthquake acceleration in the direction analyzed, as shown in the equations below:

$$\begin{aligned} F_{IV} &= -V\gamma_c a_v & (a) \\ F_{IH} &= -V\gamma_c a_h & (b) \end{aligned} \quad (11)$$

Figure 3 – Sign convention for ground accelerations and horizontal and vertical inertial forces



where,

$V$  – Volume of the dam cross section studied with a depth of 1m (m<sup>3</sup>);

$\gamma_c$  - Specific weight of concrete (kN/m<sup>3</sup>);

$a_v$  - Vertical acceleration of the dam in relation to the acceleration of gravity;

$a_h$  - Horizontal acceleration of the dam in relation to the acceleration of gravity.

The negative sign indicates that they are applied in the direction opposite of acceleration. The sign convention may be observed in Figure [3].

Since the Pseudo-static Method does not consider the oscillatory nature of the forces, what is done when calculating equilibrium is that a combination of usual forces (dead weight, hydrostatic force, uplift pressure, among others) with forces provoked by the earthquake (hydrodynamic and inertial forces). The weight of the horizontal seismic forces is 1.0 and the weight of the vertical seismic forces is 0.3. The combinations and coefficients appear in Table [1].

## 5. The Pseudo-Dynamic method

The Pseudo-Dynamic Method is an analytical procedure developed by (Chopra, [8]) as a hand calculation alternative to the more general procedures that require using a computer. It is a simplified analysis of the response spectrum which determines that response of the structure to the fundamental mode of vibration for horizontal ground movement (FERC, [7]).

This methodology considers the dam to be flexible and the reservoir water as a compressible fluid. Besides this, both the inertial forces and the hydrodynamic ones become dependent on the

structure's fundamental mode of vibration.

The additional seismic load can be determined in the following manner (Chopra, [8]):

Compute  $T_s$ , the dam's fundamental period of vibration, in seconds, without considering the influence of the reservoir, from Equation:

$$T_s = 1.4 \frac{H_s}{\sqrt{E}} \quad (12)$$

where:

$H_s$  - height of dam in feet (ft);

$E$  - modulus of elasticity of concrete in pounds per square inch (psi).

According to (Chopra, [8]), Equation (12) is an approximate expression which allows for determining the dam's fundamental period of vibration – without the influence of the reservoir – for the preliminary design criteria.

Compute  $\tilde{T}$ , the fundamental vibration period of the dam, in seconds, including the influence of the reservoir, from Equation:

$$\tilde{T} = R_1 T_s \quad (13)$$

Table 1 – Combination of static and dynamic loads

| Type of Combination | Static Forces<br>(W + Hs + U + Sed) | Horizontal Seismic Forces<br>(Fih + Fhd) | Vertical Seismic Forces<br>(Fiv) |
|---------------------|-------------------------------------|--|----------------------------------|
| Combination I       | 1.0                                 | 0  | 0                                |
| Combination II      | 1.0                                 | 1.0 (-)                                  | 0.3 (-)                          |
| Combination III     | 1.0                                 | 1.0 (+)                                  | 0.3 (-)                          |
| Combination VI      | 1.0                                 | 1.0 (-)                                  | 0.3 (+)                          |
| Combination V       | 1.0                                 | 1.0 (+)                                  | 0.3 (+)                          |

The parameter  $R_1$  concerns the values of the dam's fundamental periods of vibration considering the reservoir full and empty. This parameter is obtained graphically taking into consideration common values for the modulus of elasticity of concrete found in dams. To determine it, enter on the x-coordinate the value of the ratio between the heights of the reservoir and the dam, intercepts the curve with the modulus of elasticity correspondent to the concrete applied and then, read the  $R_1$  value on the y-axis. The values of the modulus of elasticity in the graph referred to are given in psi (Chopra, [8]).

Compute  $f_s(y)$ , which is the seismic load distributed throughout the height of the dam and takes into account additional inertial forces and hydrodynamic effects. This force can be calculated from the equation below:

$$f_s(y) = \alpha_1 \frac{S_a T'_s}{g} \left( w_s(y) \psi(y) + g \tilde{p}_1(y) \right) \tag{14}$$

in which,

$y$  – position in relation to the dam's base;

$\alpha_1$  - constant term with a value of 4;

$\frac{S_a \cdot T'_s}{g}$  - spectral acceleration of the fundamental period of the dam;

$w_s(y)$  - dam weight per unit of height;

$\psi(y)$  - fundamental deformation of the structure, obtained through an equation that will be shown further on.

$g \cdot \tilde{p}_1(y)$  - value obtained graphically which should be multiplied by the square of the ratio between the height of the water ( $y$ ) and the height of the reservoir ( $H$ ). In Equation (14) this quantity depends on the ratio between the height of the water ( $y$ ) in the reservoir and its total height ( $H$ ). This value also depends on the fundamental resonance frequency expressed by several curves (Chopra, [8]), according to Equation (15). Thus, enter the value of the  $y/H$  ratio intercepting the curve corresponding to  $R_2$  and then read  $g \cdot \tilde{p}_1(y)$  which denotes variation of the pressure due to the reservoir's depth.

$$R_2 = 1/\sqrt{T} (4H/C) \tag{15}$$

where:

$C$  - speed of sound in water. The value of  $R_2$  varies from 0,5 to 0,99.

The Equation that represents a function for the fundamental deformation of the structure is presented in the following:

$$\psi(y) = 1,2877 \left(\frac{y}{H}\right)^4 - 1,2551 \left(\frac{y}{H}\right)^3 + 0,7914 \left(\frac{y}{H}\right)^2 + 0,1718 \left(\frac{y}{H}\right) \tag{16}$$

Equation (16) was adapted by (Ribeiro, [5]) from the deformation proposed by (Chopra, [8]) through the method of least squares.

## 6. Description of the *Safebarr* program developed in MATLAB™

Bearing in mind the importance of studying dams and the analysis and computing methods of the stresses previously presented, the aim now is to comment on the *Safebarr* program developed in MATLAB™. This program allows computing the strains and stresses in a dam's cross section. The program was developed to cover more general cross sections of dams in terms of geometry which are subject to several load combinations (static and dynamic), besides silts in one of its four possible forms of action (Rankine, [13]). The *Safebarr* Program allows determining the  $\sigma_z$ ,  $\sigma_y$ ,  $\tau_{yz}$  stresses; the main stresses ( $\sigma_1$  and  $\sigma_2$ ) and the resulting position through one or more of the joints of the dam's cross section. Such stresses may be calculated for the static case (dead weight, hydrostatic pressures and weights, silts and uplift pressure) and for the dynamic case, in which are included earthquake actions, statically, through the Pseudo-Static or Pseudo-Dynamic method. The *Safebarr* program also permits the simultaneous analysis of several cross sections through variation in geometric parameters and the property of materials in the cross section base initially considered. This resource gives the user the possibility of carrying out comparative analyses.

For this purpose, next, the methodology utilized for creating its code is stated. Following this, output data and results generated from the analysis of some dam cross sections subject to several load combinations are presented.

### 6.1 Program and design code

In this topic, the *Safebarr* program code is discussed in order to illustrate its algorithms and operation.

The *Safebarr* program code is composed of subroutines that develop scripts and functions. By subroutines understand a sequence of commands in MATLAB™ that are executed and can generate output data without needing to provide input data. The functions are command sequences whose input and output data are previously determined and interdependent (Gilat, [9]). In this way, in one function, the command sequence is only executed in the event that the input parameters are previously provided.

The subroutines and functions that make up the *Safebarr* code are presented in Table [2], along with a brief description of each of them.

The organization and hierarchy among the functions and routines, broken down in Table [2], are shown in the flowchart in Figure [4]. Figure [4] shows the *Safebarr* subroutine as a top level routine, meaning that it is hierarchically superior when running the program. In this routine, specific commands prompt the user for input data. This data is duly separated and sent to the storage in the *variaveis (variables)* routine.

It is also in the *Safebarr* routine that the *calculo\_tensoes (calculate\_stresses)* function is prompted. In turn, this prompts the other secondary functions in reference to calculating the resultant forces acting in the cross section analyzed. With the result of each secondary function, the *calculo\_tensoes (calculate\_stresses)* function calculates the results of moments and then the stresses in several

joints in a single cross section given. The results of this step are taken to the *Safebarr* subroutine where they are organized to be made available at a later point for the user to confer.

As already commented upon, the program permits creating gen-

eral cross sections which allows analyzing simple cross sections, trapezoids and ever more complex cross sections. Figure [5] shows a complex cross section which may be analyzed by the program. It should be remembered that this geometry is provided

**Table 2 – Functions and subroutines of the *Safebarr* program**

|             | Name                                      | Function in the Program  |
|-------------|---|--|
| SUBROUTINES | variaveis<br>(variables)                  | Stores the variables that are used in executing the program functions.   |
|             | Safebarr                                  | Program prompt, collects data provided by the user, prompts some functions and brings together the results.  |
| FUNCTIONS   | FHid_horiz<br>(HFid_horiz)                | Computes the resultant of hydrostatic pressures exerted by upstream and downstream water, as well as their respective application points.  |
|             | peso_proprio<br>(dead_weight)             | Computes the cross sections dead weight and the coordinates for the section's center of gravity.   |
|             | Wagua<br>(W_water)                        | Computes the resultant of the weight exerted by the water in the upstream and downstream faces, as well as their respective points of application.   |
|             | subpressão<br>(uplift pressure)           | Computes the resultant uplift pressure and its point of application on the base of the dam.  |
|             | sedimentos<br>(silts)                     | Provides the horizontal resultant force from the pressures exerted by the silt on the face of the structure. This resultant force may be computed in the four forms according to (Rankine, (13)).                        |
|             | Resvert_sed<br>(Resvert_sil)              | Provides the resultant vertical force of the weight exerted by the sediments on the upstream and downstream faces.   |
|             | sismo<br>(seismic)                        | Computes the resultant forces of additional seismic forces and their respective points of application in accordance with the Pseudo-Static or Pseudo-Dynamic Method.   |
|             | Calculo_tensoes<br>(Calculate_stresses)   | Carries out the computation for the resultant vertical and horizontal forces acting on the cross section, as well as the moments and the stresses. This function also provides the design of the cross section analyzed. |
|             | resist_compressao<br>(resist_compression) | Computes the safety factor against crushing by concrete compression.   |
|             | resist_tracao<br>(resist_tension)         | Computes the safety factor against excessive cracking in the concrete.   |
|             | Desliz_bar<br>(Sliding_dam)               | Carries out the computation for the safety factor against dam sliding.   |



Figure 4 - Representative flowchart of the design of the program

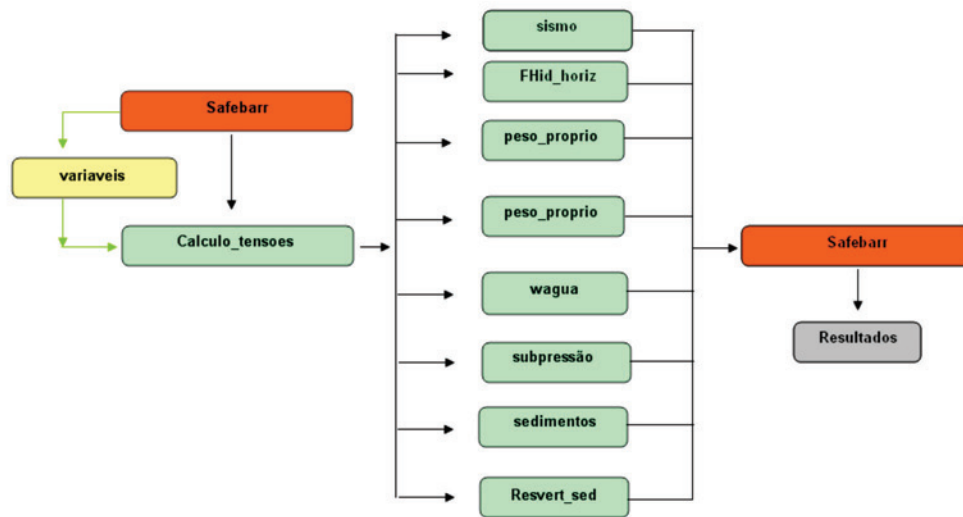


Figure 5 - Possible general cross section to be analyzed by the Safebarr program

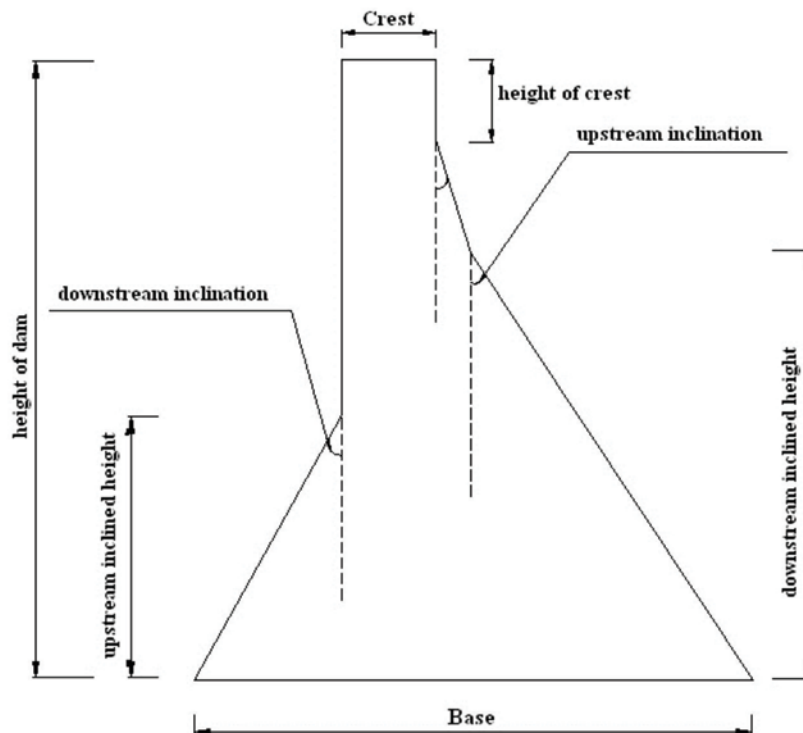


Figure 6 – Data entry window regarding cross section geometry

by the user. Figure [6] shows the data input window of the cross section's geometry.

## 6.2 Basic features of the *Safebarr* routine

*Safebarr* is a managing subroutine and program prompt. As such, there are many features concerning its code that are worth highlighting.

The first commands in this subroutine concern requesting input data from the user. In this step, all of the cross section's geometric parameters to be analyzed are requested as well as the properties of the material and the kind of analysis to be carried out: static or dynamic (Pseudo-Static or Pseudo-Dynamic Method). All of these parameters are requested through windows that provide interactivity with the user, as shown in Figure [6].

As stated previously, the *Safebarr* program makes it possible for the user to simultaneously analyze multiple cross sections by varying some geometric and operational parameters, as well as the property of materials. This is an important tool since it makes it possible to verify the impact of some parameters in the stress field acting on the cross section of the dam, meaning a parametric study.

Among the geometric and material properties, the program enables varying, for example, the width of the cross section base, the cross section height and the width of the crest in addition to be

able to also vary the level of the upstream column of water and the cohesion of the concrete.

Varying the width of the base can be done by adjusting for a new length between upstream and downstream or making all adjustments downstream. The first option allows the user to choose the proportion at which an increase to the base width will be distributed between upstream and downstream. In the second option, all of the increase takes place downstream of the cross section, which is the most common and recurrent way in the practice of dam design. It must be remembered that altering the width of the base also changes the inclination of the cross section. Note that, in general, the inclination of the upstream face is neither very big nor very variable among designs (tolerance of between 5° and 7° small angles). Therefore, it is preferable and more efficient to modify the upstream inclination, this being the reason for not having the option in the program to make all of the increase at the downstream base of the cross section.

Varying the height of the cross section of a dam is also a parameter sometimes necessary when carrying out a project. This is due to prevailing conditions of the terrain as well as construction features. In the program, this variation may be made in two distinct ways, which are: the height of the cross section may vary with or without altering the inclined heights of the faces. In the case that altering the inclined heights when varying the height has been chosen, choosing with what proportion the inclined height will vary in relation to the total increase given is still possible.

The level of the upstream column of water and the cohesion of the concrete may vary, which is a condition consistent with natural changes during the useful life of a dam. The level of the water varies between maximum and minimum levels depending on the region's rain. Moreover, the cohesion between the base of the dam and earth tends to diminish given that it is a parameter related to roughness in the concrete, which over time also loses its geometry considerably.

It must be noted that the user may choose to do all of the possible variations or only some of them.

## 6.3 Output data of the program

The *Safebarr* subroutine in addition to managing all of the analyses requested also has the function of being able to collect and organize all of the results so that they may be presented to the user. To this end, all of the results generated are recorded and saved in a Word document entitled results which can be opened for verification after using the program. In this document there is a record of the value of each force acting on the cross section, the resultant of force and moments, the normal and shear stresses, and the principle stresses for each joint analyzed. Figure [7] illustrates an example of the Word document generated.

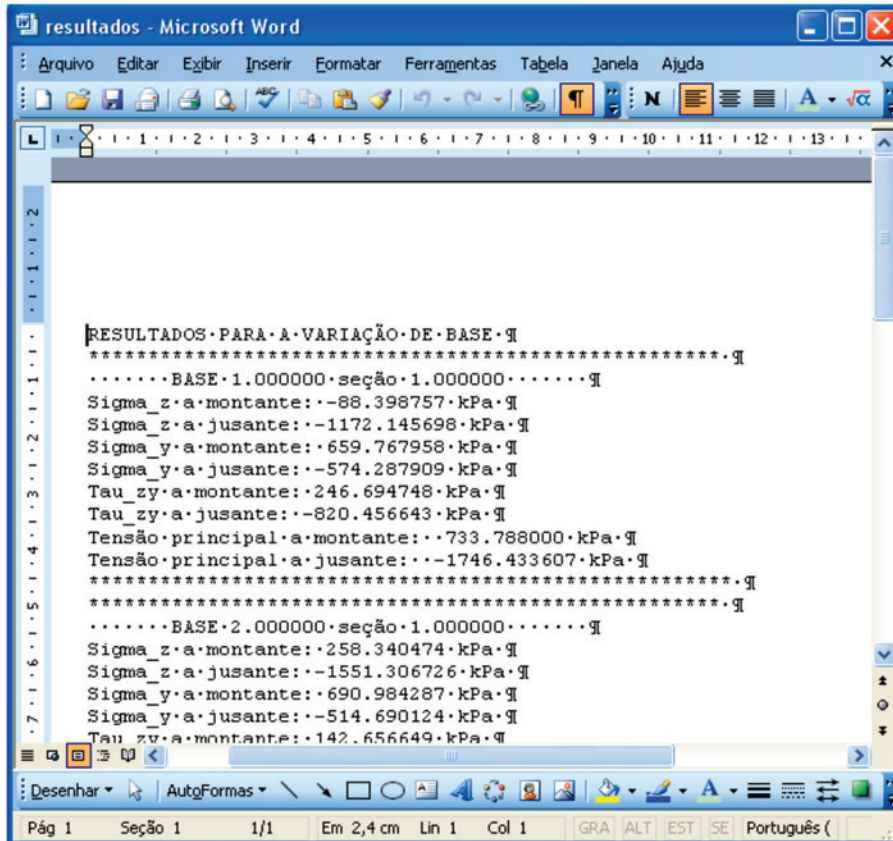
Besides the results recorded in the document mentioned, the program generates graphs, for the user, of each stresses' ( $\sigma_Z$ ,  $\sigma_Y$ ,  $\tau_{YZ}$ ) development for each joint analyzed in the cross section.

## 7. Validation of results

To validate the results, the same analysis is made utilizing two resources: the *Safebarr* program and the (CADAM, [12]) version 1.0.1 program.

A static and dynamic analysis with the Pseudo-Static method will be

Figure 7 – Word document window generated at the end of the analysis



made for a single cross section with the following geometric properties: height = 75m; base = 55m; crest = 15m; upstream and downstream inclined height respectively equal to, 20 and 65m and the upstream and downstream inclination respectively equal to 0.099 and 0,53 radians.

7.1 Static Analysis Validation

In the first analysis, a static study is made with the following configuration of requests: upstream and downstream water height respectively equal to 70m and 15m and the height of a column of silts equal to 25m. We adopted for the specific weight of the water, 9.81kN/m³, for the specific weight of the silt, 11kN/m³ and the specific weight of the concrete mass equal to 24kN/m³. For purposes of comparison, Table [3] shows the results found by the two programs for two joints in the cross section with one located on the base and the other 27 meters from the base. From Table [3], it can be seen that all of the values found from the *Safebarr* program present satisfactory compatibility with the results obtained from (CADAM, [12]), except for the resultant vertical force of silt and the  $\tau_{ZYD}$  and  $\tau_{ZYU}$  stresses. In these cases, the results found from (CADAM, [12]) were considerable greater than those arrived at by the *Safebarr* program.

To verify this incongruity from the results obtained, a step-by-step calculation was developed for the silt and stress  $\tau_{ZYD}$  results.

Resultant vertical force of silt (Section 1 – base)

The resultant vertical force of silt is calculated as being the weight of the column of silt above the upstream face, which is inclined. To do this, calculate the volume of the silt above the face and multiply this value by the specific weight of the silt. The volume to be calculated is represented in Figure [9].

With a unitary depth, the problem becomes calculating the area of the figure, which is made up of a triangle and a 2m rectangle. The total area is then:

$$Area_{sed} = \frac{2 \times 20}{2} + 2 \times 5 \Rightarrow Vol_{sed} = 30m^3 \quad (17)$$

The specific weight of the silt is 11kN/m, so then the weight of the silt above the face is 330 kN.

Stress  $\tau_{ZYD}$

According to the (USBR, [3]) stress  $\tau_{ZYD}$  is calculated by Equation (3). Thus we have:

$$\tau_{ZYD} = ((176.484 - 15 \cdot 9.81) \cdot 0.58461) \Rightarrow \tau_{ZYD} = 687.79 - 86.0253 = 601.76kPa \quad (18)$$

From the above calculation, it can be seen that the result presented by (CADAM, [12]) corresponds to only the portion ( $\sigma_{ZD} \tan \phi_D$ ), meaning that it does not take into account the portion with the water column pressure in the section. A similar problem may be observed in the calculation of  $\tau_{ZYU}$ . As for the stresses  $\sigma_Y$ , they were indirectly validated from the main stresses calculation presented by (CADAM, [12]).

## 7.2 Dynamic analysis validation with the Pseudo-Static Method

In the following, there is a dynamic analysis of the same cross section, according to the Pseudo-Static method (Figure [8]). The accelerations and the seismic coefficients adopted in this analysis are presented in Table [4].

Table [5] shows the results obtained through the (CADAM, [12]) and the *Safebarr* programs for the same two joints considered in item 7.1. From this table observe the great coherency among the results obtained through the (CADAM, [12]) and the *Safebarr* pro-

grams, except for the shear stress upstream (values highlighted in Table [5]). This case however, falls into the same situation as in item 7.1.

## 7.3 Dynamic analysis validation with the Pseudo-Dynamic Method

For dynamic analysis with the Pseudo-Dynamic method, the results obtained by the *Safebarr* program are compared to the results available in the work of (Chopra and Fennes, [10]). For this purpose, a new cross section will be studied with different geometric properties and load conditions.

A cross section of the Pine Flat Dam is studied here whose geometry is illustrated in Figure [10].

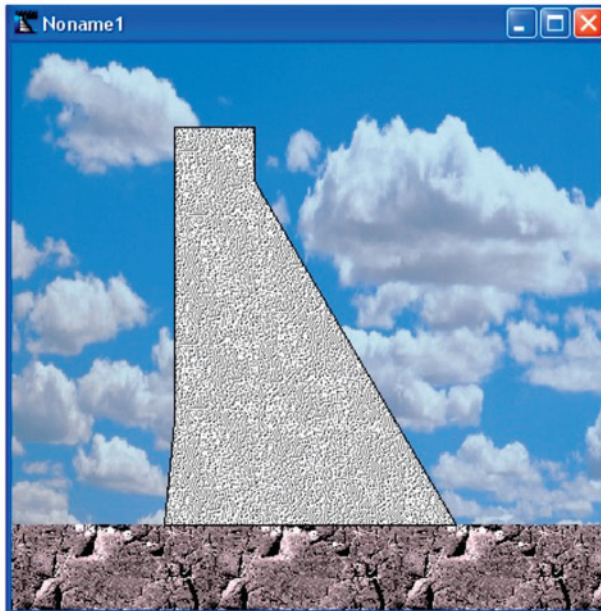
The following material properties and operational conditions were considered:

- Specific weight of water - 0,062b
- Specific weight of concrete – 0,155b
- Acceleration of gravity – 32.2 ft/sec<sup>2</sup>

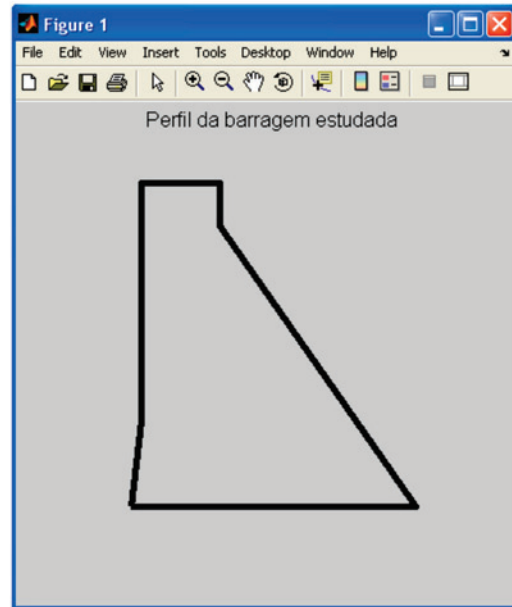
**Table 3 – Static analysis results done in CADAM™ and in the program *Calculo\_tensoes (Calculate\_stresses)***

| Source of results/ Resultant            | CADAM<br>Joint 1 (base) | CADAM<br>Joint 2 (27m) | Calculo_tensoes<br>Joint 1 (base) | Calculo_tensoes<br>Joint 2 (27m) |
|---|-------------------------|------------------------|-----------------------------------|----------------------------------|
| Self weigh<br>(kN)                      | -57120                  | -27410.2154            | -57120                            | -27410.2154                      |
| Upstream Hydrostatic<br>Pressure (kN)   | 24034.5                 | 9069.345               | 24034.5                           | 9069.345                         |
| Downstream Hydrostatic<br>Pressure (kN) | -1103.625               | 0.00                   | -1103.625                         | 0.00                             |
| Upstream Hydrostatic<br>weight (kN)     | -1177.2                 | 0.00                   | -1177.2                           | 0.00                             |
| Downstream Hydrostatic<br>weight (kN)   | -645.2                  | 0.00                   | 645.2                             | 0.00                             |
| Silts<br>(horizontal) (kN)              | 3437.5                  | 0.00                   | 3437.5                            | 0.00                             |
| Silts<br>(Vertical) (kN)                | -412.5                  | 0.00                   | -330                              | 0.00                             |
| Uplift pressure<br>(kN)                 | 22930.875               | 7849.283               | 22930.875                         | 7849.2828                        |
| $\sigma_{ZD}$<br>(kN/m <sup>2</sup> )   | -1176.484               | -634.818               | -1178.5632                        | -634.8177                        |
| $\sigma_{ZU}$<br>(kN/m <sup>2</sup> )   | -148.026                | -416.411               | -142.9467                         | -416.4106                        |
| $\tau_{ZYD}$<br>(kN/m <sup>2</sup> )    | 687.791                 | 371.124                | 602.98                            | 371.1242                         |
| $\tau_{ZYU}$<br>(kN/m <sup>2</sup> )    | 14.803                  | 0.00                   | 54.3753                           | 0.5419                           |
| $\sigma_{YD}$<br>(kN/m <sup>2</sup> )   | —                       | —                      | 2452.5884                         | 830.2646                         |
| $\sigma_{YU}$<br>(kN/m <sup>2</sup> )   | —                       | —                      | 24002.7113                        | 9069.0282                        |

Figure 8: (a) Design of the cross section generated by CADAM™  
(b) Design of the cross section generated by *Safebarr*



(a)



(b)

- Modulus of elasticity of concrete –  $3.25 \cdot 10^6$  psi
- Speed of sound in water – 4720 fps
- R1 – 1.213
- Reservoir height upstream – 381ft
- Reservoir height downstream – 0 ft
- Spectral acceleration – 0.327g

It is worth noting in this analysis that the uplift pressure was taken into account, but not the presence of silts.

The cross section shown in Figure [10] was divided into 10 sections (joints) equally spaced for which were calculated all of the parameters necessary for computing the resulting seismic force. Tables [6], [7], and [8] show the results obtained for each joint from the *Safebarr* program in addition to those available in the work of (Chopra and Fenves, [10]). It may be observed from Table [7] that the  $gp/\omega H$  portion was compared with values obtained through graphs proposed by (Chopra, [8]) and by analytical methods proposed by (Silva, [11]). It may be concluded that the results from both are valid for application in this dam cross section. Looking at Tables [6], [7], and [8] also permits the conclusion that the parameters arrived at by the *Safebarr* program for calculating the result of seismic activity are satisfactory considering the comparisons made. Once again, this shows that the results provided by the program make it valid for use, given that the analyses revealed themselves to be efficient.

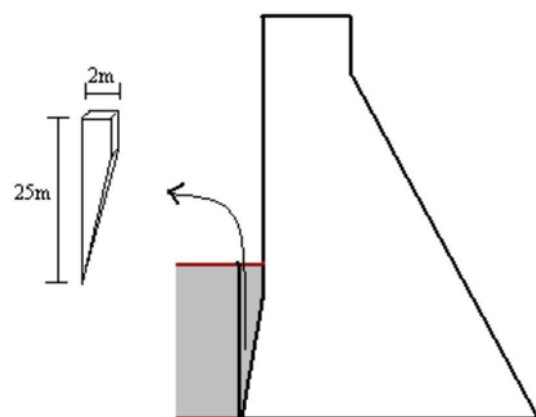
Figures [11], [12], [13] and [14], show some graphs generated by the program during the Pseudo-Dynamic analysis and the results of the normal, shear and principal stresses in six joints of the cross section considered. Figure [13] gives the variation in the dam's inertia in kN/m due to the positioning of the section/joint considered in the dam. This value is obtained by the product of the dam's

weight/m multiplied by the deformation (Figure 12). Table [9] reveals these values.

## 8. Additional analyses from Pine Flat Dam

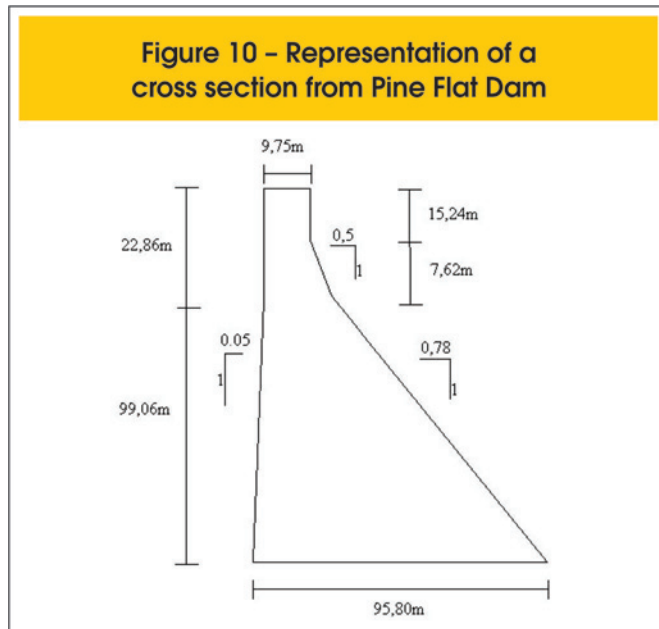
In the following, an analysis of the cross section of Pine Flat Dam

Figure 9 – Representation of the volume of sediment above the upstream face in the cross section analyzed



is presented, Figure [10], having the same material properties and load configurations utilized in the Pseudo-Dynamic analysis reported in item 7.3.

In this analysis, variation of the base is made in order to get three



**Table 4 – Acceleration and coefficient values adopted for computing stresses**

| Variables          | Values     |       |
|--------------------|------------|-------|
| Peak Accelerations | Horizontal | 0.10g |
|                    | Vertical   | 0.07g |
| Coefficients       | Horizontal | -1    |
|                    | Vertical   | -0.3  |

base-height relationships: 0.7, 0.8 and 0.9. For each base value, three joints were analyzed which are the base and sections at 30 and 90 meters.

The analysis was repeated for four types of silt behavior: fluid, active, passive, and at rest (Rankine, [13]).

It must also be noted that the specific weight of the silt considered is equal to 11 kN/m<sup>3</sup> and its angle of internal friction is 20°. In Table [10] the main stresses upstream and downstream are presented for each behavior of the silt and cross section considered.

## 9. Conclusions

Developing this work constitutes a contribution to the study of concrete gravity dams as well as to deepening the evaluation of the

**Table 5 – Dynamic analysis results done in (CADAM.(12)) x Safabarr**

| Source of results                     | CADAM Section 1 (1m) | CADAM Section 2 (27m) | Safabarr Section 1 (1m) | Safabarr Section 2 (27m) |
|---------------------------------------|----------------------|-----------------------|-------------------------|--------------------------|
| Vertical inertial force (kN)          | 1199.52              | 575.61                | 1199.52                 | 575.6155                 |
| Horizontal inertial force(kN)         | 5712                 | 2741.02               | 5712                    | 2741.0215                |
| Upstream hydrodynamic pressure (kN)   | 2578.40              | 1256.67               | 2611.7                  | 1257.4158                |
| Downstream hydrodynamic pressure (kN) | 120                  | 0.00                  | 119.925                 | 0.00                     |
| Silt horizontal inertial force (kN)   | 577                  | 0.00                  | 576.9894                | 0.00                     |
| $\sigma_{zd}$ (kN/m <sup>2</sup> )    | -1694.481            | -962.204              | -1696.5982              | -962.2601                |
| $\sigma_{zu}$ (kN/m <sup>2</sup> )    | 417.807              | -58.090               | 418.7072                | -58.034                  |
| $\tau_{zvd}$ (kN/m <sup>2</sup> )     | 973.181              | 562.519               | 905.8313                | 562.552                  |
| $\tau_{zvu}$ (kN/m <sup>2</sup> )     | 110.5                | 36.4                  | 110.5407                | 36.3796                  |
| $\sigma_{vd}$ (kN/m <sup>2</sup> )    | —                    | —                     | 3130.1153               | 1258.5194                |
| $\sigma_{vu}$ (kN/m <sup>2</sup> )    | —                    | —                     | 11611.0586              | 2063.1528                |

Table 6 – Parameter results for calculating seismic force

| Level | Elevation (ft) | Length of section (ft) |                 | Weight/ unit of height.        |                                |
|-------|----------------|------------------------|-----------------|--------------------------------|--------------------------------|
|       |                | <i>Safebarr</i>        | Chopra & Fenves | <i>Safebarr</i> ( $\omega_s$ ) | Chopra & Fenves ( $\omega_s$ ) |
| Top   | 400            | 32                     | 32              | 4.96                           | 4.96                           |
| 1     | 360            | 32                     | 33.4            | 5.18                           | 5.18                           |
| 2     | 320            | 48.69                  | 52.8            | 8.19                           | 8.19                           |
| 3     | 280            | 81.9                   | 81.9            | 12.7                           | 12.7                           |
| 4     | 240            | 115.1                  | 115.1           | 17.8                           | 17.8                           |
| 5     | 200            | 148.3                  | 148.3           | 23                             | 23                             |
| 6     | 160            | 181.5                  | 181.5           | 28.1                           | 28.1                           |
| 7     | 120            | 214.7                  | 214.7           | 33.3                           | 33.3                           |
| 8     | 80             | 247.9                  | 247.9           | 38.4                           | 38.4                           |
| 9     | 40             | 281.1                  | 281.1           | 43.6                           | 43.6                           |
| 10    | 0              | 314.3                  | 314.3           | 48.7                           | 48.7                           |

Table 7 – Parameter results for calculating seismic force

| Level | $\frac{y}{H}$   |        | gp/ $\omega H$  |                 |        | gp      |
|-------|-----------------|--------|-----------------|-----------------|--------|---------|
|       | <i>Safebarr</i> | Chopra | <i>Safebarr</i> | Analytic method | Chopra |         |
| Top   | 1.05            | 1.05   | -0.035          | -0.083          | ---    | -1.9606 |
| 1     | 0.94            | 0.94   | 0.088           | 0.088           | 0.09   | 2.078   |
| 2     | 0.84            | 0.84   | 0.1562          | 0.156           | 0.155  | 3.689   |
| 3     | 0.73            | 0.73   | 0.187           | 0.187           | 0.188  | 4.426   |
| 4     | 0.63            | 0.63   | 0.203           | 0.203           | 0.205  | 4.809   |
| 5     | 0.52            | 0.52   | 0.212           | 0.212           | 0.212  | 5.005   |
| 6     | 0.42            | 0.42   | 0.215           | 0.215           | 0.215  | 5.09    |
| 7     | 0.31            | 0.31   | 0.216           | 0.216           | 0.215  | 5.102   |
| 8     | 0.21            | 0.21   | 0.214           | 0.214           | 0.214  | 5.055   |
| 9     | 0.1             | 0.1    | 0.212           | 0.212           | 0.212  | 5.007   |
| 10    | 0               | 0      | 0.21            | 0.21            | 0.208  | 4.96    |

Table 8 – Parameter results for calculating seismic forces

| Level | y/Hs            |        | Deformation ( $\Phi$ ) |        | $\omega s \times \Phi$ |        |
|-------|-----------------|--------|------------------------|--------|------------------------|--------|
|       | <i>Safebarr</i> | Chopra | <i>Safebarr</i>        | Chopra | <i>Safebarr</i>        | Chopra |
| Top   | 1               | 1      | 1                      | 1      | 4.9                    | 4.96   |
| 1     | 0.9             | 0.9    | 0.73                   | 0.73   | 3.6                    | 3.8    |
| 2     | 0.8             | 0.8    | 0.53                   | 0.53   | 4                      | 4.35   |
| 3     | 0.7             | 0.7    | 0.4                    | 0.89   | 4.9                    | 4.95   |
| 4     | 0.6             | 0.6    | 0.3                    | 0.128  | 5.06                   | 4.98   |
| 5     | 0.5             | 0.5    | 0.21                   | 0.2    | 4.76                   | 4.6    |
| 6     | 0.4             | 0.4    | 0.148                  | 0.13   | 4.16                   | 3.65   |
| 7     | 0.3             | 0.3    | 0.099                  | 0.084  | 3.3                    | 2.8    |
| 8     | 0.2             | 0.2    | 0.06                   | 0.047  | 2.23                   | 1.80   |
| 9     | 0.1             | 0.1    | 0.024                  | 0.021  | 1.044                  | 0.92   |
| 10    | 0.0             | 0.0    | 0                      | 0      | 0                      | 0      |

Figure 11 – Graph generated by the Safebarr program during dynamic analysis by the Pseudo-Dynamic Method (Weight in kN)

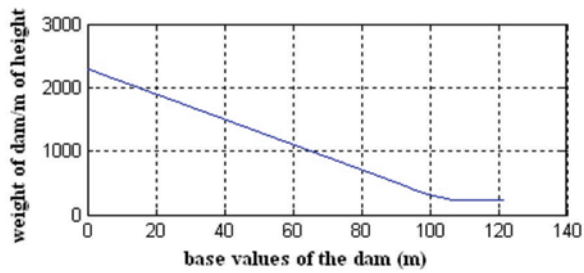


Figure 12 – Graph generated by the Safebarr program during dynamic analysis by the Pseudo-Dynamic Method

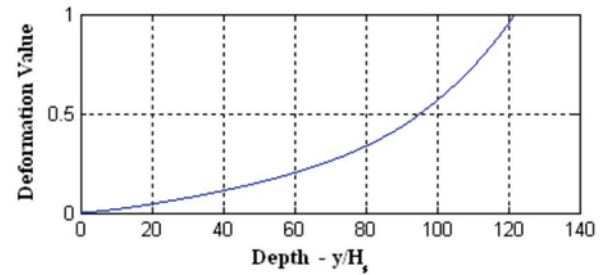


Figure 13 – Graph generated by the Safebarr program during dynamic analysis by the Pseudo-Dynamic Method

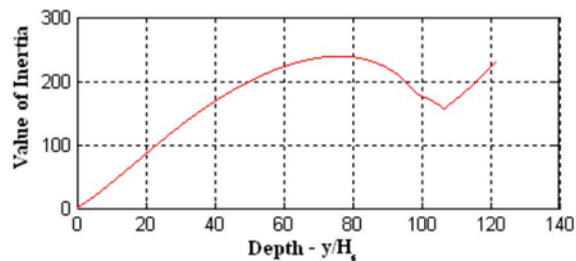


Figure 14 – Graph generated by the Safebarr program during dynamic analysis by the Pseudo-Dynamic Method

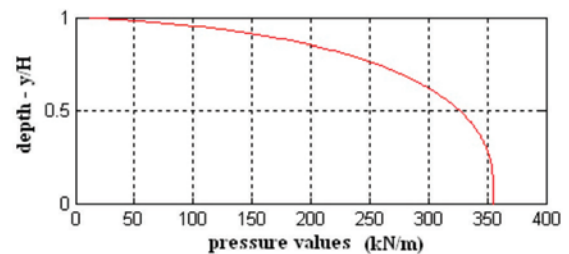


Table 9 – Shear, normal and principle stresses for sections in six heights in the cross section considered according to the *Safebarr* program

| sections   | 1<br>0.00m | 2<br>30.48m | 3<br>53.34m | 4<br>76.20m | 5<br>99.06m | 6<br>114.30m |
|--|------------|-------------|-------------|-------------|-------------|--------------|
| $\sigma_{zD}$<br>(kN/m <sup>2</sup> )                  | -21644.28  | -20862.91   | -19663.36   | -18397.34   | -20645.30   | -3760.56     |
| $\sigma_{zU}$<br>(kN/m <sup>2</sup> )                  | 19882.44   | 19529.40    | 18636.81    | 17641.73    | 19973.66    | 3412.75      |
| $\tau_{zVD}$<br>(kN/m <sup>2</sup> )j                  | -16882.54  | -16216.02   | -15226.07   | -14209.12   | -16003.32   | -2923.45     |
| $\tau_{zVU}$<br>(kN/m <sup>2</sup> )m                  | -936.06    | -929.71     | -892.85     | -852.47     | -982.74     | -168.91      |
| $\sigma_{vD}$<br>(kN/m <sup>2</sup> )                  | -13168.38  | -12648.49   | -11876.33   | -11083.11   | -12482.59   | -2280.29     |
| $\sigma_{vU}$<br>(kN/m <sup>2</sup> )                  | 1185.98    | 239.55      | -531.12     | -1012.37    | -834.58     | -671.91      |
| Upstream<br>principle stress<br>(kN/m <sup>2</sup> )   | 19929.19   | 19575.83    | 18681.40    | 17684.30    | 20022.74    | 3421.19      |
| Downstream<br>principle stress<br>(kN/m <sup>2</sup> ) | -34812.66  | -33555.91   | -31626.55   | -29590.28   | -33205.90   | -6048.48     |



field of stress in these structures when subject to diverse static and dynamic loads. To this end, a study of the main types of loads and how they act on gravity dams was carried out as well as, developing a program in the framework of MATLAB™. This made it possible to determine loads and stresses in concrete gravity dams submitted to various combinations of static and dynamic loads (earthquakes). In regards to the programming developed, the possibility for calculating and analyzing with varied cross sections from the most simple to the more complex is highlighted in addition to measuring the values of the loads and stresses in various parts of the cross section studied.

It should be underscored that varying the upstream and downstream angles, the dam height and the physical properties of the materials within the same analysis when using the *Safebarr* program is also possible. This makes it an important tool for parametric analysis.

It must also be underlined that the study of static and dynamic loads includes a review of the Gravity Method for computing stresses taking into consideration the seismic loads determined by the Pseudo-Static and Pseudo-Dynamic methods.

### 10. References

[01] ANEEL (2007). *Superintendência de Gestão e estudos Hidroenergéticos*. Relatório de acompanhamento de Estudos e Projetos de Usinas Hidrelétricas. P. 1-72.  
 [02] Stewart, L. U. e Floyd, E. D., "Proyecto de Presas Pequeñas". United States Department of the Interior, Bureau of Reclamation. Madrid, 1970.  
 [03] USBR. "Design of Gravity Dams". Denver: United

**Table 10 – Analysis with sediment in fluid active, passive and at rest forms**

**Results for Sediment in a Fluid Form**

| Original cross section | $\frac{B}{H} = 0,7$ |                  | $\frac{B}{H} = 0,8$ |                  | $\frac{B}{H} = 0,9$ |                  |
|------------------------|---------------------|------------------|---------------------|------------------|---------------------|------------------|
|                        | Upstream (kPa)      | Downstream (kPa) | Upstream (kPa)      | Downstream (kPa) | Upstream (kPa)      | Downstream (kPa) |
| Base                   | 24938.95            | -40421.53        | 20009.14            | -34963.10        | 15525.58            | -29958.65        |
| 30.00m                 | 24011.51            | -38398.14        | 19570.98            | -33567.16        | 15454.83            | -29042.32        |
| 90.00m                 | 19504.31            | -30548.52        | 17806.45            | -29624.25        | 15958.37            | -28670.59        |

**Results for Silt in an Active Form**

|        | Upstream (kPa) | Downstream (kPa) | Upstream (kPa) | Downstream (kPa) | Upstream (kPa) | Downstream (kPa) |
|--------|----------------|------------------|----------------|------------------|----------------|------------------|
| Base   | 24889.68       | -40346.93        | 19969.95       | -34900.20        | 15495.44       | -29906.57        |
| 30.00m | 24010.13       | -38396.05        | 19569.86       | -33565.37        | 15453.96       | -29040.81        |
| 90.00m | 19504.31       | -30548.52        | 17806.45       | -29624.25        | 15958.37       | -28670.59        |

**Results for Silt in a Passive Form**

|        | Upstream (kPa) | Downstream (kPa) | Upstream (kPa) | Downstream (kPa) | Montante (kPa) | Downstream (kPa) |
|--------|----------------|------------------|----------------|------------------|----------------|------------------|
| Base   | 25039.44       | -40573.64        | 20089.07       | -35091.36        | 15587.03       | -30064.85        |
| 30.00m | 24014.34       | -38402.42        | 19573.26       | -33570.82        | 15456.61       | -29045.40        |
| 90.00m | 19504.31       | -30548.52        | 17806.45       | -29624.25        | 15958.37       | -28670.59        |

**Results for Silt in an at Rest Form**

|        | Upstream (kPa) | Downstream (kPa) | Upstream (kPa) | Downstream (kPa) | Upstream (kPa) | Downstream (kPa) |
|--------|----------------|------------------|----------------|------------------|----------------|------------------|
| Base   | 24905.89       | -40371.48        | 19982.85       | -34920.90        | 15505.36       | -29923.71        |
| 30.00m | 24010.58       | -38396.74        | 19570.23       | -33565.96        | 15454.25       | -29041.31        |
| 90.00m | 19504.31       | -30548.52        | 17806.45       | -29624.25        | 15958.37       | -28670.59        |

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